Electric Circuit Analysis, KTH EI1120
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Topic 01: Basics

Each lecture/tutorial pair covers a *Topic* of the course. For each Topic there is a document (like this one) describing the subject. We used to call these documents "notes", thinking of them as rather long lecture notes; but they're more like chapters of a textbook . . . so from 2016 we call them Chapters (and sometimes we produce low-text, hand-written notes as an alternative).

At the end of a Topic's Chapter there may be an "Extra" section, going beyond the core material; these sections can be ignored with regard to the main course-content and exam, but they might be interesting if you want to think more deeply or broadly about the subject.

There is also a set of Exercises for each Topic, designed for study alone or in groups; some of these may be used in the tutorial sessions. For most of the Topics there is a homework task, whose solution will be provided soon after the submission deadline.

Together, these resources are intended to provide sufficient explanation and practice to cover the whole course content. It is expected that as well as any lectures and tutorials that you choose to attend, you will carefully read the Notes (around the time of the lecture in that Topic) and work through at least some of the Exercises (before the next lecture). Past exams and solutions are a further valuable study aid, particularly when preparing for tests.

1 Relevance of the subject

Circuit analysis is important in many areas of electrical engineering. Circuit models are used to describe and design tiny microelectronic devices, power networks that span across continents, delicate sensors, high-power converters, audio equipment, and much more.

In other branches of engineering it is also common to have some study of electric circuits as a standard part of the education: electrical sensors can be useful for making measurements, and electrical equipment is of large and growing importance in big engineering systems such as aircraft.

Similar *models* are used in other disciplines too, even if not modelling electricity: for example, models of heat-loss from a building (where temperature is analogous to electric potential, and heat-flow to current); or models of fluid flowing in pipes (pressure as potential, flow as current).

Many of us have an intuitive feeling for water circuits, that can help our judgements about electric circuits. By studying electric circuits in detail, so that we train ourselves in solution methods and theorems, we improve our ability to understand other physical phenomena that can be handled with circuit-like diagrams. However, our main purpose is to learn circuit analysis for analysing electric circuits!

2 Circuits and Models

Electromagnetic phenomena happen in space and time. The basic equations are "Maxwell's Equations", describing electric and magnetic fields. (When materials are involved, there are further equations describing the influence of material properties such as conductivity σ, permittivity ε and permeability μ. These properties all arise from the interaction of charges in materials with the fields that the materials are subjected to.)

Calculations using the concept of *fields* in space should therefore be able to give answers to the questions we might want to solve: for example, *What heat-power will be created in this piece of a loop of wire when I rotate this magnet thirty times per second this way?*, or *What relation between input and output signal will there be for this amplifier that I have built from all these resistors, capacitors, transistors and connections?*

However, these sorts of calculations easily become inconvenient or impossible, even with modern computers, unless big simplifications are made in the *modelling assumptions*. A simpler model is beneficial not just for doing the solution, but also for understanding more clearly what influences different variables will have. On the other hand, in order to be useful, a model must have enough detail to describe the modelled phenomena with acceptable accuracy.

A *circuit model* is a simplification and abstraction of a *field model*. Phenomena that might actually be field phenomena distributed over significant volumes and in nonuniform ways, are described in a circuit model by single numbers that approximate their overall effect. For example, the current through a wire may be described as one value, without needing to consider the shape and length and position of the wire, or how the current is distributed across the wire's area.

The saving of effort compared to field calculations is immense. It is hard to imagine how people would work to design complicated systems, even with computer simulations being available, without the modelling simplification that is given by discrete circuits.

3 Direct Current: Statics

In Section A of the course (Topics 01–05) we study only *direct current (dc)* (sv: *likström*). That is like the subject of *statics* in mechanics. It is suitable model for a system that has reached an equilibrium state corresponding to constant driving forces.

When starting studying dc circuits, we introduce sources and meters, and the only other components are resistors. Ideal resistors have no 'memory': for example, a resistor's voltage at time t depends only on its current at time t. That is much more simple than inductors and capacitors! For these dc circuits, the solution of a circuit at any time depends only on the values of the sources at that same time.

4 Symbolic calculation etc.

In this course we almost always represent the variables symbolically; in many questions no numeric value is given at all. We would represent a resistor as, for example, R_3 rather than as $4.7\,\mathrm{k\Omega}$.

Even when the values of components are known, we recommend writing the circuit with unique symbols for the components, then finding a symbolic solution, then putting in numbers to this solution if a numeric answer is needed. This isn't necessary for the simplest exercises, but it is generally useful for more detailed solutions.

Symbolic calculation allows us to see at all steps which components are involved, and to see more clearly how we got to each step. We can easily do dimensional analysis as a safety check. Sensitivity analysis can be used to see how each component affects the result. Values of components can easily be changed in the final numerical calculation, if we want to experiment with the solution. It is hoped that you will see some benefit of the symbolic method, after getting more experience of it!

In some situations, such as solving on paper the simultaneous equations from nodal analysis, it could be easier if the coefficients were reduced to numbers instead of big expressions of symbols. However, this is rather an artificial problem nowadays: we can get computers to do the boring work, even symbolically. Our exams do not require this type of long solution of simultaneous equations.

5 Voltage and Current

The two main *circuit quantities* are voltage and current.

The *current* is often represented by the symbol i or I, for *i*ntensity. It is the rate of flow of *charge*. Charge is the property of matter that describes its electric behaviour, akin to how mass describes the gravitational (and intertial) behaviour of matter.

