Electric Circuit Analysis, KTH EI1120
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Topic 06: More components

This is the start of Section B of the course, where we study circuits whose quantities change with time. The name *Transients* includes the most general cases of linear circuits, where sources may vary with arbitrary time-functions. In this course, however, we restrict the examinable part of Transients to situations where only first-order differential equations with constant forcing are needed.

Topic 06 introduces several new components and concepts, including time (!), the "unit step" function, switches, diodes, and capacitors and inductors.

The most important new components are the capacitors and inductors. These will also be seen throughout the subject of ac circuits, in Section C of the course. They are the way in which circuit models can include the effects of electric and magnetic fields in physical circuits. When quick changes happen to voltages or currents in a circuit, then even small capacitances and inductances may have significant effect on how the circuit behaves.

In a capacitor or inductor, the relation between voltage and current depends on time. Both components store energy: the voltage on a capacitor, and the current in an inductor, depend on how much energy is stored. The amount of energy stored depends on what values the *other* circuit variable has had during all the earlier time. This leads to circuits where the variables can depend not only on the components and connections *now* but also on what happened *in the past*. Such a circuit is a *dynamic* system. Differential equations are then used to describe how the circuit behaves in time; this will be considered as the final topic in 'Transients'.

The step function and time-dependent switches are only introduced as a way to produce changes in a circuit, so that the dynamic behaviour of capacitors and inductors can be studied.

The diode is presented just as a light introduction, using ideal and more detailed models. It belongs really in *nonlinear* circuit analysis, outside the scope of this course; it is a subset of the scope of *analog electronics*. The reasons for including it at all here are that it is related to switches, and the course's final lab can be better understood with some slight knowledge of diodes.

In this topic we also start with the concept of *equilibrium* (sv: *jämviktsläge*) in circuits that contain sources, resistors, capacitors and inductors. This is studied more thoroughly next time, along with *continuity* (sv: *kontinuitet*). The principles of equilibrium and continuity are useful in their own right, since equilibria and sudden changes are frequently of practical interest. They are also a common way to find *initial conditions* (sv: *begynnelsevärde*) for differential equations in a dynamic system, which is what we will need for Topic 08 at the end of the Transients Section of the course.

1 Time-dependence

The topics up to now have not mentioned *time*. We have had components — U, I, R — connected in a circuit, and found solutions that satisfy the constraints imposed by the components (Ohm's law, or fixed voltage or current) and their connections (KCL, KVL). At any point in time, all these constraints must be satisfied.

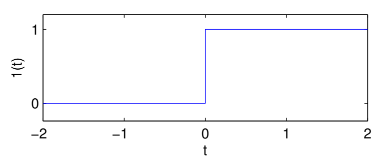
One can imagine a case where the circuit is *time-dependent*, so that a voltage source has a value that is a function of time $U(t)$, a resistor is $R(t)$, a dependent source has gain $k(t)$, and switches keep reconfiguring the circuit's connections. But we can still use our existing methods to find the voltages and currents *at any given time* such as '$t = t_1$', by using the values that all the components have *at that time*, such as $U(t_1)$, and then performing our familiar dc calculations with those values. This type of circuit is *static*, not *dynamic*. It does not have memory of the past; its solution *now* depends only on the values of its components *now*.

Before changing this by including capacitors and inductors in the circuits, we will look at the two ways in which (in this course) we can introduce changes to a circuit. The "unit step" allows us to change a component's value at a particular time. Switches allow a circuit's connectivity to be changed, by joining or separating nodes.

1.1 Unit step

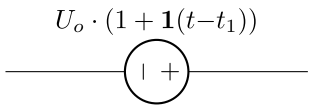
The *unit step* (sv: *enhetssteg*) is a function of one variable. It is defined as being 0 if this variable is < 0, and 1 if the variable is ≥ 0; if we're practical people we probably don't care what it is when the variable is *exactly* zero.

We will find this function useful with time as its input, to let us make components change their value suddenly. Here I will call the unit step function $\mathbf{1}(t)$.[1]

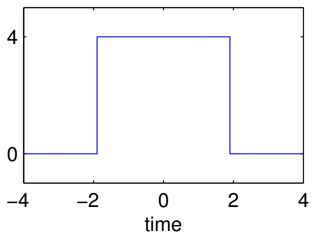


[1] We could write the **u**nit step as $u(t)$, but that would look like voltage. Or u(t) to distinguish the u as not being a variable ... still perhaps too much like voltage. Or $H(t)$ ("Heaviside step") ... but we will be using H for something else later! Choosing 1(t) (the number one!) is quite distinctive, as we seldom use numeric values in this course; we will use a **bold** symbol $\mathbf{1}(t)$ for extra clarity.

