This topic seems hard to introduce. We used this document in VT15, then a revised, handwritten version in VT16. We change it a little each time, trying to get the most important points across in an accessible way. In spite of these efforts it always feels that this is simply a surprisingly hard topic. Perhaps this is because of having more connection to practical questions, or too much terminology to get familiar with. There is in fact no new theory, but just a set of phasor calculations and power calculations. Learning to see and exploit symmetries, and various formula based on them, is an important part.

Topic 13: Three-phase AC systems

All our power-circuit examples so far have been based on power flowing in a pair of wires, where the currents in the two are equal and opposite. This gave us a simple relation such as $S = ui^*$ for the complex power flow (assuming rms values).

You probably have realised that most power lines you see have *three* main conductors, with similar levels of insulation on each.[1] In this Topic we look a little at why "three-phase ac" is practically universal for electric power systems, and then we consider some definitions (terminology) and calculations.

The actual calculations do not *require* any more than your existing knowledge. However, there are convenient ways to handle three-phase calculations, so it is useful to get familiarity with these. We've already become used to the factor $\sqrt{2}$ turning up everywhere ... now it's joined by the factor $\sqrt{3}$. We also need to be familiar with some terminology, such as line- and phase-voltages, and wye and delta connections.

1 Why three-phase ac systems

Some justification of "Why ac" has been given in the Extra part of Topic 12, on transformers. If an ac generator (*alternator*) with a single coil[2] is connected to a resistive load, the output power will vary between zero and a peak value. We have seen this idea already in Topic 10, where the instantaneous power of a sinusoidal source is an oscillation at twice the frequency of the ac voltage or current.[3] This variation of the electrical power coming out of the generator will result in a mechanical oscillation. This time-variation of instantaneous power can be reduced by having another coil, or several coils, at equal spacing around the generator, then connecting loads to each: the varying powers then sum to a constant, so the total power output is smooth. In other situations, such as modern power electronics, it is also useful to have this smoothness of power that is given by a three-phase system, so that converters can give smooth output power without needing large energy-storage components.

Another feature of polyphase ac is that by applying the phase-shifted voltages to a set of coils equally spaced inside *another* machine, a magnetic field is set up that appears to "rotate", i.e. its direction changes smoothly in time. A metal object in this field will have currents induced in it that cause it to be dragged round! This allows a simple, reliable, but quite efficient design of motor (*induction motor* (sv: *asynkronmotor*)), which has been the main form of industrial motor for over a century. That idea came in at least two places around 1890, and was originally based on two phases shifted by 90°.

It isn't actually necessary to have a pair of wires for each phase: one of the two wires from each coil can be connected to a common point (e.g. "ground"), usually called a neutral. It turns out that the currents in the different phases cancel, due to their equal phase-shifts. So the neutral isn't even needed in many cases. This gives a saving of materials, as three wires can do the job that six would have done. Three phases is the minimum number where one gets a cancellation of the currents *and* the smooth power and "rotating field".

2 Typical components

The main source of three-phase power is still three-phase alternators.[4] These are machines with rotating magnets and three sets of stationary coils, which are positioned so that there is a 120° phase-shift between their induced voltages.

For our purposes in circuit analysis, we will see an ideal three-phase voltage source as having three *phase* terminals and possibly a fourth terminal, the *neutral*. The neutral's potential is the mean of the phase potentials at any time. In a public power system this is almost always deliberately connected to earth. We will normally have neutral points connected to ground (reference potential) in our circuits.

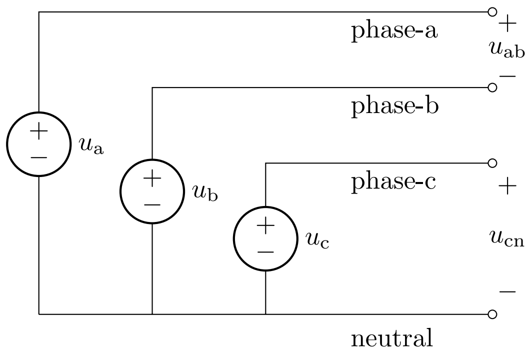
[1] The main exceptions to three-phase ac for high-voltage lines will be HVDC (high-voltage direct current), which usually consists of a balanced positive and negative conductor; these are very unusual compared to ac lines. Another exception special to Sweden, Germany, and a few other countries around them, is rail traction supplies at what is now called 16.7 Hz (it used to be called $16\frac{2}{3}$ Hz, i.e. $\frac{1}{3}\times 50$ Hz) `[link]`. These traction supplies are in some regions linked together by a grid at a higher voltage, e.g. 130 kV, than is used for the actual railway wires, using just a pair of conductors.

[2] "Winding": complete set of turns of wire in a machine such as a motor, generator or transformer, with two wires coming out to connect. (The word is used in various ways, e.g. sometimes for all the wires in some region of the machine.)

[3] The output doesn't have to be a sinusoid; it won't be perfectly so. But there are practical constraints against making or using near-squarewave voltages and currents.

[4] Another source is power-electronic converters, where three output voltages with controlled magnitude and phase are synthesised by rapidly switching them between different dc levels, and filtering away the high-frequency changes due to the switching. Converters are increasingly important for interfacing dc sources such as photovoltaic cells to the grid. They also interface the ac system with high-voltage dc systems that are used for transmitting power over long subsea cables, or very long land distances: hot-topic examples are offshore wind farms in the north sea, remote hydro power in India, and new long-distance connections in China.

The following diagram shows how we can draw an ideal three-phase source, including a neutral terminal, using just the normal two-terminal voltage sources that we are familiar with.



