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Topic 02: Simplifications

Topic 02 is mainly about ways of manipulating a circuit diagram into a form that is easier for us to solve. That is a bit like rearranging and simplifying a mathematical expression into a form that you know how to solve. Indeed, a circuit diagram showing ideal components *is* really just another way of expressing equations.

A common task in circuit analysis is to find a particular subset (perhaps just one) of the currents and voltages in the circuit. The known data might be all the values of the components in a circuit (voltage for a voltage-source, resistance of resistor, gain of a controlled source, etc). Or it might be some of these values, and some other marked voltages and currents in the circuit. A *design*-problem often requires us to *choose* a value of one or more components, in order to satisfy a specification of particular voltages and currents in the circuit. Even this type of task can be approached by finding an equation for the specified values in terms of the components, then using the equation backwards to find the component values.

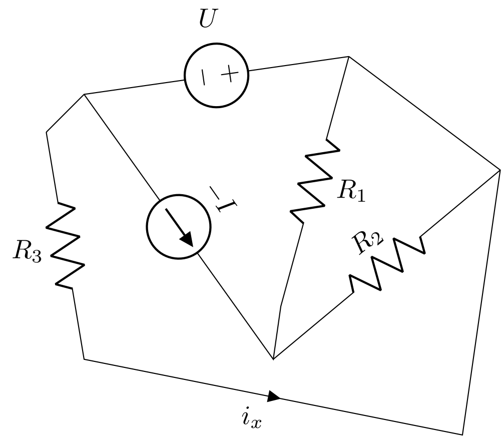
Therefore, a common task in circuit analysis, and in this course, is to start from a circuit diagram with components' values given, and to find an equation that uses these values to find a particular voltage or current (or power, etc). Having "given" values does not have to mean we know numbers: we can use a symbol. In many cases, not all of the components will even be relevant to the particular voltage or current we are studying; other components might be able at least to be combined, to simplify the analysis.

We can choose to make simplifications *in* the diagram *before* we write the equations, or instead to write more complicated equations based on the original diagram, in which case the simplifications have to be done in the algebra. It is often not obvious which option will be better; this depends on the circuit, and even on how reliable *we* are at manipulating algebra compared to diagrams. As with so many subjects, making the best choice it is something we get better at with experience, but never perfect.

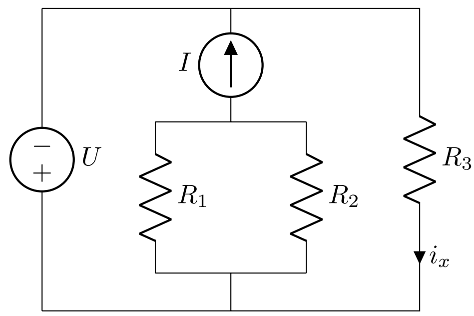
In Topic 03 we will consider more general and automatable methods of analysis, suitable for finding equations that could be solved for any solvable circuit containing the linear components that we have met. However, this Topic 02 includes some principles that can be useful for avoiding unnecessary calculation in any circuit problem that you are solving by hand, even if you then use a more systematic method from Topic 03 as a later stage in the solution. For a practical user of circuit analysis, it is useful to train one's ability to see which parts in a circuit are not relevant, or are most relevant; and when numeric values of components are given, we might be able to identify even more components that do not have much effect on the result that is being sought.

1 Re-draw the circuit

We get used to conventions. A map with north at the bottom can be confusing. It is common to draw circuits where any ground node is at the bottom, lines are vertical and horizontal, and sources are mainly on the left with their + terminals upwards. If you try to solve a circuit diagram that is very different from what you are familiar with, it might be worth drawing exactly the same circuit in an easier form, as your first step towards solving it.



By the *same* circuit, it is meant that there are the same *components*, with the same *connectivity* relation between them (the nodes), and the same directions of any marked voltages and currents relative to the nodes and components. Thus, any quantity that we are trying to find – such as i_x in this example – will be the same as in the original circuit.