A charge q passing some point in a circuit in a time t means an average current of $i = q/t$. If you are happy with derivatives, $i(t) = \frac{\mathrm{d}q(t)}{\mathrm{d}t}$ is a better description, avoiding the need to say 'average'. In the SI system, current and time are base units: amperes and seconds. Their product is a derived unit, the coulomb.

A *voltage* is defined *between* two points; it describes how much energy is lost or gained by a charge moving through the component between those points. Whether energy is gained or lost depends on the relative directions of voltage and current. Often we talk of the 'voltage across a component', meaning the difference in energy for charge moving through the component from one terminal to the other. We will use the symbol u or U for voltage.

If we define a particular point in a circuit as a reference, then we can measure voltages between any other point and this reference point. This voltage, relative to a reference, is often called a *potential*, in the same way as one talks of potentials in electrostatic and gravitational fields. Kirchhoff's voltage law (section 7.4) gives the same *conservative* condition for circuits as for those fields — i.e. that there is zero overall gain or loss in energy when going round any closed loop back to the starting point. We will use the symbol v or V for potential.[1]

So: the current through a circuit component describes the flow-rate of the charge, and the voltage across that component describes how much energy change each unit of charge has within the component. The product of voltage and current is therefore a power: it is, in SI units, "joules per coulomb" / "coulombs per second", hence "joules per second". Alternatively, we can think directly of voltage as being "watts per ampere".

An analogy with water is a rather useful one. Current is the flow rate, and voltage the pressure difference. Charge is therefore the amount of water, and potential is the pressure. This can be very helpful to some people for a more intuitive grasp of the concepts. The analogy works well also for power, resistance, voltage and current sources, and even can extend to inductors and capacitors.

5.1 Defining Quantities on Components

We define various *components* for circuit analysis. In this course, almost all[2] of these are *two-terminal* (sv: *tvåpol*) components.

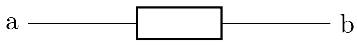
For each two-terminal component, the voltage between the terminals, and the current passing through the component, are the important variables to know. Whatever current passes in to one terminal must also pass out of the other.

Let us therefore start by how we define current and voltage. There are various conventions ('standards') for this. Sometimes that can cause confusion: *"The nice thing about standards is that they are so many of them to choose between!"* The following is what we'll be using.

[1] The symbol v or V is often also used for voltage, particularly in English texts, but we will try to be specific. It is also common to talk of a 'voltage at a point', which means the same as the 'voltage between this point and the reference', or in other words the potential!

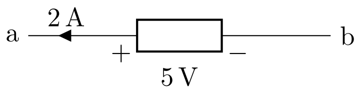
[2] The exception in this course is just the operational amplifier, whose model could have anything from three to six visible terminals. Later courses about electronics may include important nonlinear devices, such as transistors, which are commonly modelled with three or four terminals.

Consider a generic two-terminal component, shown as a box. It could represent a source, or a resistor, or a complicated circuit with two external terminals labelled 'a' and 'b'.



5.1.1 Concrete voltage and currents

Next we see how it could look when marked to show a current and voltage as concrete numbers and units.



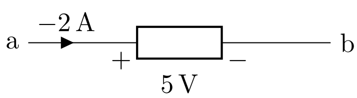
A current of 2 A is passing through the component in the direction *from* 'b' *to* 'a'; the arrow symbol near the left terminal is very clear at indicating the reference direction of the current..

There is a voltage of 5 V between ends 'a' and 'b'. The + and − symbols near the voltage text tell us that the voltage is measured as the potential at the side marked +, *relative* to the potential at the side marked −. If we define potentials v_a and v_b at the terminals 'a' and 'b', then $v_\mathrm{a} - v_\mathrm{b} = 5\,\mathrm{V}$.

In this case, notice that the direction of voltage and current are such that the current *gains* energy when going through the component. The component is supplying power *to* the circuit, as charge comes out at a higher potential than it went in at. This component therefore cannot be a resistor; a resistor's current is driven through it by the voltage, from high to low potential, so it can only consume power from a circuit. This component might be a voltage source or current source, which can both supply or consume power.

This *same* physical situation can be shown in a total of *four different ways*, depending on the reference directions chosen for current and voltage! Depending on the relative directions, we might need a positive or negative sign when calculating power. We now calculate the power in to the component from the circuit for each of these cases; remember that this was a negative power. This should give the same result in each case, or else the diagrams could not be describing the same physical circuit.

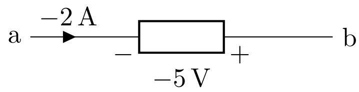
In the diagram below, the reference direction (arrow) for the current has been reversed, so the *sign* of the current has also been reversed in order that the diagram will show the same physical situation as before.



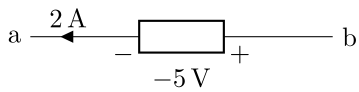
The power into the component can be calculated by considering that the actual current is 2 A going up a potential-difference of 5 V, so the component is giving 10 W to the circuit, or in other words the power into the component is −10 W.

Alternatively, one can think that the power to the component is the produce of the voltage and the current that flows into the higher-potential side; this current is negative, so the power into the component is $5\,\mathrm{V} \times -2\,\mathrm{A} = -10\,\mathrm{W}$.

In the next diagram, the voltage has also had its reference direction reversed, and its sign changed in order to describe the same physical situation.



The one remaining case is where the current has the same reference direction as in the first diagram.

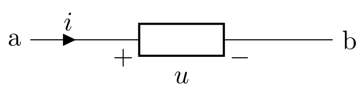


Here, we could say that 2 A goes down in potential by −5 V, or equivalently that it goes up in potential by 5 V. If we want to find the power into the component, we want to see how much the current went down in potential, so we want the product $2\,\mathrm{A} \times -5\,\mathrm{V}$, giving −10 W.