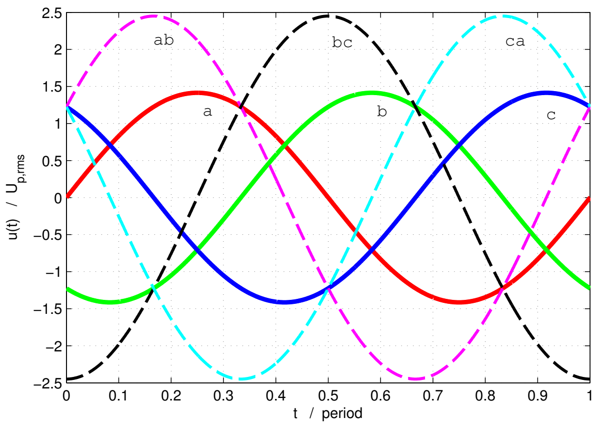
The three source-voltages can be described by the following phasors,

$$\begin{aligned} u_{\mathrm{a}} &= U_{\mathrm{p}}\angle 0^\circ \\ u_{\mathrm{b}} &= U_{\mathrm{p}}\angle -120^\circ \\ u_{\mathrm{c}} &= U_{\mathrm{p}}\angle -240^\circ \end{aligned} \tag{1}$$

where U_{p} is the rms magnitude of the "phase-voltage" between one of the phase outputs and the neutral. Using rms values is strongly the convention in power subjects, so that we don't have to keep writing a factor $1/2$ when calculating power.

The set of voltages defined above (1) has *phase-sequence* (sv: *fasföljd*) abc.[5]

The following figure shows these voltages as time-functions over one cycle. The values are normalised to the rms of the individual sources, U_{p}.



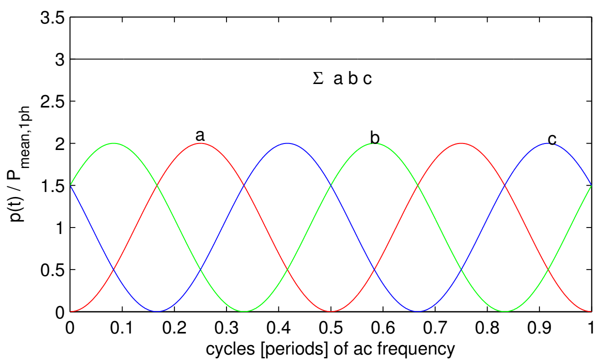
After phase-a has had, for example, a positive peak, then the phase that has its positive peak soonest afterwards is phase-b, and then phase-c. There are only two possibilities for phase-sequence: abc and acb (you could also write e.g. cba, but this is the same sequence as acb if it keeps repeating).

The voltage between phase-c and the neutral is marked on the diagram as u_{cn}. It is common to use this double subscript to make clear that it is phase-c relative to the neutral; a shorter form of u_{c} is sometimes used. The voltage between phase-a and phase-b is marked as u_{ab}. There are six different voltages that we can measure between pairs of these four conductors.

By symmetry, there are only two different *magnitudes*. One magnitude U_{p} has already been defined, for the voltages from any phase to neutral. The other is the magnitude of voltage between different phases. However, the voltages all have different *phase* (angle).

Loads, in our calculations, are usually represented as impedances. It is often reasonable and useful to model a three-phase load as three similar impedances, which form a *balanced load*. Machines designed for three-phase operation will usually have a quite similar impedance on each phase. Loads consisting of lots of single-phase loads will usually have a quite similar amount of load on each phase at a given time.

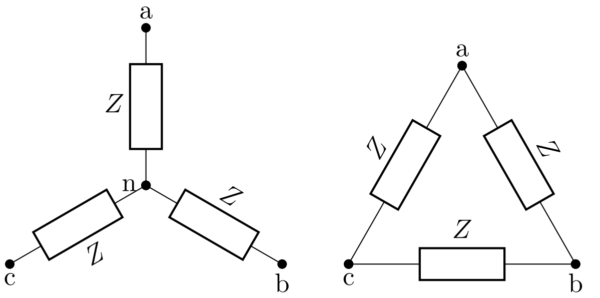
Consider three equal resistances connected between the three phases and neutral, of a source with voltages defined as in the previous plot. The instantaneous powers in the resistors, and the total instantaneous power, are shown in the following figure.



We can note how the individual phases have powers oscillating between zero and twice their mean value, but the total power input at any time is a constant of three times the mean value of each phase.

A further question is then how the impedances are connected. There are two ways to connect three similar impedances to a three-phase source in a symmetrical (balanced) way.

[5] Various sets of letters are used to denote the phases: the abc choice, or colours red/yellow/blue or red/green/blue are common particularly in English-language use. One common European choice is RST. For marking the two ends of each phase-coil in a motor, one sometimes uses RST (one end) and UVW (other end).



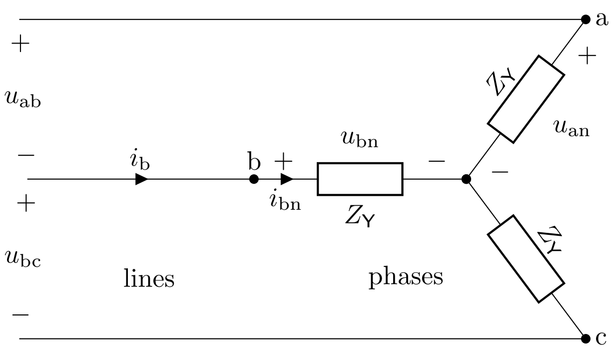
The one the left is the same as was used for the voltage sources that we looked at earlier. This type of connection can be called *star wye* (Y), based on its shape. Each terminal connects to one end of just one of the impedances. The other ends of the impedances connect together, to a point that has been labelled 'n' for neutral: this is sometimes called the *star point*.