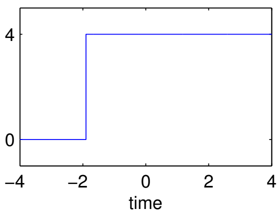
To cause a voltage source to double its value at time t_1, we could write its value in the following way.

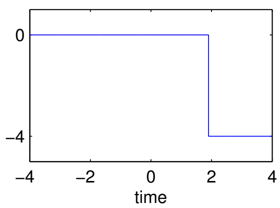


Multiple steps can be added. Let us try to produce a square pulse that goes from zero to U at time $-T$, then back to zero at time $+T$. The following figure shows this with $U = 4$ and $T = 2$.



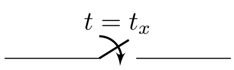
That function can be made from two unit steps, $U \cdot (\mathbf{1}(t{+}T) - \mathbf{1}(t{-}T))$. The positive step happens at time $t = -T$, then a negative step happens at $t = T$, which will cancel the first step for times $t > T$. The two parts are shown in the following figures, with $U \cdot \mathbf{1}(t{+}T)$ on the left and $-U \cdot \mathbf{1}(t{-}T)$ on the right.

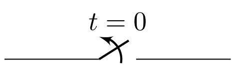




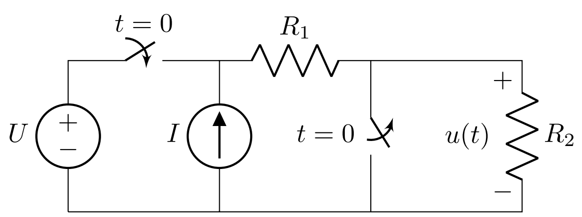
1.2 Switches

A switch can be a short-circuit or an open-circuit, depending on the time. The symbols that we use are shown below. On the left is a “closing switch”, which changes from an open circuit to a short-circuit at time t_x. On the right is an “opening switch” which changes from short to open circuit at time 0.

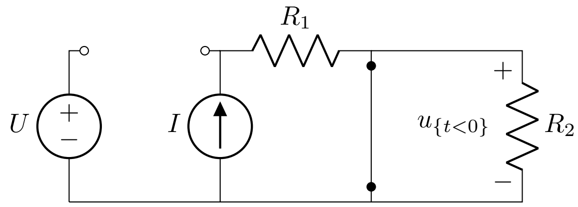




The switch allows us to make the structure of a circuit change with time. Consider the following circuit, where both switches change at $t = 0$. Note how we often choose to define as “zero” the time when a particular change happens.

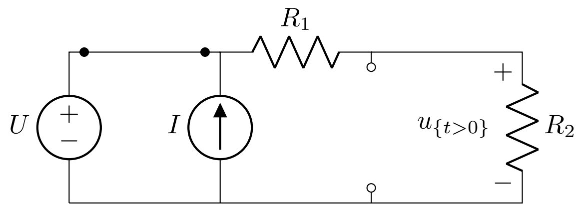


At times $t < 0$, this circuit is the following,



so we see that when $t < 0$ the marked voltage is $u_{\{t<0\}} = 0$, and a current of I flows in resistor R_1, and no current flows in the voltage source.

At times $t > 0$ the switches change the circuit to the following,



so we see that the marked voltage is $u_{\{t>0\}} = U \frac{R_2}{R_1+R_2}$, and a current of $\frac{U}{R_1+R_2}$ flows in resistor R_1, and a current $\frac{U}{R_1+R_2} - I$ flows in the voltage source.

This may not seem a very difficult or meaningful thing to do with circuits: its main relevance is when the circuits also contain capacitors and inductors, which take time to adjust to changed conditions in the circuit.

1.3 Times before and after a change

A step function or switching event happens instantaneously, in our idealised circuits. When we consider calculations with equilibria and continuity, it will be useful to consider the times just very very slightly before and after the event.

If a change happens at time $t = 0$, we define the concept of $t = 0^-$ as the time just before the change, and $t = 0^+$ as the time just after. These correspond to the old and the new conditions in the circuit.

(The change could instead happen at a time with a name like t_1, in which case we could use t_1^- and t_1^+; but we often choose to define the time-point of the change as the convenient $t = 0$, when studying a single change.)

The difference in time between 0^- and 0^+ is very small, so no significant change can happen to the energies stored in capacitors and inductors, unless there are very unrealistically large powers.