In summary: there are lots of ways to think of this. You must decide whether you want to know the power from the component to the circuit, or vice versa. By thinking about how the component changes the current's potential as the current passes through it, you will find whether the sought power is positive or negative.

5.1.2 Symbolic voltages and currents

In this diagram, the voltage and current are marked as symbols, not as concrete quantities of number and unit.



The symbols u and i represent quantities that might have negative value. We don't know if they will be negative or positive. Often, we choose the reference directions before we know what some of the voltages and currents will be.

This might sound bad, but it's actually easy. Just imagine both quantities are positive. Do your thinking based on that. If you want to calculate power into the component, then for the above choice of reference directions you can say "the current i loses a potential of u in the component", from which you see that the power into the component is $P = ui$. If the power out had been wanted, or the current or voltage had the opposite reference direction, then the expression would instead be $P = -ui$.

But what if one or both of the quantities actually turns out to be negative? It's not a problem: the negative sign will then come in to the equation, and the algebra will sort everything out so that the answer has the correct sign. If you're doubtful, then try some examples with positive and negative quantities.

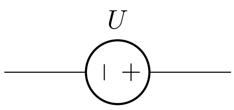
What is particularly important here is that the *reference directions* in which the voltage and current are marked do *not* have to be the same as the direction of the actual voltage or current.

6 Components

Now we introduce the ideal circuit-components that are needed for Topics 01 to 04.

6.1 Voltage source

The classic source in circuit analysis is the voltage source, which we will show with the following symbol.



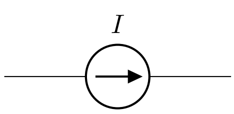
The + and − show that the source's voltage U is the difference in potential of the + side relative to the − side.

A voltage source always has the specified voltage between its terminals. The current through an ideal voltage source is not specified at all: it depends on the rest of the circuit to which the voltage source is connected.

An ideal voltage source is often a good first approximation of a battery or electric generator, as long as it is not too heavily loaded (too high current). To improve the model, one can include a series resistance to model how the voltage from a real physical source goes down when current is coming from the source.

6.2 Current source

The current source defines a particular current. That current must pass through it, regardless of what it is connected to: that's its definition.



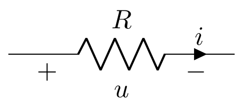
The *voltage* across the current source might be anything. It is not at all defined by the source; it is determined by what the source is connected to. A 1 mA current source connected directly to a 16 kΩ resistor will have a voltage of 16 V or −16 V, depending on the definitions of current or voltage direction!

One of the most common mistakes in circuit analysis is to assume that a current source has *zero* voltage, instead of just undetermined voltage.

Compared to a voltage source, it is harder to find a common, simple physical effect that is well modelled by a current source. Very good approximations, over a range of voltage such as up some tens of volts, can be made with transistor circuits. Phenomena involving a limited rate of ionisation in a gas, with ions flowing away in the field generated by a voltage between some electrodes, can also give a quite constant current over a limited range of voltage. An amplifier with feedback to force its current to a specific value could also well be modelled as a current source, but if you look inside, it would probably seem more like a voltage source with feedback.

6.3 Resistor (Ohm's law)

The ideal resistor gives a direct proportionality between voltage and current. One common way of showing a resistor is a box, like the 'generic component' we used earlier. The symbol we will use is the zig-zag line, shown below.



If the voltage and current are defined so that the current goes in at the terminal that is the positive voltage reference, then $R = u/i$. If one of these definition directions is reversed, then $R = -u/i$.

Note that there are four ways of drawing the u and i definitions, but when both directions are swapped the sign of resistance is not changed.

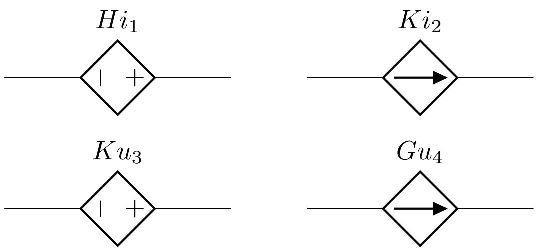
Warning: be careful about writing, for example, $u = Ri$ until you have checked the relative directions in which u and i are defined; the correct equation might instead be $u = -Ri$. If the 'passive convention' is used, then $u = Ri$. You can see this by thinking that the voltage across a resistor is what pushes the current through it, from high to low potential (or 'pressure').

6.4 Dependent (controlled) sources

The voltage and current source that we already have seen are *independent sources*. This means that their *value* of voltage or current (respectively) does not depend on anything else in the circuit.

Dependent sources (sv: *beroende källor*), sometimes called controlled sources, may also be voltage or current sources. The only difference is that their voltage or current is not a fixed value. It is instead proportional to some chosen voltage or current somewhere else in the circuit. This other voltage or current is called the *controlling variable*.

We use a symbol where the outer box is a diamond instead of a circle, to indicate a dependent source. The following symbols show the four possible combinations of current-controlled voltage source, current-controlled current source, etc; these names are often abbreviated to *CCVS*, *CCCS*, *VCVS*, *VCCS*.



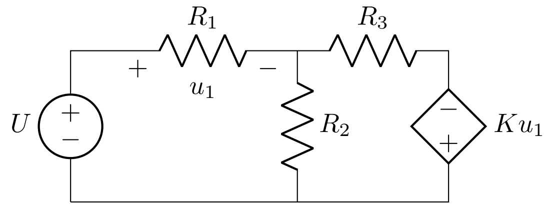
Next to the source, instead of writing a constant value, we write the controlling variable and a multiplier. The multiplier is the relation between the controlling variable and the source's output.