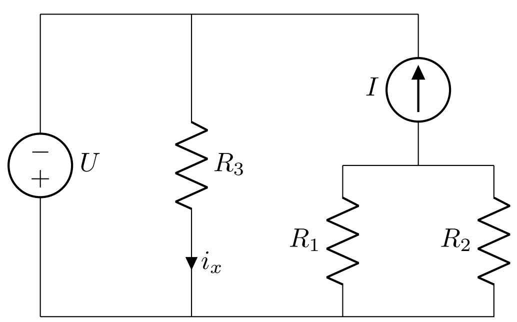
Sometimes a circuit becomes more easily solved in this way, with less chance of making a mistake when converting the circuit diagram to equations. On the other hand, you might make a mistake when re-drawing, and thereby solve the wrong circuit! There is, as usual, some compromise between the advantages and disadvantages of a method. It is important to be careful: double-check that the original circuit and the re-drawn circuit *are* the same, so that you don't waste time on the wrong problem.

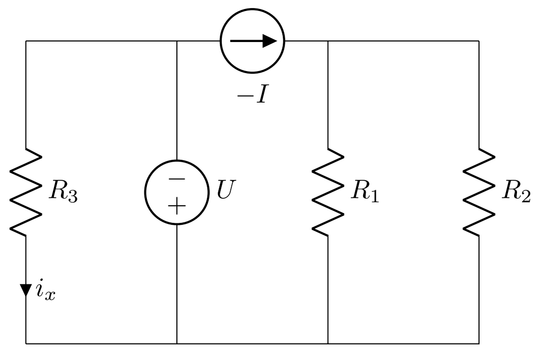
Here is a *suggestion* for a method of redrawing:
1. Identify the *nodes*: draw a ring all around each node, going up to where it joins to the components.
2. Give each node its own number or letter.
3. If there are sources with negative values ($-U$) consider reversing the source direction and writing a positive value. The meaning is identical, but it is sometimes helpful to avoid the negative value.
4. Choose one node to start with: if one node clearly connects to many components, consider starting with this and having it as a line along the bottom of your diagram.
5. Select a component that connects to this first node; draw the component either vertically or horizontally, connected to the node, being careful to get the direction right if the component has a specific direction. Do the same for each component that connects to the first chosen node.
6. Then move to the next node: it's sensible to choose a node that is the other end of one of the components you already have drawn. Keep doing step 5, on each node, until all the nodes, components and marked quantities have been transferred to the new diagram. 7. *Double-check* that all the information is equivalent between the two diagrams!

Like rearranging an equation or playing a strategy game, you will get better results if you think ahead. Look at the whole circuit and make judgements about where you want the main parts to be in your final diagram; the goal is to get a final form that you are comfortable with.

Both of the following circuit-diagrams are further ways of re-drawing the previous one. They actually look quite similar compared to some less conventional forms that could have been found by choosing a different node to go at the bottom, or rotating everything by 90°, etc.

Look at them and try to think in *both* of the following ways to check the similarity:
1) Convert by the steps listed above, such as labelling nodes.
2) Imagine the circuit made of bendable, stretchable wires, so that you can pick up a diagram, then turn, twist and bend it into another shape. See if you can, in this way, with pictures in your imagination, see how to go between the diagrams. When you get familiar with re-drawing circuit diagrams, you might not need to think through steps, but just to think about the shape. People seem to differ a lot in their liking of this sort of "visualisation".



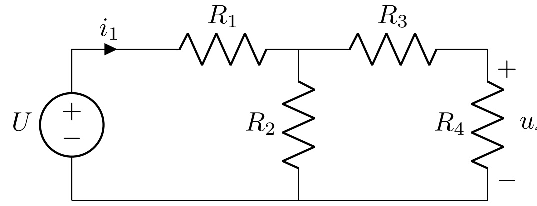


Notice how parallel branches, such as R_3 and U, can be drawn in different order, or on different sides of the other components: they are still connected to the same nodes, which is all that matters.

2 Resistors in series and parallel

Multiple resistors in series or parallel connection can be combined to give an *equivalent resistance*. This is probably well known from school.

The concept of equivalence can be useful in reducing a more complex circuit to an easier form for solving. For example, assume that all components' values in the following diagram are known, and i_1 or u_x is sought.



If we want to find i_1, we simply need to reduce all four resistors to one equivalent. The *only* resistors that are in series or parallel with each other are R_3 and R_4. If they are converted to their series equivalent, then this becomes parallel with R_2, so it can be converted to a parallel equivalent which is then in series with R_1.

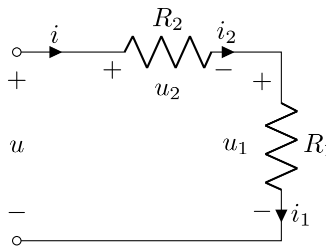
If instead the aim is to find u_4, then it actually becomes more difficult. An equivalent can be made as above, to find i_1. Then look at the original circuit, and find the voltage across R_2, which is $U - i_1 R_1$. From this, the current in the equivalent of R_3 and R_4 can be found by Ohm's law, then the voltage u_4 can be found by Ohm's law.