The other type of connection is shown on the right. This is called a delta (Δ) or mesh connection. Each impedance connects between a pair of phases. There is no possible neutral-point. Each impedance is exposed to the full line-voltage. The current into each terminal is the phasor sum of the currents in the two impedances that connect to that terminal.

Now we can look at some terminology, which also will help to show the relations between voltages and currents across the impedances and in the supply wires. In the following diagrams, there is a balanced load modelled by three impedances, and there are three wires (nodes) supplying this load from a three-phase source that is assumed at the left.

The impedances can be called the *phases*; the voltages across them, and the currents through them, can be called the phase values. This terminology is normal for people who work with the loads: each impedance is one "phase" of their device, such as a coil in a motor, or a resistance-wire in a heater. The voltages between the supply wires, and the currents through these wires, can then be called the *line* values, referring to the power line.

The diagram below shows three equal impedances forming a Y-connected load, with an ideal three-phase voltage source assumed at the left.



The current through each phase of the Y-connected load is clearly the same as the current in the supply line that is connected to it,

$$i_b = i_{bn}$$

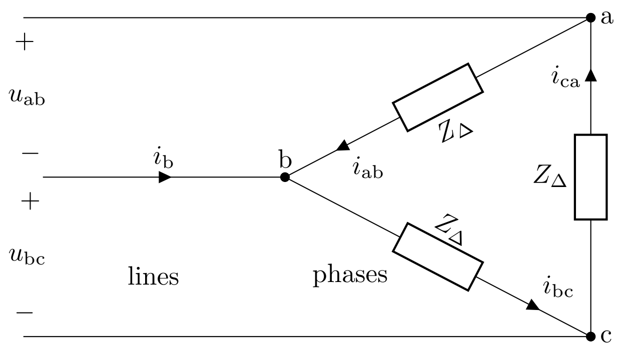
The voltage between two supply lines is different from the voltage across any phase of the load. For example

$$u_{ab} = u_{an} - u_{bn} \quad \text{(KVL)}$$

This can be seen in the earlier time-plot of voltages. The magnitude $|u_{an} - u_{bn}|$ is greater than $|u_{an}|$ or $|u_{bn}|$, due to the angle between these voltages. Noting that the star point is at neutral potential (due to symmetry), the magnitude of the voltage is the same across each phase, and is the value U_p that was defined when we defined the source. The voltage between two lines can be calculated by phasor subtraction. It turns out that the magnitude of this line voltage is $U_l = \sqrt{3}U_p$. So

$$|u_{ab}| = \sqrt{3}|u_{an}|.$$

The next diagram shows three equal impedances forming a Δ-connected load.



Now the situation is reversed: each phase of the load has a voltage equal to the line voltage. But each line current is a phasor sum of the two phase-currents of the impedances that connect to the line. For example,

$$i_b = i_{bc} - i_{ab}.$$

This again gives a factor of $\sqrt{3}$: for example,

$$|i_b| = \sqrt{3}|i_{bc}|$$

The complex power to each impedance in a Y-connected load is $U_p^2/Z_Y{}^*$. The total complex power to a balanced Y-connected load is therefore

$$S = \frac{3U_p^2}{Z_Y{}^*} = \frac{U_l^2}{Z_Y{}^*}.$$

The complex power to each impedance in a Δ-connected load is $U_l^2/Z_\Delta{}^*$. The total complex power to a balanced Δ-connected load is therefore

$$S = \frac{3U_l^2}{Z_\Delta{}^*} = \frac{9U_p^2}{Z_\Delta{}^*}.$$

Two useful results come from the above.

One is that by setting $Z_\Delta = 3Z_\mathsf{Y}$, the balanced delta or star connected load will have the same power.

The other is that, if we know the magnitudes of balanced line voltage U_l and line current I_l to an unknown (but balanced) load, the apparent power can be calculated as $|S| = \sqrt{3}U_\mathrm{l}I_\mathrm{l}$. This is a standard part of three-phase calculation: for example, what maximum design current would be expected from each terminal of a three-phase transformer that is rated at 1 MVA, 400 V (line-voltage)? Answer, $|i| = |S|/(\sqrt{3}U) = 1.44\,\mathrm{kA}$. The complex power can be found if the phase angle between a voltage and current can also be measured.

4-wire or 3-wire Y connection. A Y-connected load can have its star-point (centre) floating (connected only to the three impedances) or connected to a neutral that leads back to the source. In the latter case, this point is held to a fixed potential even if the load or source is not balanced. Calculation is easier if we know this point's potential: then we know each Y-connected impedance's voltage. If the load and source are balanced, then by symmetry we can argue that the star-point is at the same potential as the neutral. To be more formal, node analysis can be used: define the start-point of a balanced load as having potential V. KCL gives $0 = \frac{u_\mathrm{a}-V}{Z} + \frac{u_\mathrm{b}-V}{Z} + \frac{u_\mathrm{c}-V}{Z}$. So $V = \frac{u_\mathrm{a}+u_\mathrm{b}+u_\mathrm{c}}{3}$, and the definition of a balanced three-phase voltage source with grounded neutral gives us that $V = \frac{U_\mathrm{p}}{3}\left(1\angle 0 + 1\angle{-120°} + 1\angle{-240°}\right) = 0$, with the zero coming from simplifying the summed complex numbers.