The CCVS, top left in the above symbols, is controlled by a current i_1. This current must be marked in some part of the circuit where the CCVS is used. It could of course have any name: i_1 is just an example. The CCVS component's *value* is the multiplier that relates its output voltage (let's call this u) to its controlling variable i_1. This is $H = {}^{u}\!/_{i_1}$, which has the dimension of resistance. A resistance that is the ratio of a voltage to a current in *different* parts of a circuit is sometimes called a *transresistance*. That name is therefore sometimes used for a CCVS. It is quite common to use the symbol H for the value of a CCVS, but it's not a strong convention in the same way as using R for a resistance!

Similarly, the component value of a VCCS relates the output current (let's call it i) to a controlling variable of voltage defined in a different part of the circuit. This is $G = {}^{i}\!/_{u_4}$ for the VCCS shown above. Such a relation is sometimes called a *transconductance*.

The other two dependent sources, VCVS and CCCS, have dimensionless values that could be called just *gain* or *transgain*. We could use symbols E and F respectively, to distinguish these: in a common circuit simulation program the dependent sources are denoted by single letters E, F, G, H. In this course we typically just use K.

The following is an example of a complete circuit with a controlling voltage u_1 marked across component R_1. This circuit can be solved to find what u_1 must be in terms of the components' values U, R_1, R_2, R_3 and K.



Dependent sources are often useful in models of transistor circuits. They could also model amplifiers. Rather strange things can happen in a circuit when one allows dependent sources to be included: for examples, parts of the circuit can behave like negative resistances.

6.5 Short-circuit, Open-circuit

It is sometimes useful to consider short- and open-circuits as components.

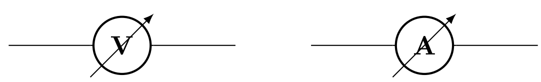
The short-circuit can be shown as a line, like part of a node. Like a part of a node, the short-circuit has the same potential at both ends, and can carry any current. It is thus a zero resistance. A voltage source with a voltage of zero is identical to a short-circuit.

The open-circuit can be shown as a gap, rather like the space between two nodes that are not connected. It has no current, and the voltage between the two sides can be any value. It is thus an infinite resistance. A current source with a current of zero is identical to an open-circuit.

In Topic 4 (Equivalents) and the Transients section of the course, we will often replace other components with short- and open-circuits to make some steps in the analysis clearer.

6.6 Meters

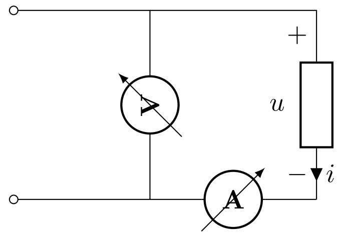
Measurement of voltage and current in a circuit is done with meters called voltmeters and ammeters. A common type of hand-held or workbench meter is a multimeter, which can measure voltage, current, and resistance. These meters have two terminals. They can be shown by V and A (volt and ampere) symbols in circles, possibly with a 'pointer' to indicate that they make measurements.



For voltage measurement, the wires must be connected between the nodes where the voltage is to be measured. Ideally, the meter would behave like an open-circuit so that it takes no current from the circuit, and therefore does not change the circuit's voltage by its presence.

For current measurement, the wires must be connected so that the current to be measured flows through the ammeter. Ideally, the meter would behave like a short-circuit, so that it behaves like a part of a node and therefore does not introduce a voltage that could affect what the currents are in the circuit.

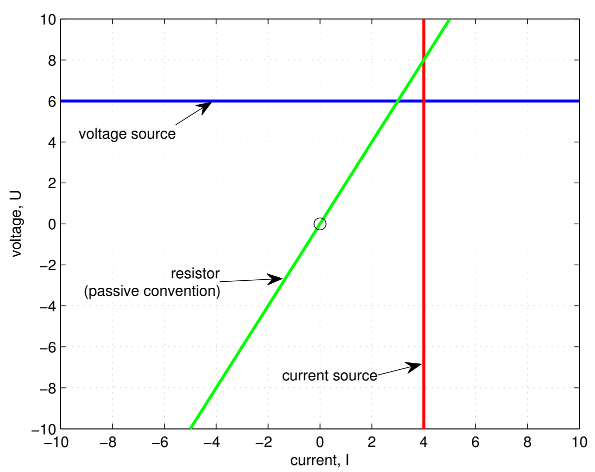
The following circuit is therefore one way in which the two circuit quantities could be measured for a two-terminal component. (Real meters have clear marking of the positive and negative terminal, so they can be connected to match whatever reference direction is desired.)



The terminals at the left are connected to a circuit that allows some current i to flow in the component. The reading on the voltmeter will be the same as the voltage across the component, u, as long as the ammeter has negligible resistance. The reading on the ammeter will be the same as the current through the component, i. The ammeter could be inserted in the 'wire' at the left of the voltmeter, instead of at the right. In that case, the current would be measured correctly if the voltmeter has negligible conductance (very high resistance).

6.7 Graphical view: the u-i plane

A helpful way to think of the behaviour of different basic two-terminal components – voltage source, current source, resistors – is to plot the relation between their voltage and current.



The resistor has a slope dependent on its resistance. Notice that if the current or voltage definition on the component were reversed, then the slope would be negative. The ideal sources have vertical or horizontal lines.