This seems a bit complicated for such a simple-looking circuit. The principle of voltage division is a shortcut for the last step. (And nodal analysis, in Topic 03, will solve the whole problem more neatly.)

Using the rules from Topic 01, we can now *derive* the equations for equivalent resistors. Voltage and current *division* are related to series and parallel resistors, so these rules will also be derived. Some effects of nonideal meters (voltmeter, ammeter) will be shown, since these are related to voltage and current division.

2.1 Series resistors

Consider two resistors, R_1 and R_2, in series. The total voltage and current between the ends of the series pair are defined as u and i. The voltages and currents on the separate resistors are defined using subscripts 1 and 2.



With the chosen reference directions of current,

$$i_1 = i_2 = i.$$

You might consider this obvious: "the current is the same for each component in a series circuit".[1] If we want to be more formal, by using just the basic circuit rules from Topic 01, we could apply KCL at each node; for example, $i_2 - i_1 = 0$ at the junction between the resistors.

With the chosen reference directions of voltage,

$$u = u_1 + u_2.$$

This can be seen from KVL around the whole loop, $u - u_1 - u_2 = 0$.

2.1.1 Series equivalent

In the above case of two series resistors, the quantities u and i are what are directly 'seen' by any circuit that we connect to the two terminals on the left. It is therefore these two quantities that must behave in the same way for any equivalent that we try to make for the two resistors.

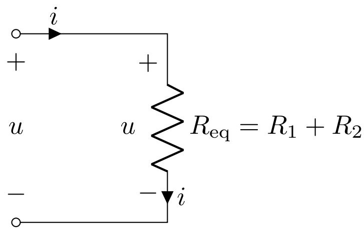
By Ohm's law, $u_1 = R_1 i_1$ and $u_2 = R_2 i_2$.

Substituting i instead of i_1 and i_2, gives

$$u = i(R_1 + R_2).$$

This tells us that the relation of u and i at the terminals is a direct proportionality, with a factor of $R_1 + R_2$. That is the behaviour of a resistance of $R_1 + R_2$! A single resistor $R_{\text{eq}} = R_1 + R_2$ connected between the terminals will therefore behave in exactly the same way as the series pair did.

[1] 'Current is the same for each component in a series circuit.' That's true if we *follow* the current through the series group of components, noticing that there is no way for current to enter or leave the path; but when we are writing equations like $i_1 = i$ we need to be careful about the direction in which the currents are defined: the correct equation could be $i_1 = -i$ if one of the reference directions is reversed.



For any number of resistors all in series, the equivalent resistance is just the sum of the individual resistances. This is easy to remember, as the resistance is the difficulty of pushing a current through: if all the current has to go through one resistor then another, the difficulty is increased by each.

2.1.2 Voltage division

We often want to find a voltage across one of two series resistors, Voltage division is a direct relation for finding this voltage as a proportion of the total voltage across both resistors, without having to calculate the current. Voltage division is used *very* often as a step in circuit analysis.

From the above diagram of two series resistors, we found that $i = i_1 = i_2 = \frac{u}{R_1+R_2}$.

By Ohm's law we can then easily find the voltage across just one of the resistors, in terms of the current: for example, $u_1 = i_1 R_1 = i R_1$.

If we know the resistances R_1 and R_2, and the voltage u across both resistors together, then the current can be eliminated, $u_1 = R_1 i = R_1 \frac{u}{R_1+R_2}$.

This is *voltage division*:

$$u_1 = u \frac{R_1}{R_1 + R_2}$$

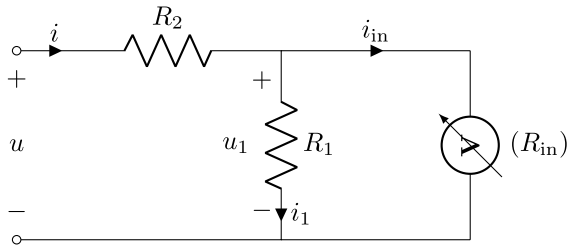
The total voltage is divided between the resistors, in proportion to their resistance.

2.1.3 Voltage measurement

It is important to remember that voltage division and series equivalents are *based on the assumption of series connection.* True series connection means that all the current out of one series-connected component must be going in to the other component.