> Confusing names.
>
> The following description is for background information. It should *not be a problem in our course*. It will be made very clear in exams which voltages or currents are being discussed, by marking them on diagrams and referring to them as e.g. u_bn.
>
> We've just looked at a common meaning of *phase* and *line* values of voltage and current, typical when distinguishing between loads and the electricity network.
>
> Sometimes, people working with systems (networks) instead call the voltage from one phase to neutral the phase voltage. In this way, the terms *phase voltage* (sv: *fasspänning*) and *line voltage* (sv: *huvudspänning*) are used to distinguish what we'd have called the line-neutral and line-line voltages in our earlier definition. The two definitions would only disagree if discussing a Δ-connected load. Part of the reason for this network-oriented definition is that it's common to call the different wires the phases (a synonym for lines). The word *phase* is already confusing, as we've used it for the angle of a sinusoidal signal, or for a coil or set of conductors!
>
> Summary: the words line-voltage and line-current have clear definition; phase-voltage and phase-current might be used differently by different groups. Introductory textbooks usually use the first of the definitions above, that we used when studying the Y and Δ connections.

3 The toolkit

As was already mentioned, there's nothing fundamentally new about "three phase systems" when you've already studied ac circuits and power. There's just a bit of extra terminology, which has already been described: Y and Δ connections, phase- and line-voltages and currents.

There are a few methods that can be helpful as shortcuts.

3.1 Symmetry

Symmetry is probably the most important shortcut.

If we have balanced loads (same impedance) from each phase to neutral, then we know the currents are of similar magnitude but displaced from each other by 120°. Then we can immediately say they sum to zero in the neutral.

If we have a balanced load with a particular complex power, it does not matter whether the load is actually three impedances Z in Y connection, or three impedances $3Z$ in a Δ connection: the currents into the loads will be the same. This can be useful when solving a system with Y and Δ loads, where it's easier to combine them with line (connection) impedances and each other if they are, e.g. in Y.

A balanced Y load does not actually "use" its neutral. By symmetry you can see that no neutral current will flow, and even if the connection to the star-point (the centre node of the Y) is removed, that point's potential will stay at zero. That potential can be calculated by node analysis, or just seen from symmetry. Node analysis is useful if needing to calculate neutral potential for an unconnected neutral in an unbalanced load.

3.2 Power

It is often simple to think in terms of complex power; this is a "conserved quantity" (what goes in must come out). So the sum of active powers consumed by loads, and consumed in any resistance of the connections between the loads, must equal the active powers provided by any sources. The same is true for reactive powers.

A very suitable case for power-based calculations is when lots of load impedances are connected in various ways to a three-phase source. If the voltage *magnitude* is known, across a known impedance, that is enough to calculate the complex power in the impedance, i.e. $S = |u|^2/Z^*$. The same is true if current magnitude is known. In this way, the actual phase-angle of the current or voltage in each element of a Δ or Y load is not important, as the complex power can be found by just knowing the voltage magnitudes.

In balanced three-phase systems, the complex power in one of the three impedances of a balanced load can be calculated then multiplied by three, to find the total complex power.

This is a convenient way to, for example, add powers of several balanced three-phase loads, then design power-factor compensation in Δ or Y. When series impedance in the supply system has to be included, then it can be more difficult to apply a power-based calculation with rigour, since the voltages might not be the same for all loads. Voltage division can be done between series impedances and loads, but the complex numbers have to be considered: phase angles are important for finding the magnitudes of voltages and currents. It is not hard with a computer, but it would quickly become messy using pure symbolic manipulation.

In Q7 of the 2013-03 [Exam] it was convenient to think in terms of power or a Y-Δ transformation to find the part if i_b due to the balanced Δ load. If working from normal circuit analysis, you would find the currents in the two impedances Z that connect phase-b to phases a and c. By the quick method, it's enough to see that a star with impedances $Z/3$ is equivalent, and therefore that the current into phase b of the balanced load is $u_\mathrm{b}/(Z/3)$.

3.3 Splitting balanced and unbalanced

The symmetry methods are based on balanced conditions, but there are cases where there is some unbalanced load. It may still be profitable to analyse the balanced part in a simple way, then add the unbalanced part.

3.4 Common factors, angles and surds

For the cases where you have to do some phase-based calculation by paper and pen, instead of using symmetry and power and computers, it is useful to be familiar with the commonly occuring angles and their sine and cosine. For example,
$\cos(60^\circ) = \sin(30^\circ) = 1/2 = 0.5$
$\cos(30^\circ) = \sin(60^\circ) = \sqrt{3}/2 \simeq 0.866$
$\cos(45^\circ) = \sin(45^\circ) = 1/\sqrt{2} \simeq 0.707$.

The line-line voltages (or line voltage) are $\sqrt{3}$ the magnitude of the line-neutral voltage (sometimes called phase-voltage). The currents into a set of 3 Δ-connected elements are $\sqrt{3}$ the magnitude of the currents in the separate elements.

For some people, phasor diagrams (draw phasors in the complex plane) are very useful, for thinking about angles and addition or subtraction of phasors.