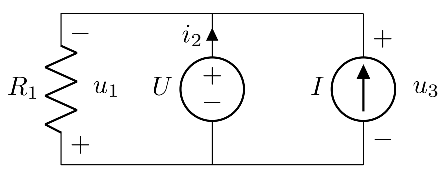
When two of these components are connected together at both ends, the point on the u-i plane where their lines meet will be the point describing the voltage and current in that circuit, where the demands of both components are satisfied. It is then necessary to be careful with how the voltage and current are defined.

This sort of u-i-plane view (sometimes shown i-u instead) will be more important later when we study equivalent sources.

6.8 Known and Unknown

We will mainly be studying what one could call the 'forward problem' of circuits: we are told what all the components are and how they are connected, then we have to find some other circuit quantities that would result.

Each component has a value written next to it, which is often symbolic in this course. It can be assumed, unless told otherwise, that a resistor marked as R_1 has this known ratio between its voltage or current; any marked source such as U or I has this known value of its voltage or current respectively. Our task is then to express a sought *unknown* quantity such as i_2, in terms of the *known* quantities that describe the components. On the other hand, it is similarly assumed that quantities simply marked on the diagram, such as the current through a resistor, the potential of a node, or the voltage across a current source, are *not* known, but are just definitions.



Sometimes the important inverse-type problem occurs instead, where we need to find what component value is needed in order to give a specified circuit quantity at some point in the circuit. This is quite typical of design situations; even the connections of the circuit might be able to be chosen to help towards the design goals. This type of problem often does not have a unique solution, unless the available variables for choosing are very limited.

7 Connectivity and Nodes

The components by themselves do not have their voltages and currents fully defined. A source defines *one* of these variables, leaving the other unknown. The resistor defines a *ratio*, but not an actual value of voltage or current. The connections *between* components impose further conditions on the voltages and currents.

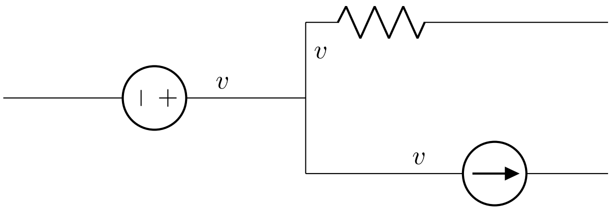
By putting together the conditions required by the components *and* by the way that they are connected, we usually get a well determined system, where exactly one possible value exists for each variable. Then we can solve to find all the voltages and currents. There are some special situations where a circuit model is overdetermined (it demands something impossible, like $2 = 3$) or underdetermined (the conditions are satisfied by a range of values, not a unique value).

7.1 Nodes

A *node* is a connection between some component terminals. In concept, it is an ideal connection — a perfect conductor — so the potential should be the same at all points on the node. We can thus say that all the terminals connected to a particular node are at the same potential.

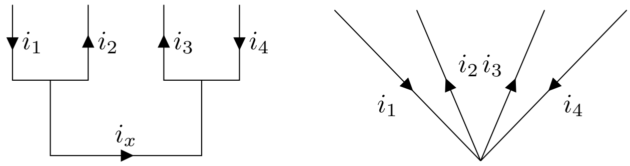
In a diagram we draw a node as a set of lines connecting some components' terminals. The node tells us that all those connected terminals have the same potential. We also know from Kirchhoff's current law (next section) that the currents into the node from the component terminals must all sum to zero,

In the diagram below, the middle region is a node that connects one terminal of each of the two sources and the resistor. The connections to components are marked with a potential v, indicating that the node demands the equality for all the terminals that it connects. The other terminals are assumed to connect to other components "outside the page".



Sometimes we think of the lines in a diagram as representing wires, like the copper strips that connect components on a circuit board. The strips are in this case being treated as nodes. However, the lines in the diagram do not *have* to copy the shape of connections in the real circuit. The solution of the components' voltages and currents is not affected by the shape in which the nodes are drawn, but only by what sets of terminals are connected to each other.

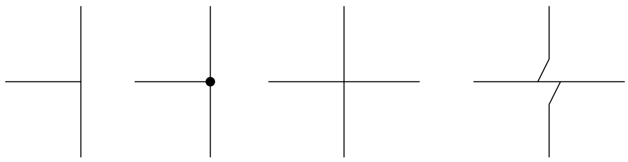
On the other hand, keeping some similarity between the physical circuit and the diagram may help us to remember what the diagram's components represent. If we want to mark a current that corresponds to the current in a particular wire, it could be a problem if the node has been reshaped so that no part of it actually has a current of that value! The following example has current i_x marked in the node on the left, but not found in the equivalent node on the right.



If a wire has significant resistance, it probably needs to be modelled as a resistor between two nodes, instead of as a single node. Wires on a circuit board might have very low resistance compared with the other components, in which case they are well approximated as parts of nodes. On long circuits carrying significant power, the resistance of the wires is significant, and the wires are modelled with a resistor.

The case of a node connecting just two things is sometimes called a *trivial node*. A line links the two components. The current out of one component into the trivial node must all go into the other component (section 7.3).

There are varied conventions about how to show points where three or more lines join. For example, two lines appear to cross over each other in the third example in the following figure: does that mean that four parts of a node are joined, or that two separate nodes cross over each other in the diagram?



We will use the convention that all touching lines are connected. We will avoid having two lines cross, since that always makes people wonder if they are joined or not; all four-line junctions will be made into two three-line junctions, as shown at the right. In our diagrams, all the four situations shown above mean a connection of all lines. But we will use the methods shown on the left and right, for three- and four-wire junctions.