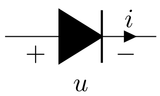
2 Diode

Diodes are NOT included after VT2015.
This Section is for interest only.

An *ideal diode* is like an ideal one-way valve for fluid flow: it allows the current to pass easily in one direction, but does not allow it in the other.

The following symbols are commonly used for diodes: current can flow in the direction suggested by the triangle arrow, i.e. left to right in these cases. The filled symbol on the left is common to denote an ideal diode, and the unfilled symbol an nonideal diode.

The ideal diode can be modelled as being either a short circuit or open circuit. The same is true for an ideal switch: but what is special about the *diode* is that its state — open-circuit or short-circuit – is determined by the current or voltage at its terminals.



If a current passes in the positive direction (following the diode-symbol's arrow), then the diode behaves as a short-circuit,

$$i > 0 \longrightarrow u = 0,$$

but if a negative voltage is applied, the diode behaves as an open circuit,

$$u < 0 \longrightarrow i = 0.$$

In other words, if we make the current 'try' to go in the forward direction (*forward bias*), the diode becomes a short-circuit; but in the other direction (*reverse bias*) the diode will become an open circuit and let us apply a negative voltage u without any current flowing.

2.1 Inideal diode

A real semiconductor diode is much better approximated by the diode equation (which you *don't* need to try to memorise!),

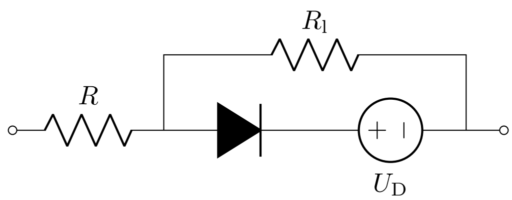
$$i = I_\mathrm{s}\left(\mathrm{e}^{u/V_\mathrm{T}} - 1\right),$$

where I_s and V_T are quantities that depend on the diode and temperature.

This equation shows *approximately* the key features of an ideal diode. The saturation current I_s is many orders of magnitude smaller than the current that a practical diode is capable of carrying in the forward direction; therefore, the reverse current, when negative voltage is applied, is 'nearly zero'. The exponential term makes the forward current able to reach even very large values for small changes in the voltage, so it approximates a short circuit.

However, the above equation also shows some of the ways in which an ideal and nonideal diode differ. Although small, some current I_s *can* flow even in the reverse direction. The current in the forward direction is small if $u \ll V_\mathrm{T}$, but rapidly becomes large when $u \gg V_\mathrm{T}$. Thus, some significant voltage is needed to get any significant current. When designing circuits with diodes, it is normal to assume that there will be about 0.6 V in the forward direction on a silicon-based diode if that diode is carrying any significant fraction of its rated current.

By adding three normal linear components to an ideal diode, as shown in the following diagram, one can get a relation of u and i that is much closer to the diode equation than an ideal diode is.



The series voltage source models the forward voltage drop, such as the 0.6 V to 0.7 V that is normal for a silicon diode. The series resistance models actual resistance in the construction of the diode, as well as some of the the slight slope of the u-i curve during forward conduction. (Note that a real diode doesn't exactly follow the diode equation: amongst other things, it contains some resistance.) The parallel resistance allows some reverse current to flow; this is proportional to the reverse voltage, which does not fit well with the diode equation ... but at least there *is* some reverse current!

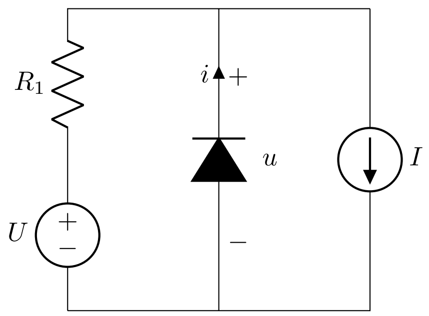
As with opamp models, you will see that there are several distinct differences between the ideal and nonideal models. There are therefore many ways of modelling an nonideal diode. None of the models is a perfect description of the real component. Some go even further than the diode equation, considering that the real diode doesn't behave like just an ideal semiconductor junction. Some are just an ideal diode with one other component. The best choice of model for a real-world problem depends on the diode and on how it will be used.

2.2 Circuit solution with a diode

To solve a circuit of linear components and an ideal diode, one only needs to decide whether the diode is a short-circuit or open circuit. Then replace the diode with that component, and solve.