This is often forgotten. For example, we see a diagram like the following one, with two resistors joined end to end, and we write the voltage division equation without thinking that *another* component also connects to the node between the two resistors.

A classic practical case of this is when adding a voltmeter or oscilloscope to a circuit to measure the voltage across one part of a voltage divider.



The results from the voltage divider equation will then be slightly or very wrong, depending on whether the current in this extra component is small or large compared to the current that would flow through the two resistors by themselves.

The input resistance of a digital voltmeter is typically 1 MΩ or 10 MΩ. This means that, for example, when measuring a voltage of 1 V, there will be a current of $\frac{1\,\text{V}}{1\,\text{M}\Omega} = 1\,\mu\text{A}$, passing through the meter.

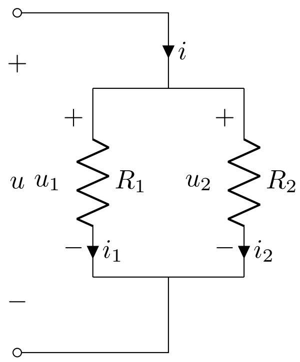
Very simple analog voltmeters that you might see on an old car's dashboard use magnetic coils to move a pointer, or use a heater to move a bimetallic strip; these would have much lower resistance. Special laboratory electrometers can have much higher input resistances when measuring voltage, for example 100 TΩ! A meter with lower resistance will more strongly *load* (draw current from) the circuit that it is measuring.

If the voltmeter's resistance is so low that we *cannot* assume $i_{\text{in}} \ll i_1$, then *the voltmeter, by being connected to the circuit, is changing the voltage that it was supposed to measure!* If the voltage we want to know is u_1 in the simple series circuit of R_1 and R_2, then by connecting the voltmeter we are changing the measured value. The voltmeter measures the voltage that is there, but the voltage is different from what it would be if the voltmeter were not connected.

A car lamp might have a resistance of a few ohms, and be supplied from a battery and cable that have a total impedance of less than one ohm. The voltage across the lamp will *not* be significantly changed by a 1 MΩ meter being connected in parallel with it! The meter is practically an open circuit compared to the resistances in the circuit. On the other hand, some sensitive electronics, or insulation samples in a high-voltage lab, have far higher resistance than 1 MΩ. For these, the normal voltmeter is practically a *short-circuit*, and connecting this meter across two nodes will make the measured voltage come almost to zero.

2.2 Parallel resistors

When two resistors are truly in *parallel*, they each have one terminal the *voltage* across both of them must be the same.



[The following should be able to be obtained by applying duality to the case of series resistors.]

With the chosen reference directions of voltage,

$$u_1 = u_2 = u.$$

This could come from applying KVL around various loops, or just because "it's obvious these three voltages are connected in the same direction between the same two nodes".

With the chosen reference directions of current,

$$i = i_1 + i_2,$$

which can be found from KCL at either of the nodes.

2.2.1 Parallel equivalent

The quantities u and i for the parallel connection can be related by including what Ohm's law requires on the two resistors.

Substituting u instead of u_1 or u_2,

$$i = \frac{u}{R_1} + \frac{u}{R_2} = u\,\frac{R_1 + R_2}{R_1 R_2}.$$

Thus, like the series case, the relation of u and i is a direct proportionality, like a resistance. The resistor that would behave equivalently is $R_{\text{eq}} = \frac{R_1 R_2}{R_1 + R_2}$.

When more than two resistors are in parallel, it is often neater to think of the conductances which are the reciprocals of the resistances. Conductance describes the easiness of pushing a current. It becomes more easy when there are multiple parallel paths. So, $G_{\text{eq}} = G_1 + G_2 \cdots + G_n$, or equivalently but less neatly, $\frac{1}{R_{\text{eq}}} = \frac{1}{R_1} + \frac{1}{R_2} \cdots + \frac{1}{R_n}$. However, when there are just two parallel resistors, it is often convenient to use the rearrangement shown above, $R_{\text{eq}} = \frac{R_1 R_2}{R_1 + R_2}$.

2.2.2 Current division

If we know the total current i through the parallel pair of resistors shown above, the voltage u across the pair can be found by the parallel equivalent resistance and Ohm's law, $u = i\frac{R_1 R_2}{R_1 + R_2}$. The current through one of the resistors can be found from this voltage, by Ohm's law again. By eliminating the voltage from the equations, we get an expression for the current through one of the two parallel resistors as a fraction of the total current.

$$i_1 = \frac{u}{R_1} = i\frac{R_1R_2}{R_1+R_2}\Big/R_1 = i\frac{R_2}{R_1+R_2}$$

This is *current division*: the total current is divided between the parallel resistors in proportion to their conductance. It looks suspiciously similar to the equation for voltage division, but the resistance in the numerator is the one where we are *not* calculating the current.