4 Final word!

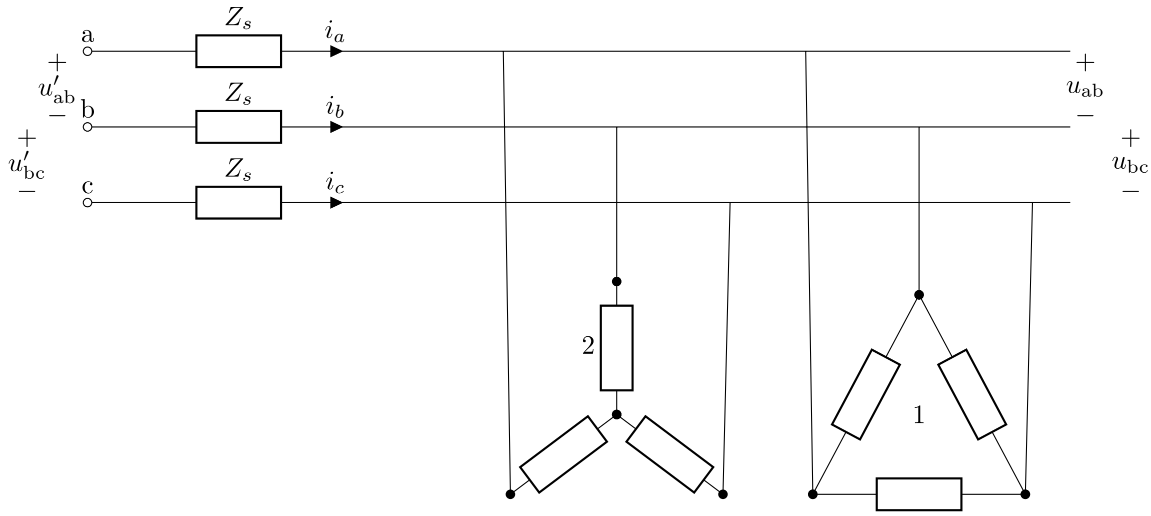
However simple the basic concept of 'three-phase system' might be, as an add-on within the already existing background of ac, ac power, etc, it is quite difficult to get used to the power calculations in some strange unbalanced cases.

You can try writing everything from first principles, and thereby create a lot of equations; or you try thinking of how to make it simpler, but realise that it's hard and not always obvious what *is* the right thing to do. It's not always easy, even for people who've done it for years. So don't be too disheartened if you think it's hard: you're not alone. People who use the concepts regularly in their jobs will tend to know well the solutions of the few special cases they tend to meet, which can be different for different types of study (and they probably use computers for much of the work).

5 Example: three-phase solution

This solution was requested in 2014, for a question in a textbook that provided no solution.

The circuit (problem 12.7-7 8th-ed *Introduction to Electric Circuits*, Dorf/Svoboda) is the following:



Load 1 is: Δ-connected, 1.8 MVA apparent power at PF = 0.9 lagging.
Load 2 is: Y-connected, 2.4 MW active power at PF = 0.96 lagging.
The supply's series impedance is $Z_s = (0.48 + 0.96\mathrm{j})\,\Omega$ in each phase.
We'll assume balanced loads, and a balanced three-phase voltage source at terminals a,b,c.
The loads "require 5 kV rms". I'll assume that's a line-voltage.
It seems we're supposed to assume the line-voltage *at* the load is this value, so $U_{\mathrm{L}} = |u_{\mathrm{ab}}| = |u_{\mathrm{bc}}| = |u_{\mathrm{ca}}| = 5\,\mathrm{kV}$.

Find:
a) The required magnitude of line-voltage at the supply (e.g. $|u'_{\mathrm{ab}}|$ in the diagram above).
b) The active power from the supply.
c) The proportion of the active power from the supply (part 'b') that reaches the loads.

It is useful to find the loads as complex powers; then the current phasors can easily be found. Although we only care about voltage *magnitudes* for the final answer, the calculations have to be done as phasors, since the magnitude of a sum of complex numbers (e.g. $|x + y|$) depends on the angles as well as the magnitudes of these numbers.

```
% load 1
s1  = 1.8e6   % apparent power
pf1 = 0.9   % power factor
P1  = pf1 * s1   % active power
Q1  = sqrt(s1^2 - P1^2)   % reactive power
% (note: Q1 should be positive because pf1 was lagging, so it consumes
%  reactive power: if pf1 had been leading, we'd make Q1 negative)
S1 = P1 + 1j*Q1   % complex power

% load 2
P2 = 2.4e6
pf2 = 0.96
s2 = P2 / pf2
Q2 = sqrt(s2^2 - P2^2)
S2 = P2 + 1j*Q2

% total complex power of both loads
Sl = S1 + S2

% define other given information:
% line voltage at loads (rms, of course...)
Ul = 5e3
% supply impedance
```

```
Zs = 0.48 + 0.96j
```

5.1 First method

a) The voltage at the loads is their correct voltage, so they should be consuming the stated power. From this, we can immediately work out the currents taken by the loads together. This current, in the supply's impedance, causes a voltage across the supply impedance, which is added to the voltage at the load to find the voltage at the source.

```
% take voltage from phase-a to neutral, at the load, as
% the reference phase (purely real)
uan = Ul/sqrt(3)

% current ia, based on line-neutral voltage u_an as reference phase
ia = conj( (S1/3) / uan )
% the above should make obvious where this expression comes from:
% one third of the complex power is supplied in each phase, and the
% phase-neutral voltage is 1/sqrt(3) of the line voltage;
% phases b and c will be the same, with 120degree shifts (balanced)

% voltage at source (left of diagram) is sum of load voltage and ia*Zs
uan_s = uan + ia*Zs

% line voltage magnitude at source is thus
Uls_s = abs( sqrt(3)*uan_s )
% which is about 5.7 kV with the given numbers
```

b&c) The real power from the supply is the sum of real powers in all phases; these are equal as the system is balanced, so we can calculate in phase-a alone then multiply by 3.