If there are multiple sources, it might not be obvious which direction the current is 'trying' to go. One approach is to guess: replace the diode with a short or open circuit, then solve the circuit and see if the guess fits with the solution. For example, if you guess open-circuit, but then find a voltage in the forward direction, you know the diode would actually be forward biased and therefore a short-circuit. On the other hand, if you guess open-circuit then find a reverse voltage, you know you were right, and your solution is the correct one for the circuit.

What condition is there on the component values R_1, U and I in the following circuit, in order for the diode to be forward biased (behaving as a short-circuit)?

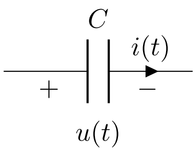


Another method, in a circuit with just one diode, is to model the rest of a circuit as a two-terminal equivalent such as a Thevenin source. Then, if numeric values are known, it is immediately clear which direction the current would 'try' to go in the diode, and therefore whether the diode is conducting or not.

3 Reactive components

3.1 Circuit definitions

The *capacitor* (sv: *kondensator*) that we consider is an idealised circuit model — as usual! It is a component with two terminals, drawn with the following symbol, where C denotes the *capacitance* (sv: *kapacitans*).

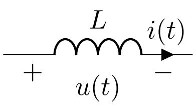


The relation between its current and voltage is

$$i(t) = C\,\frac{\mathrm{d}u(t)}{\mathrm{d}t}. \qquad (1)$$

In this symbol and equation, the current and voltage are explicitly shown as time-functions. Later we might get more sloppy (slarvig) and not bother always writing this "(t)".

The ideal *inductor* (sv: *spole*) is also a component with two terminals, drawn with the following symbol, where L denotes the *inductance* (sv: *induktans*).



The relation between its current and voltage is

$$u(t) = L\,\frac{\mathrm{d}i(t)}{\mathrm{d}t}. \qquad (2)$$

These components have a similarity to a resistor, in that the component value does not specify a particular voltage or current, but specifies a relation *between* these variables,

$$u(t) = R\ i(t).$$

The important difference compared to the resistor is that these relations now include a time-derivative.

Notice an important detail about the above diagrams and equations for $u(t)$ and $i(t)$ in a capacitor and inductor: the current's reference direction was defined into the positive side of the voltage's reference direction. For a resistor, this definition results in a positive resistance, $R = u/i$. For a capacitor or inductor this also gives positive equations such as (1) and (2). If one of the definition directions had been swapped, then a negative sign would be needed in the equations. You should be able to check the sign by thinking physically, without just having to remember a rule about arrows. The potential of one terminal of a capacitor will rise, $\frac{\mathrm{d}u}{\mathrm{d}t} > 0$, if more positive charge is put on it. The voltage induced in an inductor will be in the direction to "oppose the change of current that causes it" (Lenz's law), so an increasing magnitude of current will result in a higher potential on the terminal where that current goes in to the inductor.

Swapping which circuit quantity is expressed in terms of the other, we could write

$$\begin{aligned} u(t) &= u(t_0) + \frac{1}{C}\int_{t_0}^{t} i(x)\,\mathrm{d}x && (3)\\ i(t) &= i(t_0) + \frac{1}{L}\int_{t_0}^{t} u(x)\,\mathrm{d}x && (4)\\ i(t) &= \frac{1}{R}\,u(t), \end{aligned}$$

where t_0 is some chosen time at which we know the value of the sought quantity, and x is a help-variable for integrating.

The time-derivatives describing the capacitor and inductor have been integrated out. The maths tells us to add an integration constant, such as $u(t_0)$ for the capacitor. Physical reasoning explains that (1) and (2) only tell us how much the quantities have been *changing* over some period; the actual value at a point in that period is also needed in order for (3) and (4) to define exactly what values the capacitor's voltage or inductor's current have.

The equations (3) and (4) clearly show that the capacitor voltage and inductor current depend on the *past* values of capacitor current and inductor voltage. In contrast, the equations (1) and (2) show that capacitor current and inductor voltage are determined only by the *present* rates of change of capacitor voltage and inductor current.

Sometimes capacitors and inductors are called *reactive* components. They cannot just continue supplying power for ever, in the way that sources can. But they are not just passive: they *can* supply energy, if that energy was earlier supplied to them. In this way, these components "react" by being able to accept and return energy.

3.2 Physical details

The capacitor and inductor symbols in a circuit diagram represent electric and magnetic fields in a physical circuit. Electric fields in a physical circuit correspond to a build-up of charge "at the ends" of the field; a change in this charge is a current in the capacitor model. Magnetic fields exist around currents, and a change in the magnetic field causes a voltage in the loop that the current flows in; this relation is described by the inductor model. Please see the 'Extra' section in this Chapter for more about the physical nature of capacitors and inductors.