> Note the word *truly* in the above cases. When applying division formulas, be very careful that the components are in series or parallel, *and* that you do know the voltage across the two, or the current through the two. One common error is to use the voltage division equation even when a current is coming in or out of the middle point of a voltage divider. Another common error is to use the current division equation when seeing a current coming into node and having to pass out through two resistors; but that is only valid if the other sides of those resistors are guaranteed to be at the same potential, as is the case if the resistors are truly in parallel.

3 Reduction of multiple components

We have now studied equivalents for multiple resistors. It is time to move on to the other possible combinations of two components in series or parallel, and how they behave when "seen from outside".

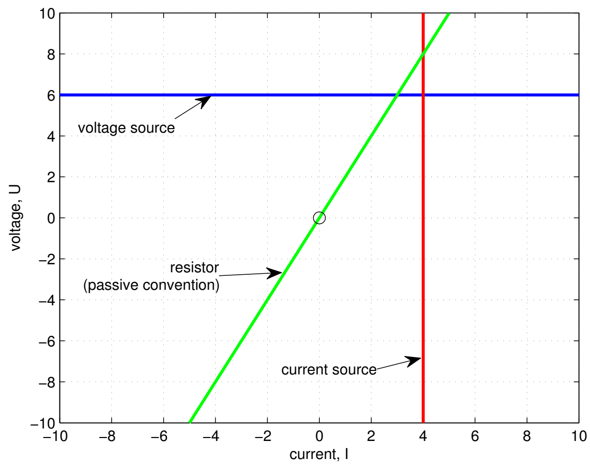
When the aim is to find a quantity somewhere *else* in the circuit, replacing components by an equivalent is a useful simplification. Even when the aim is to find a quantity within the region that is being replaced by an equivalent, it can be useful to find an external quantity based on the equivalent and then to use the external quantity to find quantities *within* the region that the equivalent represents. Such a procedure was seen with resistors in the example at the start of Section 2, when finding u_4.

We will use the term *branch* to describe a part of a circuit that connects through just two terminals into the rest of the circuit; a branch could be a single component, or a pair, or a far more complicated mixture of series and parallel connections.

In the first of the re-drawn circuits of Section 1, we could say that between the top and bottom nodes there are three branches. One branch is the voltage source U, one is the current source and two parallel resistors (I, R_1, R_2), and the other is the resistor R_3. This shows several type of branch (single component, series, series and parallel components).

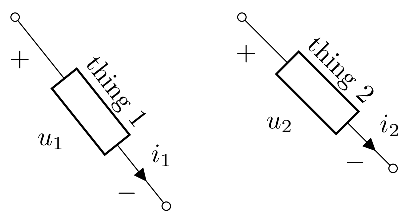
In this section we generalise the idea of equivalents for pairs of components in series or parallel that are connected to the rest of a circuit by two terminals (nodes). In some cases a principle is found that allows a reduction of very complex branches into a single component. In other cases, it might be necessary to have at least two components to make an equivalent of the branch.

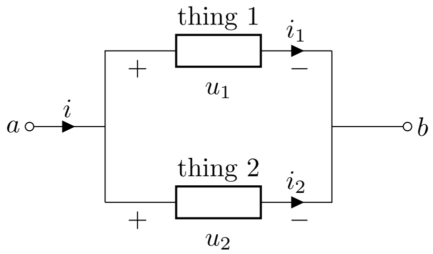
We have considered (Topic 01) the relation between the two circuit-variables, current and voltage, for the three main two-terminal components. These all give straight lines in the i/u or u/i plane.

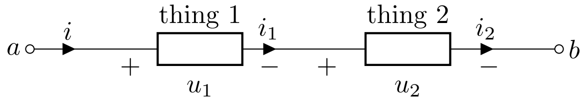


In the following, it will be seen that *except for two special cases*, all the possible pairs of two components in series or parallel behave like just *one* of these basic types of component. In some cases it will therefore be possible to reduce an apparently complex branch to a single component, by successively combining pairs of components in the way we have already done for resistors. Topic 04 generalises this idea further, by claiming that any linear circuit with just two terminals can be equivalently modelled by a single pair of components, with regard to its terminal quantities of voltage and current.