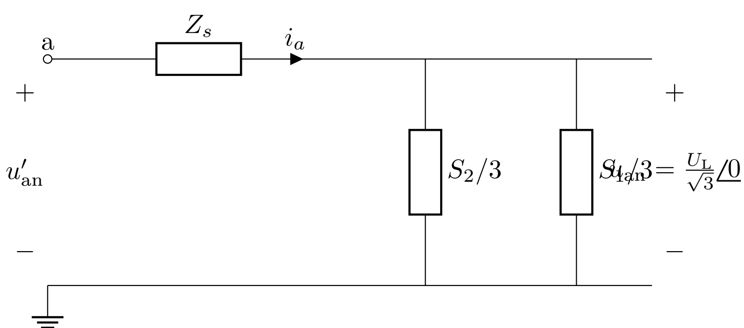
```
Ssa = uan_s * conj(ia)  % complex power into phase-a at source
Ss = 3 * Ssa    % total complex power at source
Ps = real(Ss)   % real power at source

eta = (P1+P2) / Ps
%    about 92% of the source's power gets to the load in this case
```

5.2 Alternative method

This is very similar to the above, but is more explicit about using a single-phase equivalent for the balanced three-phase calculation. This might help you to understand better why the calculations are made in this way.

To get a single-phase equivalent, we can replace the above diagram with the following:



Let's name the diagram's variable u'_{an} as `uan_s` in our computer program. We assume the variables `S1` and `S2` are already defined, as we did at the start. Then,

```
uan = 5e3/sqrt(3)
ia = conj( (S1/3 + S2/3) / uan )
uan_s = uan + ia*Zs
```

```
% line-voltage at source (in actual, 3-phase circuit)
Ul_s = sqrt(3) * abs(uan_s)
% power from source (for all three phases)
P_s = real( 3 * uan_s * conj(ia) )      % ~4.37 MW
eta = (P1+P2) / Ps    % ~92%
```

Notice how the power-based balanced three-phase calculation didn't require us to use the information about Y- or Δ-connection of the loads.

And another method: Impedances

The above diagrams and calculations did not define load impedances; they just showed generic two-terminal loads with a given power. We were not told that the loads behaved like fixed impedances; for example, some motor loads can take almost the same active power even if the voltage drops by 5%, which is not the behaviour of a simple impedance. The reason we didn't need to care is that we were told the voltage and the power at the load, so we can find the current.

We could have got the same result by assuming the loads to be impedances, given the assumption of the known voltage at the load.

But then it's important to remember that if we start asking questions about "what happens if the voltage at the load changes", then the results will be wrong unless we were correct to assume the load to be an impedance.

For example, we might want to assume the *supply* voltage u' is fixed, and we want to find the load voltage. Then we would need to know how the loads' currents depend on voltage. When loads don't act as impedances, these types of question can (in some cases) lead to nonlinear equations.

Anyway: let's assume impedances, and work with Y-connected impedances so that we can directly use them in the single-phase equivalent. Then

```
% reminder of the rated phase-voltage of the loads
Up = 5e3/sqrt(3)
uan = Up
% load impedances (per phase, if Y-connected)
Z1 = conj( Up^2 / (S1/3) )
Z2 = conj( Up^2 / (S2/3) )
% total parallel impedance (of one phase)
Zl = 1/( 1/Z1 + 1/Z2 )

% voltage-division equation
% uan = uan_s *  Zl / ( Zl + Zs ) ... therefore,
uan_s = uan * ( Zl + Zs ) / Zl

% line-voltage at source (in actual, 3-phase circuit)
Ul_s = sqrt(3) * abs(uan_s)     % ~ 5.71 kV (line)
```

And for a little extra sport, we see how easy (with the help of a computer) it is to treat the more difficult problem of a fixed supply voltage and unknown load voltage, assuming the loads behave like impedances,

```
uan_s = 5e3/sqrt(3)
uan = uan_s *  Zl / ( Zl + Zs )
Ul = sqrt(3)*abs(uan)    % about 4.38 kV (line)
```