7.2 Reference / Earth / Ground node

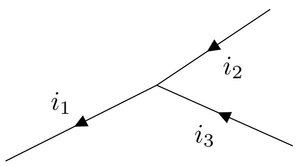
Potentials are only meaningful relative to a reference level, which needs to be defined. If we use the concept of potentials in circuit analysis, we usually define one of the nodes in the circuit as the reference. In that case, the reference node must have a potential of zero ... because the voltage between a node and itself is zero!

A common name for this reference node is the *earth* (sv: *jordnod*) or *ground*. The chosen reference node is shown by a special symbol: we will use the symbol on the left here, but the one on the right is quite common too.

By itself, this symbol only defines the reference node, of potential 0 V. No current can flow through it, since it has only one terminal, so KCL requires a zero current. But beware! We will see in Topic 05 that people sometimes write multiple earth nodes that are assumed to connect to each other in a hidden way.

7.3 Kirchhoff's current law, KCL

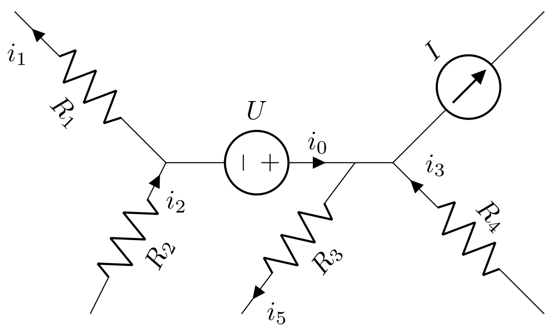
The total current into some part of a circuit equals the total current out of that part. The studied part is often a single node; in the following diagram, KCL at the middle point gives $i_2 + i_3 = i_1$.



Another way of thinking of KCL is to express all the currents in the inward direction, in which case they should sum to zero; in the example above, $-i_1 + i_2 + i_3 = 0$. Alternatively, express all the currents in the *outward* direction and they should (of course) still sum to zero, as $a + b = 0$ is the same as $-a - b = 0$: hence, $i_1 - i_2 - i_3 = 0$.

KCL is not only relevant to a single point where multiple currents meet, or a single node. For *any* region of a circuit the sum of all the currents going into (or out of) that region is zero. The diagram below shows a *part* of a bigger circuit; the marked currents such as i_1 are continuing into rest of the circuit, which is not shown.

KCL in the node at the left of the voltage source gives the relation $i_1 - i_2 + i_0 = 0$, and in the node on the right of the voltage source it gives $-i_0 + i_5 - i_3 + I = 0$. But we can also look at *all* currents coming out of this visible part of the circuit, to say that $i_1 - i_2 + i_5 - i_3 + I = 0$.

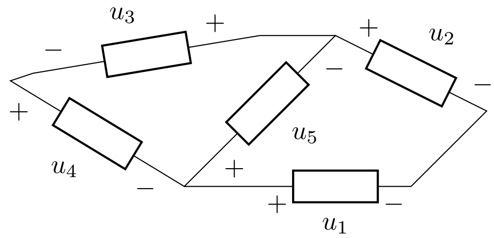


Sometimes we only want to study KCL for a larger region of a circuit: in this case we can avoid dealing with 'internal' variables like i_0, by directly writing KCL for the whole studied region.

Not all the KCL that we could find are independent. For example, the KCL expression for all the current out of the region shown above was just the sum of the expressions for the two nodes separately (check!).

7.4 Kirchhoff's voltage law, KVL

The total change in potential around any circuit loop is zero. So if you sum all the voltages as you go through various components and finally come back to your starting-point, the sum should be zero if you took care about the direction of each voltage.



In the above case, KVL around the outer loop tells us that $u_4 + u_3 - u_2 + u_1 = 0$. This was derived by starting at the bottom, going clockwise, and treating as positive the voltages that are defined as going from the $-$ to $+$, i.e. increases of potential. Like the application of KCL with inward or outward current, we can choose arbitrarily whether to count increasing or decreasing potentials as positive or negative changes as we go around the loop; it's only important to use the *same* choice all the way around.

8 Summary

The main *circuit quantities* are voltage and current. It may help to think of these as, respectively, pressure difference and flow of a fluid.

Other quantities of potential and charge are strongly related to the voltages and currents in a circuit. Power can be calculated from voltages and currents, being careful about the relative directions. Think of the voltage across a component as a 'change in energy' of the charge that moves through the component. Or divide by time, and think of the power taken from or given to the current through the component.

Several two-terminal components were defined here. Each has a voltage and current: two 'degrees of freedom'. Each fixes only *one* of these degrees of freedom, such as its voltage or the ratio of voltage to current.

Nodes show the connectivity of components. Kirchhoff's laws put further conditions on the circuit, based on this connectivity. These conditions remove the remaining degree of freedom of the circuit quantities. Except for a few special cases (next Topic!), there is a unique solution for the circuit quantities in a linear circuit where the components and their connections are known.

9 — Extra —

[Remember: from here to the end of this document is the "just for interest" part.]

9.1 Links

The following Wikipedia pages are very good sources of more detail, examples and references, about our main components. They contain lots of further references.

`[CircuitDiagram]`

`[Resistor]`

`[CurrentSource]`

`[VoltageSource]`

You are not recommended to get lost in all of this, but seeing some pictures and examples of physical things that are approximated by the idealised circuit components. These pages are quite strongly towards the subject of 'analog electronics', dealing with lots of transistors.