These effects that we can represent with capacitors and inductors in a circuit diagram may be intentional: some physical components are *designed* to have capacitance and inductance ... those are the capacitor and inductor. The effects may instead be unintentional, and perhaps undesirable, as consequences of the construction of a physical circuit: voltages between different points in space, and currents around loops, have *some* extent of electric and magnetic field associated with them. It is particularly when voltages and currents are changing quickly that the effects of capacitance and inductance become more significant; this is implied by the time-derivative terms in the equations (1) and (2).

Reasons for wanting capacitance and inductance in a circuit include energy storage, and 'smoothing' of circuit quantities so that they don't change very rapidly. In Section C of the course we see more general cases of *filters* that achieve special frequency-dependent properties. Reasons for *not* wanting capacitance and inductance in a circuit are generally that they reduce the speed at which changes can happen, and that circuits with both types of component can result in oscillation of energy between the two. The speeds at which digital circuits switch have increased a lot in the recent decades: a significant limitation has been the capacitances and inductances between the signal conductors and to ground.

3.3 Energy

The stored energy W in a capacitor depends on its voltage,

$$W = \frac{1}{2}Cu^2 \tag{5}$$

For the inductor, the same principle applies, with dual variables (swap voltage with current, and capacitance with inductance):

$$W = \frac{1}{2}Li^2 \tag{6}$$

These equations have an appearance that could remind us of kinetic energy. The underlying principle is similar: the more of "something" (charge, or momentum) we put in, the more energy it takes to put in any more.

Consider charging a capacitor. When we start to put in charge, there is no voltage, so we do not need to give energy to the charge to put it there. As the charge on the capacitor increases, the voltage on the capacitor increases, so each new bit of charge needs more energy to force it into the capacitor. In this way, doubling the voltage means doubling the charge ($q = Cu$) *and* doubling the average voltage that the charge had to be given to go into the capacitor: hence four times the energy, as can be seen from the u^2 term.

The expression for energy (5) can be found by integrating the energy involved in charging the capacitor from zero to u. When the charge on the capacitor C is q, the capacitor's voltage is q/C. The energy needed in order to put a small extra charge $\mathrm{d}q$ on the capacitor is then $\frac{q}{C}\mathrm{d}q$. Integrating this from zero up to the voltage u, we get

$$W = \int_0^{uC} \frac{q}{C}\,\mathrm{d}q,$$

which is

$$W = \frac{1}{2}\frac{q^2}{C}\Big|_{q=0}^{uC}$$

and therefore

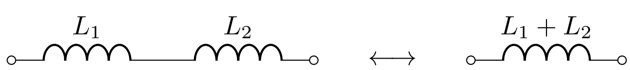
$$W = \frac{1}{2}Cu^2.$$

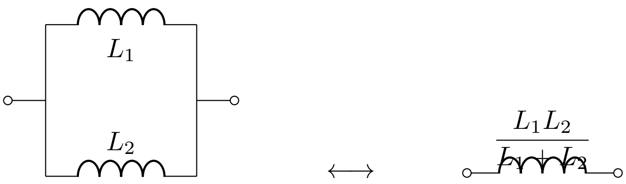
A similar reasoning applies to the inductor.
Have a go!
Invoking the principle of duality is sort-of cheating, although it's actually a very sensible choice of method.

3.4 Combining similar components

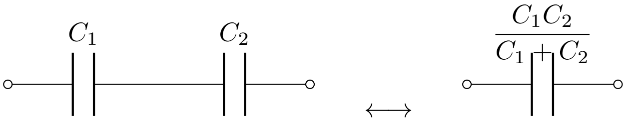
It is useful to be able to combine series or parallel resistors to simplify a circuit. The same idea can be used for inductors for capacitors.

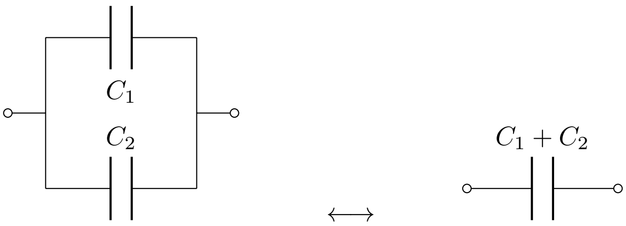
Resistance is $R = u/i$, and inductance is $L = u/\frac{\mathrm{d}i}{\mathrm{d}t}$. The only difference is the time derivative. In order to find an equivalent for series or parallel inductors, the same reasoning can be used as for series or parallel resistors. The only difference is to have "$\frac{\mathrm{d}i}{\mathrm{d}t}$" instead of "$i$" throughout the proof. The result is therefore that series inductors have their inductances added to give an equivalent inductor, and parallel inductors have to be added in reciprocal.