6 — Extra —

6.1 Some example waveforms

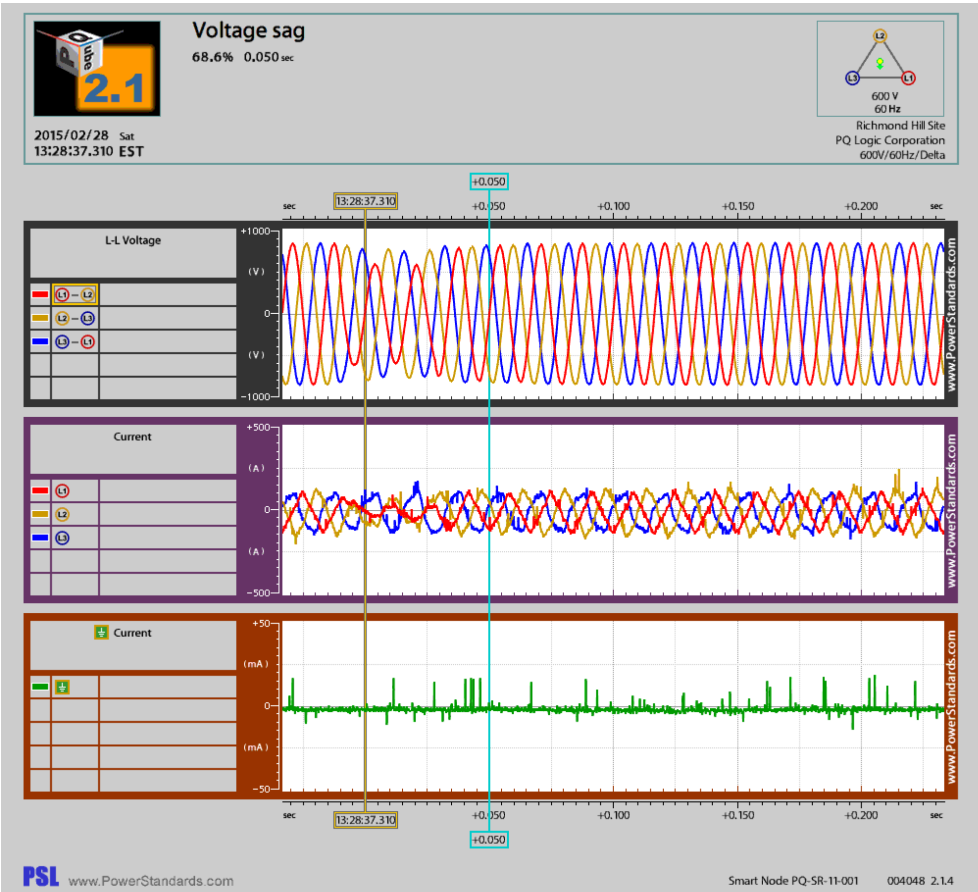
This is rather fun. A company that makes "power quality monitors" has a few customers that make their measurements public. So you can see some different voltage standards (230 V, 120 V, etc), see some waveforms of voltage and current, and see these in several different languages. It's admittedly not very helpful to the exam in our course, but it gives some indication of how three-phase concepts of "L1-L2 voltage" and so on are fundamental in practical discussions of power networks.

See the Map. Yellow dots with a number indicate multiple sites that can be seen if you zoom. Click on a site with green or flashing-yellow light.

The link 'Meters' will then show some rms values such as line voltages (e.g. L1-L2) and phase voltages (e.g. L3-N), then possibly currents (if it monitors current), harmonic distortion (how nonsinusoidal the ac voltage or current is), and some powers. Generally you'll see how well balanced the voltage is between phases.

The link 'Events' might show some further links, in which case (if lucky) there might be some images with names ending in `Waveform.gif` or `Waveform.png`. These get produced when some abnormality is detected: commonly this is a brief reduction in voltage magnitudes (a 'sag'). Many users are interested to analyse why their machines or computers have problems: the instrumentation helps them find out what the problems are and where they probably came from. If you find a lucky set of waveforms, you'll see three-phase voltages and currents, be able to see that the peaks are about $\sqrt{2}$ of the declared rms, see that the normal currents tend to be not good sinusoids (distortion from nonlinear loads such as computers, modern lamps, etc), and perhaps see the 'event' that the instrument had detected.

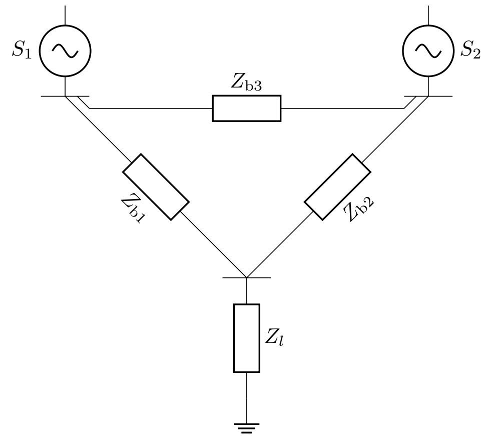
One recent example is shown below, with all these features, on a system rated at 600 V.



6.2 Single-line diagrams

A *single-line diagram* (SLD) is a common way to represent connections and loads in a balanced three-phase system. It uses just one line to represent all three phases and a possible neutral. It is assumed that all loads and generators have balanced three phase voltage and current; all series impedances representing cables, nonideal transformers, etc, are assumed to be similar impedance in each phase.

A small example is shown below, where a three-phase generator is shown as a wave inside a circle. The "nodes" really represent three or four separate connections (each phase and a possible neutral) to each line or component that terminates at the node.



In this way, one makes calculations as if on a single-phase circuit. It's just occasionally necessary to remember that in fact the system is a three-phase one – for example, if you wonder why a meter tells you it's 230 kV to earth, whereas you were calling it a 400 kV line. By a normalization called the power system "per-unit" method, even the factor of $\sqrt{3}$ when calculating powers is not needed.

6.3 Balanced and Unbalanced

Many power-system calculations are made on the assumption of balanced three-phase operation. When the calculated system is big, it naturally becomes unlikely that the loads would become very different between the phases; so assuming balanced loads is often very reasonable. Most lines and transformers do not behave quite like balanced impedances (which would look exactly the same for each phase) but they can be approximated as balanced for a lot of studies.

Major reasons for needing a more general type of solution, for unbalanced conditions in a three-phase system, are faults (e.g. short-circuit) that affect phases unequally.

An old and widely used way to calculate on unbalanced three-phase systems is a *symmetric (sequence) components* transformation. In a commonly used basic form, some arbitrary state of sinusoidal voltage and current (*not* necessarily balanced) is represented as a superposition of three sets of voltage and current: one is ideal balanced, another is ideal balanced but with the opposite sequence (e.g. acb instead of abc for the time-order of the phases), and the other has no phase-shift between the phases.