Consider series or parallel *pairs* of components, which can connect to a circuit through just two terminals, a and b. We are interested in how each pair behaves when treated as a two-terminal component itself.







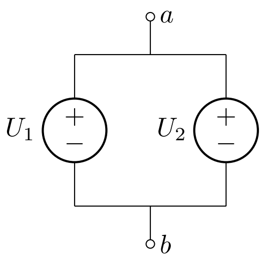
One good way of reasoning is to think what *requirement* each component puts upon the terminals of the pair of components. In this way we can try to find the u/i relation of the pair: if this matches the u/i relation of a known type of component, we can replace the branch with that component, which will be *equivalent* when seen at the terminals a and b.

There are several possible combinations. If we consider just the three basic components of U, I and R (independent voltage- and current-sources, and resistors), then there are six different possible pairs: UU, II, RR, UI, UR, IR. Each pair could be connected in series or parallel, giving a total of *twelve* cases.

To try to learn this sort of thing as a random list (like irregular verbs!) does not seem a good use of time. We should be able to learn how to *deduce* these properties: then we have also increased our intuitive understanding of circuits, and will see the right answer quickly.

3.1 Sources

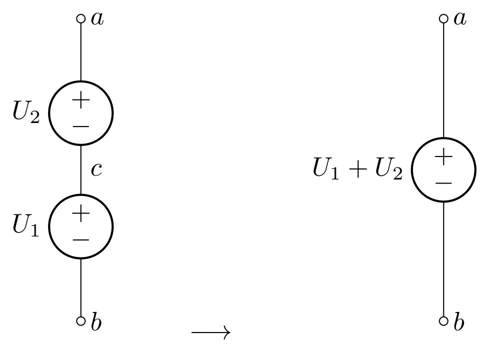
Take a strange combination such as the parallel voltage sources shown below.



Think of what a voltage-source determines. It can have *any* current that the rest of the circuit wants. All it sets is the voltage: it rigidly requires the difference in potential (energy) between its terminals to be a set amount.

A node defines that all the terminals connected to it have the same potential. So the left source requires that the voltage u_{ab} (potential at a, relative to at b) is U_1. And the right source requires that u_{ab} is U_2. This can only be true if $U_1 = U_2$. So we see that parallel voltage sources are in general a *contradiction*: unless they have exactly the same value, then the circuit diagram is contradicting itself, like an equation that tells us $x = 2 = 3$. You will therefore not see parallel voltage sources in a typical question in a circuits course. It would be meaningful only in the special case that the sources have the same value and direction, and there is even then a problem in defining how any current between a and b is shared between the sources.

Consider now two *series* voltage sources.



The lower one, U_1, requires that the potential at c must be U_1 greater than the potential at b. The upper one, U_2, requires that the potential at a is U_2 greater than the potential at c. Together, they therefore require that $u_{ab} = U_1 + U_2$. Neither of them cares about the current: it can be any amount. The series connection of course requires that the two sources have the *same* current: but nothing about this connection or the sources determines how much this is. From the above description, this combination clearly behaves exactly like a single voltage source of $U_1 + U_2$.

The same style of thinking can be applied to current sources, with the opposite result. In series they will contradict unless they both have the same value and direction. In parallel they will add to be equivalent to a single current source.

Any two *different* ones of the three main components will produce a combination that is not contradictory (as long as resistor has a value that is not 0 or ∞).

Whenever a voltage source has something else (resistor or current source) in *parallel* with it, the combination is the same as the voltage source. This means the other component is *irrelevant* to the rest of the circuit. Why? Well, it's enough to say that the voltage source fixes the voltage of the combination. But you might wonder how this works if a parallel current source keeps injecting a huge current into the nodes, or if a very low resistance is connected. The answer is simply that the *ideal* voltage source will provide whatever current is needed, to hold the output voltage fixed. If a parallel current source adds a current, the voltage source will let this current pass through it, back to the other side of the current source.

Whenever a current source has something else (resistor or voltage source) in *series* with it, the combination is the same as the current source. The voltage of the current source will simply adjust to whatever it needs to, to ensure this current keeps passing through the total branch.

> A common mistake: Do not assume that a current source has zero voltage across it! A current-source simply does not determine its voltage. For a particular circuit that it is connected to, we can calculate this voltage as one of the unknowns... it might happen to be zero as a particular case, but in general it *isn't*! In the dual case, a *voltage* source does *not* necessarily have a zero current. For some reason, almost everyone seems to realise this: it is only the current-source's voltage that commonly causes people to have problems in tests.