Capacitance, however, is the other way up: current is on the top in $C = i/\frac{\mathrm{d}u}{\mathrm{d}t}$, which is similar to conductance $G = i/u$. Equivalent capacitance therefore follows the same rules as conductance, where the parallel case is simple addition, and the series case requires reciprocals.





This is still quite conceptually easy, if one thinks of capacitance as "coulombs per volt": parallel capacitors will all see the same voltage, and will all store their charge Cu, so the total charge is larger than for just one of the capacitors.

3.5 Inidealities of Capacitors and Inductors

Differences between our ideal components and real things are not a major part of this course, even though they can be of great importance in practice.

But it can be pointed out here that devices *designed* to be capacitors and inductors often include a material that increases the capacitance or inductance for a given size of the component by having permittivity (ε) or permeability (μ) considerably greater than the values for air or vacuum. The presence of materials introduces such features as time-delays, temperature-dependence and energy losses.

Apart from this, any physical construction of capacitor or inductor will have some capacitance (there are points that can have different potentials), some inductance (currents produce magnetic fields), and some resistance (charges are moving through a material such as a metal). It is well known by people working in high frequency electronics that one has to be careful about the design of components. For example, a device that appears to be "just" a capacitor at low frequencies like 50 Hz or 1 kHz may look like basically an inductor and resistor at 10 GHz. This is true even for a component designed to be a resistor. Special care is needed when designing a component that will behave, for example, quite similarly to an ideal capacitor even over a wide range of frequencies.

4 Equilibrium and Continuity

This part is mainly for Topic 07. Here we just introduce the idea of equilibrium.

What is special about the capacitor and inductor is that as well as their *value* (which describes their capacitance or inductance), they can have a stored energy. This influences the circuit, as the energy corresponds to voltage or current (respectively). In this way the reactive components have *two* important properties to consider when doing circuit calculations: the value, *and* the amount of stored energy.

4.1 Equilibrium calculations

Consider a circuit with sources, resistors, capacitors and inductors, where all sources and other components have constant values, and the circuit has been in this situation for "a very long time". We then assume that all the currents and voltages will have reached constant, *equilibrium* (sv: *jämvikt*) values.

This *assumption* is clearly *not* a certainty in the world of idealised circuits: for example, an inductor connected in parallel with a constant voltage source will have a current that just keeps changing forever, $\frac{\mathrm{d}i(t)}{\mathrm{d}t} = U/L =$ constant! Even worse, an inductor or capacitor might be connected to a circuit that behaves as a Thevenin or Norton source with negative resistance: this could result in a change, $\frac{\mathrm{d}i}{\mathrm{d}t}$ or $\frac{\mathrm{d}u}{\mathrm{d}t}$, that keeps getting *bigger* with time.

But such cases are really just amusements found in idealised circuits. In practical cases there will be some resistance between and within the components. This will limit how large the capacitor voltages and inductor currents can become. If an inductor is connected to a voltage source through a resistor, the current will not increase beyond the level where the full voltage of the source is being used to keep pushing this current through the series resistance; at this point, KVL tells us there is no voltage across the inductor to cause a further change of current. If a capacitor is connected to a current source with a parallel resistor, the voltage will not increase beyond the level at which all the source current passes through the resistor and therefore is not charging the capacitor. Idealised circuits, particularly when including controlled sources and negative resistance, are usually only good approximations of real circuits within a moderate range of the circuit quantities. If a voltage or current keeps increasing, then resistances that seemed negligible at low voltages and currents will have to be included in the model, and opamps or transistor outputs will reach supply-voltage limits. In most power-oriented calculations we wouldn't have negative resistance in the model in the first place. The only plausible way for a real circuit to fail to reach an equilibrium is if some circuit quantities *oscillate* instead of reaching a steady value; a permanent oscillation in a linear circuit with constant sources and non-zero resistance requires some sort of controlled source.

We can therefore justifiably make the assumption of steady final values of circuit quantities, if a circuit contains resistances between other components and does not contain dependent sources. Many other circuits even without so many resistors or including controlled sources can also reach steady equilibrium values of voltages and current.