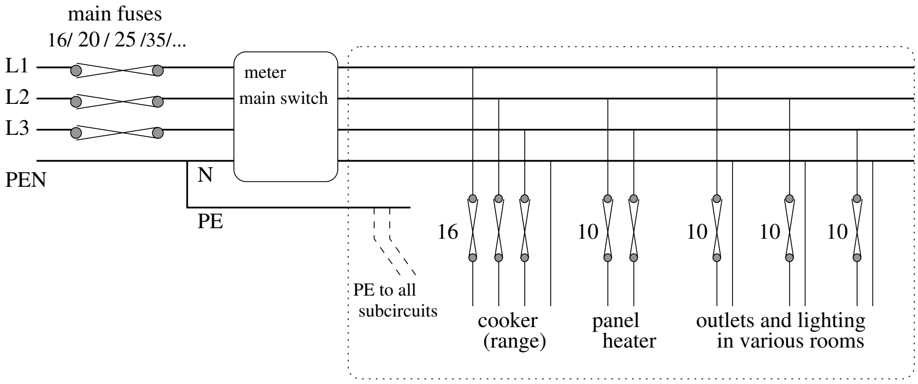
One rather nice simple visualiser to allow comparing the actual abc quantities with the positive-, negative- and zero-sequence sets is here `[link]`. You might like its beauty even if you don't quite get the point! (This method is very central to a lot of analysis of power systems and fault conditions.)

6.4 Supply to homes

There are quite big variations between countries about the choice of how many conductors (and what voltage) to bring to a home.

In Sweden (and I think all the other Nordic countries, and Germany and several others in Eastern Europe) it is normal to bring three phases and a neutral into each home. Then the main fuses can each be quite small (e.g. 20 A) and some larger loads can be supplied between two phases (to get a higher voltage and therefore a lower current) or from all three phases together.

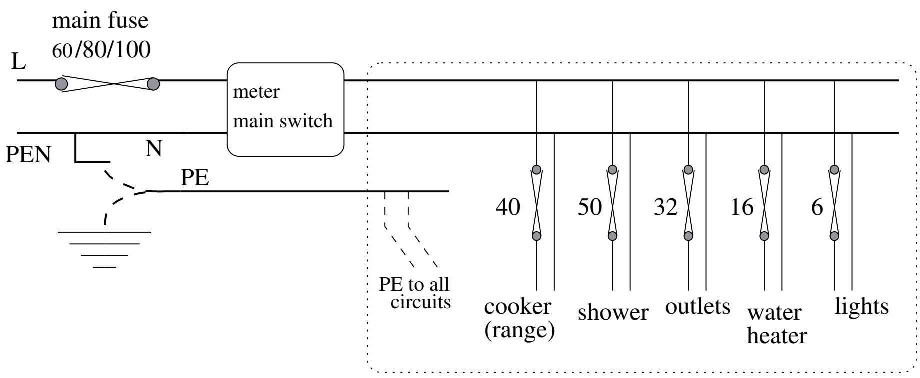
In the following diagram, 'L' denotes line conductors, 'N' the neutral (mid-point: *sv: nolledare*), and 'PE' the protective earth (*skyddsjord*). In the aforementioned countries it's common to keep the neutral and earth as one conductor until the intake to a building, or even as far as the fuseboard (*elcentral*).



As a matter of interest, Norway has a high proportion of public supply networks with the unusual feature of being three-phase without a neutral. The *line*-voltage is about 230 V, so a normal socket outlet for '230 V' is connected *between* phases. The voltage between any phase and earth could be about 130 V. ('About' is written here because they also do interesting things with not directly earthing any conductor of the supply, which means that the potential of all three phases might not be symmetric around earth potential, depending on what impedances, such as cable insulation capacitance and leakage current through damaged insulation, are connected.)

(In the laboratory room that we use in this course, a similar system is used, from a transformer at the back of the room. This converts the normal 400 V line voltage to a 230 V line voltage. The workbench sockets are then supplied with two phases, so 230 V is available to the things we plug into the sockets, but there is only 130 V to earth. An advantage is that there is less of a shock risk. In contrast to the public supplies in Norway, the lab's source has an earthed neutral. In the lab there are also unusually sensitive earth-leakage breakers (*jordfelsskydd*) that detect 10 mA 'going missing' in the circuit. By 'going missing' we mean passing back to the transformer through a path that is not one of the intended live conductors: e.g. it may be through the protective earth wires, or through someone's body. They were very thoughtful in designing that lab: safety beyond the normal levels!)

In some other countries, including France, UK, Australia and New Zealand, it's common to have a three-phase system in the street, but take one phase and neutral to each home. Then the main fuse is bigger. Wiring looks simpler, but large loads have two thick wires instead of three thinner ones.



Note that a given load on a three-phase connection would need each of its three wires to carry only 1/3 of the current compared to each of the two wires if the same load power is supplied from a single phase and neutral (from a system with the same three-phase voltage). In the above diagram, many countries would have multiple 16 A circuits for socket outlets, but in the UK it's common to have one to three bigger circuits to many sockets, then to have smaller fuses in the plugs.

In North America and some of its surroundings (central, parts of southern America, Japan), it's usual to have 120 V instead of 230 V as a normal rating for socket outlets, lamps, etc. This is commonly provided by a 240 V single-phase transformer with the centre of its winding connected to ground (neutral). Then some things are connected from one wire to neutral, and others from the other wire to neutral. Their currents can cancel in the neutral, as these two other wires have 180° phase-shift. Larger loads can connect between the outer wires, to get 240 V. So a supply with two 'hot wires' each with a 100 A fuses, can give the same power as a single-phase supply with 100 A fuse in e.g. Europe, or a three-phase supply with 32 A (ok, 33.3 A, but 32 A and 35 A are common actual values of fuses). Now they sometimes have 200 A fuses on the supply in the US: there's a lot of electricity use!