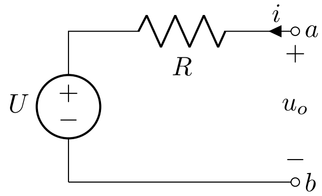
The only remaining combinations are voltage source with series resistor, and current source with parallel resistor. These do not behave like any single component: they are treated in the source transformation section.

3.2 Controlled [dependent] sources?

Dependent voltage- or current-sources could also be considered as components in the branch that we simplify. If the "controlling variable" of the dependent source is *outside* the branch, then we can treat the branch in the same way as if the source were independent. Then, if the dependent source is still present in the simplified branch after we remove irrelevant components, the external controlling variable is included in the expression for the branch voltage and current. If the controlling variable is *inside* the branch, we should be able to find a simplified branch where this variable does not have to be considered, because it has been substituted in terms of other known quantities. Even quite simple circuits with dependent sources can do strange things, depending on the relation of the source and its controlling variable: for example, a branch could be made to look like a negative resistor, or an infinite resistor (open circuit) etc. We can leave these issues for Topic 04, where the general case of simplifying a two-terminal circuit is considered.

4 Source transformation

In the earlier section we saw that a voltage source and *parallel* resistor appear identical to the voltage source (to the rest of the circuit): the resistor is irrelevant. In fact, most pairs of components can be reduced to a single component. There are two exceptions. A voltage source with a *series* resistor does not behave like any of the three basic components that we have considered. Nor does a current source with a *parallel* resistor.



Applying KVL around this loop, starting at the bottom left, we have that

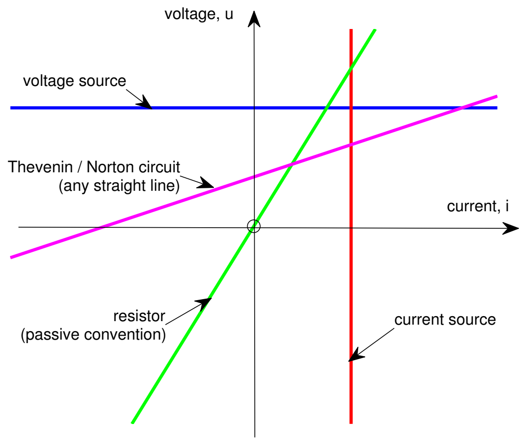
$$U + iR - u_o = 0$$

from which

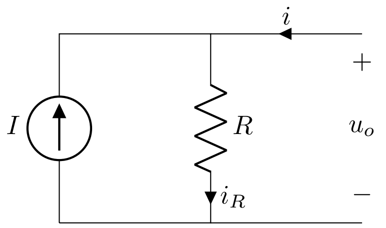
$$u_o = U + iR$$

The values i and u_o are not known: they are the quantities that are seen about this branch of series U and R components from the rest of the circuit. The equation gives a relation *between* the two quantities, allowing either one to be found from the other. This, of course, is all that we can expect: without specifying the circuit outside the branch, we cannot know *both* the current and voltage of the branch (or of any component or two-terminal circuit). If the branch behaved like an ideal source, then one of u_o or i would be fixed, and the other completely undetermined; if instead the branch behaved like a resistor, then neither u_o nor i would be fixed, but the relation between them would be a direct proportionality.

In the above case, we see that neither of these cases is true. We have an expression relating the two quantities, but it is not a direct proportionality: it is a general straight-line graph, $y = mx + c$, so it can have any gradient m and any offset c.. This combination of series U and R is known as a *Thevenin source* (Topic 04), and one example of its u/i line is shown here:



Now we can consider the dual of the above case, where we swap current with voltage, and series with parallel. We note how a current source with a *series* resistor is just like the current source: the resistor is irrelevant. But a current source with a *parallel* resistor is different: it turns out to behave similarly to a voltage source with series resistor.