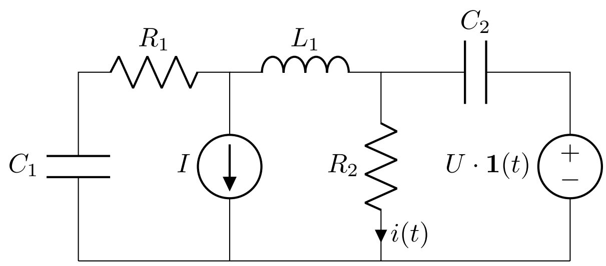
Can we, with our existing knowledge, calculate all the currents and voltages in an equilibrium state, even when there are capacitors and inductors in the circuit? Will this require complicated new methods? (Answers: Yes and No, respectively.)

The method for calculating equilibrium values is to notice that a constant value of a variable means a zero rate of change. If all circuit quantities have reached steady values, then $\frac{\mathrm{d}u}{\mathrm{d}t} = 0$ and $\frac{\mathrm{d}i}{\mathrm{d}t} = 0$ for all the voltages and currents. From the equations defining a capacitor and inductor (1) (2), we see that if $\frac{\mathrm{d}u}{\mathrm{d}t}$ and $\frac{\mathrm{d}i}{\mathrm{d}t}$ are zero, then the corresponding i and u must be zero. For a capacitor, equilibrium will mean that no *current* flows in it: this means we can consider it an open circuit, or a zeroed current source. For an inductor, there will be no *voltage* across it: so we can consider it as a short circuit, or a zeroed voltage-source.

In this way, calculation of equilibrium just requires us to replace capacitors and inductors with open- and short-circuits respectively, and to calculate to find currents and voltages in the circuit. The solution will let us find the capacitors' voltages and inductors' currents, which are the continuous variables.

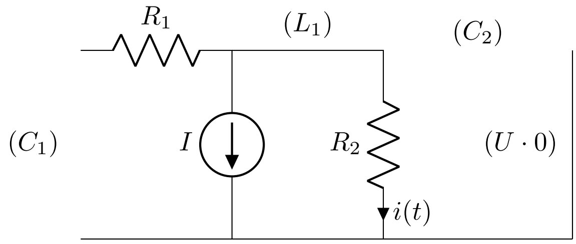
This is very easy! It uses just our existing knowledge of circuit solutions. Replacement of components with open or short circuits results in a simplified circuit-diagram.

The following circuit provides a simple example. Only one change happens, by the voltage source having a step at $t = 0$. Only one variable is being sought: $i(t)$ down resistor R_2.



There are *two* equilibria we could look at. The time "$t = 0^-$" is before the voltage step, and it is assumed that the circuit has been standing "since $t = -\infty$". Just after $t = 0$, the earlier equilibrium has been disturbed by the voltage step, so the equilibrium assumption is not true. But after a very long time, $t \to \infty$, another equilibrium is reached.

In order to solve for the marked current at the initial equilibrium, $i(0^-)$, we can redraw this circuit for time $t = 0^-$ in the simplest possible way. All reactive components can be replaced by open or short circuits, on the assumption of a constant equilibrium state. The voltage source is set to zero (short-circuit) because $\mathbf{1}(0^-) = 0$.



After this redrawing it is very clear what the sought current must be: $i(0^-) = -I$. We see how all the continuous variables could have been found: the current in L_1 is also I (plus or minus, depending on the chosen definition); the voltages across C_2 and across C_1 are the same as across R_2, which is IR_2. Be careful about the current source: remember that its voltage is not known until we calculate it based on what the rest of the circuit demands.

5 — Extra —

5.1 Links

Some Wikipedia links, as usual: [Diode] and [DiodeModelling].

[DiodesChapter] from the Irwin & Nelms book is a good overview of some nonideal properties and simple ways of modelling them by adding linear components to an ideal diode (similar to the description in Petersson).

Classic uses of diodes, capacitors, inductors and controlled switches (transistors or thyristors) are found in many modern electric loads, including "low energy" lamps of the compact fluorescent type. Some examples of the physical construction and the circuit diagram are shown here, [CompactFluorescent]. The function of the electronics is to start the lamp and keep it running. Starting is done by providing current to heat the electrodes at both ends of the glass tube, and generating a high enough voltage to start a discharge in the low pressure gas therein. For continued running, a lower voltage is sufficient. This is provided by rapidly switching (e.g. 30 kHz) a supply to a transformer: the transformer can be very small when used at this high frequency, and any generated sound or light-flicker is not noticed.