9.2 Duality

We mentioned duality in circuits. This is the idea that if you exchange certain pairs of properties, components and types of connection, you can make different circuits with similar equations. This Wikipedia page gives a list of circuit duals `[Duality]`. Examples are resistance↔conductance, voltage↔current, series↔parallel.

Another way to see duality is that a true statement about a circuit will still be true if all the words are exchanged for their duals: for example

> A short circuit in parallel with one resistor, in a series circuit of several resistors fed by a voltage source, will cause a higher voltage on the other resistors.

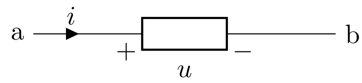
can be changed to

> An open circuit in series with one conductor, in a parallel circuit of several conductors fed by a current source, will cause a higher current on the other conductors.

(We don't really have to swap the words 'conductor' and 'resistor' here, because these are describing the same physical thing, although the properties of conductance and resistance are reciprocal, $G = 1/R$.)

9.3 Active and Passive Convention

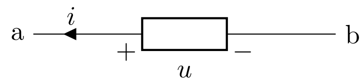
The situation where current is defined going in to the terminal where the voltage has its positive reference, is sometimes called the *passive convention*.



The product of voltage and current, when they are defined using the passive convention, gives the power *in* to the component from the circuit: $P = ui$. The name arises from the relevance to *passive components* such as a resistor, which only ever can consume power. For these components, one typically wants to calculate the power in. Then it is convenient to be able to write the expression for power in without needing a negative sign. A negative sign would be needed if the current reference did not go into the positive voltage reference.

If both reference directions are reversed, the passive convention is still being used. You could explain this as being that the positive voltage reference and the 'in' direction of current are still together. If you prefer an equation-based way of thinking, you could say that an extra negative sign has been given to the values of u *and* i, so the result is unchanged.

When just one of the reference directions is reversed, the passive convention is no longer being used: this is now the *active convention*, where the current reference is out of the positive voltage reference.



The name 'active' suggests that this is a convenient choice of direction for active components such as sources. These are often used to provide power (although they also *can* consume power). With this choice of reference directions, the power *out* of the component *in* to the circuit, can be calculated by multiplying the voltage and current values: $P = ui$. With a negative sign, this expression gives the power that goes *in* to the component.

The relative direction of voltage and current is also relevant to calculations involving resistance. Consider that a resistor's current is pushed through it by the difference in potential: it flows from high to low potential. Therefore, if the current reference goes in at the positive voltage reference, the resistance is the ratio $R = u/i$. But if one of the reference directions is reversed (to give the 'active convention'), then for a normal resistor where the actual current flows from high to low potential, exactly one of the values u or i would have to be negative. The expression $R = u/i$ would then give a negative resistance. In this case we must write Ohm's law as $R = -u/i$.

Some people prefer consistency: for example, they might like to use passive convention always. This choice is made for all components in a well-known circuit simulation program, SPICE. Consistency can be more convenient than avoiding a few negative numbers, particularly when computers are handling the numbers.

Some of us think it would be simpler just to ignore the words passive and active, and remember the physics: a current going down in potential gives power *to* a component. From that, we can decide whether to write the equation for power with or without a negative sign. A similar principle applies to resistance, where one can think of pressure difference driving a flow from high to low pressure.

That is why the idea of 'conventions' has been moved to the Extra part of the Chapter. The definitions are relevant to understanding some textbooks about circuits, and to understanding what a manual for a circuit solution program means when it says something like "the current in each voltage source is defined according to the passive convention". But the concept of active or passive convention seems to cause more trouble than benefit when it appears as a superfluous concept in the circuits course introduction. It perhaps makes people focus on remembering terminology and lots of different 'if' cases, instead of just thinking physically.

9.3.1 Practical examples

The following was considered as an example of a circuit model being a very good approximation, in spite of making the modelling very much simpler. In many practical applications the circuit model is a superb approximation.

For example, a circuit-board in an audio device may contain resistors that each are several kilohms, connected by short strips of copper, separated by air, and operating at quite low frequencies of some kilohertz where the speed of change of electric and magnetic fields around the currents is too low to generate significant current or voltage. Then it is a very good assumption that between the metal parts joined into a piece of copper strip, which have resistances of much less than one ohm, the differences in potential are almost zero compared to the changes in potential between the ends of the multi-kilohm resistors. And practically zero current flows through the air: it all flows through the metal and the components. In this case the chunk of metal is well modelled as a node.

On the other hand, a chunk of metal in a welding (sv: svets) machine might need to be modelled as a resistor instead of a node, because all the currents in that circuit are large, and all the resistances are low, so the potential in the piece of metal can change by a significant proportion of the circuit's total potential. The same idea is true for long pieces of good conductor. A power line of 100 km length will usually need to have its resistance (and inductance) considered: it cannot be modelled well as a node.

What we learn from the above is that the choice of what things to model as nodes, and what to model as components, is not absolute. It is relative to the other parts in the circuit. In some realistic cases, it requires significant skill to make suitable choices to form a good circuit model.

9.4 Voltage and Current

The textbook "The analysis and design of linear circuits", Thomas and Rosa, 1994 (Prentice Hall), is rather good. that book distinguishes itself by having a few that are actually relevant and interesting. Here's the first:

The electromotive action manifests itself in the form of two effects which I believe must be distinguished from the beginning by a precise definition. I will call the first of these 'electric tension', the second 'electric current'. Attributed to André-Marie Ampère, 1820.

That's a very important point. I hope I will some day get a chance to read about how he came to that conclusion. One must consider how difficult it must have been when people knew so little about what electricity was, and did not have good voltage sources, meters, or even nice insulated wires available. They had only some information from magnets, static electricity and electrochemical cells: there were not the sensitive instruments that we are used to, nor all our background knowledge about electricity.