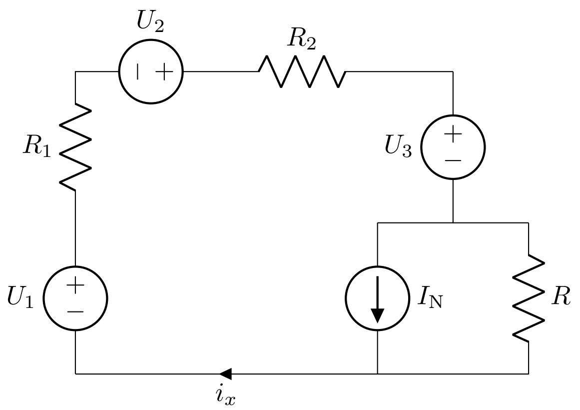
A good way of "attacking" this diagram is to see that the output voltage u_o must equal the voltage across resistor R, as they are in parallel, and that this voltage is Ri_R. By KCL in the top node, we can write $i_R = i + I$. Therefore,

$$u_o = R\,(I + i) = IR + iR$$

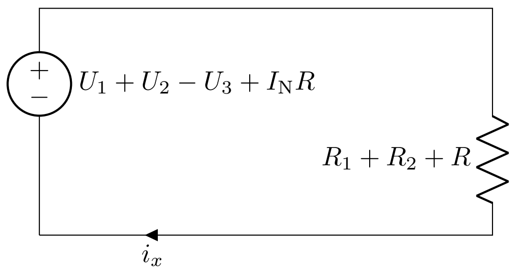
Again, this is a straight line in the u/i plane, with arbitrary slop and offset. This combination of parallel current-source and resistor is sometimes known as a *Norton source*. For these two circuits — the Thevenin or Norton source — we can therefore get exactly the same behaviour between the terminals: for this, we need the same R in both, and the source-values must fulfill the condition $IR = U$. Check this in the above equations.

> In the above diagrams and equations, the definition of i followed the passive convention with respect to u. This was done to be similar to the previous section about other series or parallel combinations. When handling Thevenin or Norton sources it is more usual to expect them to be used as sources that supply power, meaning that the current would come out of the positive voltage terminal. It is therefore common to use active convention, by swapping the direction of current definition. This would mean that the + in our equations becomes −, and the line in the u/i plane slopes downwards.

This ability to swap between a series and parallel pair of components can be useful in simplifying a circuit problem. For example, if you have a Norton source of I_{N} and R within a bigger series loop in which you want to find the current,



you could reduce the circuit to a single loop by transforming this Norton source to the equivalent Thevenin source of $U_{\mathrm{T}} = RI_{\mathrm{N}}$ and R, then combine all the series components into a single voltage source and resistance,



5 — Extra —

5.1 Links

Various types of `[Multimeter]` for measuring voltage, current and resistance, ranging from analogue handheld to large benchtop. More generally, `[TestEquipment]`.

5.2 Regarding parallel voltage sources

Even if parallel voltage sources have the *same* voltage, so that they are not a contradition, the currents in the two are not uniquely defined by the (idealised) circuit equations.

One could argue intuitively by symmetry, that the total current is equally shared if we believe the voltage sources to be identical; in the idealised world of circuit theory, any ideal voltage-source of size U is identical to any other of the same size. But that requires further logic (programming) in the common case where we want to get a numerical implementation of a solution.

In reality, one can argue that realistic sources always have some internal resistance, so they are like an ideal source *and* a series resistor (a Thevenin source) or perhaps like a more complicated nonlinear function. But there are some sources with feedback loops that could try to maintain a very constant voltage. If two such are connected in parallel, they could share the current very unevenly, or even get into an oscillation, with the sharing of the total current "moving" between the two. Even with no other components connected in parallel with them, there could be large currents circulating between the two sources. For this reason, parallel-operated sources often have deliberate "droop" to make them more like a source with resistance. The same principle is true in many physical contexts, such as the sharing of mechanical load between very stiff springs.[2]

Similar concepts apply to the case of series current sources, where the currents must all agree, and the voltages across the sources are not well defined by the circuit equations.

The problem of contradictions is more general than just pairs of current or voltage sources in series or parallel. Any complete loop that consists only of voltage sources will pose a problem, as Kirchhoff's voltage law puts a hard condition on the sum of the source values; this is true regardless of what other components are also connected branching off from this loop. Any node or groups of nodes at which Kirchhoff's current law demands a zero sum of a group of current source values is also a problem if the source values do not happen to agree with KCL.

[2]Within electrical engineering this situation with mechanical load is also similar to sharing a big current between two short wires: if the wires have a slight difference in length, and therefore different resistance, then most of the current will pass through just one, which may overheat. To a circuit-theorist this could be seen as a subset of the voltage-source problem, since a low-resistance wire is like a zero voltage-source with a small series resistance!