5.2 Physical Capacitors and Inductors

Based on equation (1), capacitance has the dimension of $\frac{\text{charge}}{\text{voltage}}$, i.e. (q/u). In SI units this is coulumbs per volt ([A][s][V^{-1}]) which has the special name of farad [F].

Around a charge there is an electric field. In a capacitor, the current that we see coming out of one terminal and going in to the other is causing negative charge to build up in one region inside the capacitor, and positive charge to build up in another region. The electric fields from these two regions can be superposed on each other to find the total electric field in the region between the charges inside the capacitor: the result from this superposition is a stronger field. If we consider a path between the two terminals of a capacitor, it will pass through this region of electric field. Integrating the electric field along this path, we find the difference in potential between the two sides of the capacitor, which is the capacitor's voltage. By another application of superposition, we see that this voltage is proportional to the charge: if the charge density on the electrodes is scaled by some factor, then the voltage must scale by that same factor if superposition holds.

This proportionality of voltage and charge was already suggested by the dimensions of capacitance. It is also seen in the equation $q = Cu$, which is commonly used to define capacitance in school-books. Bearing in mind that $i = \frac{dq}{dt}$, this clearly corresponds to our equation (1).

The two-terminal capacitor symbol obeys KCL: the current into one terminal equals the current out of the other, at any point in time. But this does not mean that *charges* are moving *through* the insulating layer between a physical capacitor's electrodes. In a capacitor with metallic connections, electrons come out of one side (leaving positive charge) and go into the other (accumulating negative charge). So it *looks* as if a current of charges goes through the capacitor. (In fact, even the internal region, with the changing electric field but no charge motion, *behaves* like a current with regard to producing magnetic fields: this is *displacement current*.)

In some cases a capacitor in a circuit model represents a physical component that was deliberately made in order to have capacitance. A design aim of these capacitors is usually a small volume, subject to the constraints of tolerating the required voltage. Some possible reasons for designing circuits to have capacitance may be that energy storage is needed, or that a voltage needs to be prevented from changing quickly (the two are related). In other cases, a capacitor in a circuit model may represent capacitances that exist only unintentionally. Examples are the capacitance between metal tracks on a circuit board, or between wires in power-lines. Another example was shown in the introduction to the course, where a bus parked under a high-voltage power line forms a capacitor between the conductors in the air and the metal body of the bus; another capacitor is formed between the bus and the [quite] conducting ground.

The dimension of inductance, from (2), can be seen to correspond to SI units [V][s][A^{-1}]. This combination has a special name of henry [H]. There is not a widely used word or obvious physical description for "[V][s]" in the way that 'charge' can be used for "[A][s]".

A current causes a magnetic field surrounding it, proportional to the current; this is known as Ampères law, one of Maxwell's equations. The time-derivative of the magnetic field passing through some arbitrary surface in space causes a proportional voltage to be generated in the complete path encircling the edge of that surface; this is Faraday's law, another one of the Maxwell equations. Thus, a changing current in a loop will cause a voltage to be '*induc*ed' in that loop.

In this way, every circuit has inductance! A current will produce a magnetic field; changes in the current will cause changes in the magnetic field; these changes will induce a voltage around the circuit.

In some cases an inductor in a circuit model is modelling a component that was deliberately designed to have high inductance. This is typically a coil with many turns of wire around a magnetic material, in order to get high inductance with small size. In other cases, an inductor in a circuit model may be modelling an inductance that is not there by intention. An example is the inductance that surrounds the conductors of a circuit. As with unwanted capacitances, this is relevant in power lines and cables, and in computer circuit boards and many other places.

Undesired properties of capacitance, inductance and resistance are sometimes called *parasitic* components when included in a circuit model.

5.3 Reverse breakdown of diodes

Something that the simple exponential diode equation doesn't show is that a strong reverse voltage can "break down" a diode and cause a high current to flow. Normal diodes will never work again after this, but special types of diodes called zener and avalanche diodes are designed to start conducting in the reverse direction at a quite precise voltage level [ZenerDiode]. These are commonly used as voltage references, because their reverse voltage is quite similar over a wide range of reverse currents.

An application example is a zener diode supplied with reverse current through a resistor from a poorly controlled voltage source; the quite accurately known voltage across the zener diode can then be used as the reference of an opamp, to force the output of a voltage supply (as used at a laboratory workbench) to this well known value. Protection of sensitive circuits from 'surges' (brief overvoltage) is another common function of diodes with intentional reverse breakdown.