So — *tension et courant*; tension and current; pressure and flow; spänning och ström; which we will be calling *voltage* and *current*. These are, as Ampère had realised, two quantities that together allow us to give a full description of what is happening in an electric circuit. They are the two main *circuit quantities* that we will be dealing with. Other electrical quantities such as power and resistance can be derived from them.

Each of these two main circuit quantities is quite commonly expressed in another way.

Charge Current is a *flow-rate*: it describes the flow of *charge*. In other words, charge is the time-integral of current: if a current $i(t)$ flows past a point, then the charge that has flowed past that point up to time T is $\int_{-\infty}^{T} i(t)\mathrm{d}t$. Sometimes one chooses to talk of charge instead of current: this is particularly natural in the context of capacitors. Charge basically *is* electricity: it is how we describe the property of matter that is associated with electric and magnetic behaviour.

The voltages and potentials in a physical circuit are determined by the distribution of charges in that region. This could make us wonder whether Ampère was really right about needing two different quantities to describe the circuit. What is particularly convenient about how we treat *circuits* instead of *fields*, is that we reduce the problem to discrete components and connections; a circuit diagram does not include all the details of components' sizes, and of how the components are distributed in space. In order to find potentials from charge distributions we would need to know details of the geometry. By defining the two different circuit quantities, voltage and current (or potential and charge), we can avoid these geometric details. In most practical circuits, the amount of surplus charge that has built up in any region of the circuit to create a potential is utterly tiny — e.g. $\frac{1}{1 \times 10^{20}}$ — compared to the amount of charge flowing past a point in a second.

Potential We talk in circuits about potential. We have a rule about changes in potential summing to zero around a closed loop. This is true of the electrostatic field due to the charges that build up around a circuit.

Even the voltages in a circuit are consequences of the behaviour of charges: a charge creates forces on other charges, leading to a *field* of electric force: in moving through this field, a charge gains or loses potential energy as it moves through a force. In a circuit, other forces also affect the moving charges: examples are interaction of charged particles with the material in which they move (electrons in a metal, where potential energy is lost to the resistance), or forces due to magnetic fields that change in time, or that the charge has relative motion to.

There are varied "opinions" about the correct word and symbol for voltage! In circuits, the matter of interest is often the difference in pressure between two sides of a component: this describes the energy that a charge will lose or gain (depending on its direction) when passing through the component.

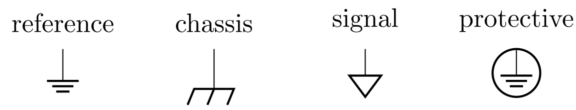
When just these differences between potentials at points in a circuit are of interest, one often simply says "voltage" in English, particularly when one is a practical user of circuits. This is often represented by the symbol V or v in English texts, and often by U or u in other European languages.

Another term for voltage, that seems most used by people who don't actually work much with circuits, is *potential difference*.[3] When we want to describe *potential*, which can be thought of as a voltage between a particular point and a specified reference, the symbol V or v is normal, and in English the word "voltage" is sometimes also used. In that case, one might use prepositions to distinguish the case of "voltage *at* this point" (potential) from "voltage *across* this component" or "voltage *between* these nodes" (voltage).

9.5 Earthing

The earth-symbol that we use in diagrams in this course is shown at the left of the following list of symbols. It hints at a connection into the slightly conductive soil around us: this often is the most obvious reference conductor we can think of, as it extends all around us and many metallic objects that we have contact with are well connected to it.

There are several other symbols called an earth or ground. The next one is more indicative of a connection to the metal case (chassis) of an electrical device, which seems a good choice of reference for the circuits in that device. Others indicate special purposes such as a local reference for sensitive electronics signals, or a *protective earth* (sv: *skyddsjord*) connection.



A protective earth connection holds nearby conductive surfaces to be at similar potentials so that there will not be electric shock on contact with these surfaces. It also provides a low-resistance path so that if a conductor at dangerous potential comes into contact with these conductive surfaces, the surfaces cannot reach a dangerous potential without a current flowing that is big enough to make the circuit's fuse or circuit-breaker quickly disconnect the supply.

The above sounds like contradicting the claim that no current flows in an earth connection (KCL for a 1-terminal component). The explanation is that the supply source of the 'dangerous potential' is assumed to have a connection to earth also; all the points marked as earth are considered to be part of the same node, so currents can flow between the two connections. In that case, there can be currents between the earth symbols: KCL should be drawn around all of them together as they are assumed to be one physical thing but split up in the diagram for convenient drawing.

In practical circuit diagrams one often avoids mess by accepting that popular nodes such as earth and power-supply connections can be split into different regions of the diagram. You might see lots of points marked as (for example) a power-supply node V_{cc} in different parts of the diagram: these are all assumed to be the same node.

The common symbol for an ideal opamp, seen in Topic 05, has an implicit earth node inside it, which can exchange current with the other earth node[s] in the circuit. A metal-bodied car normally uses its body (chassis) as one conductor of its electrical system. In modern cars it is the $-$ pole of the battery and alternator (generator) and all the other electrical equipment, that is connected to the metal of the car body and the engine. In a circuit diagram there will be many chassis-earth symbols, which are all assumed to be one big node!

[3]This is intended to mean "difference in potential", as opposed to "chance that there is a difference", which would be a reasonable interpretation if we didn't know the context: perhaps we should always hyphenate it and emphasise the first word ... or perhaps we should usually just say voltage, as so many people have done for well over a hundred years!