Tentamen: EI1120 Elkretsanalys (CENMI), 2013-06-03, kl 08–13

Hjälpmedel: *Ett* A4-ark med studentens anteckningar (båda sidor). Dessutom, pennor!

Tentan har 7 tal: 2 i del A (10p), 2 i del B (12p) och 3 i del C (18p).
Obs: Samma tal står här först **på engelska** (s.1–s.3) och sedan **på svenska** (s.4–s.6).
Du får välja mellan dessa språk för svaren.

Läs varje tal noggrant **innan du försöker svara**.
Tänk på att **använda återstående tid till att kolla på varje svar**: man kan göra dimensionsanalys, rimlighetsbedömning (t.ex. "är det rätt att y går ner medan x går ner?"), och lösning genom en alternativ metod.
Lösningar ska **förenklas** om inte annat är specificerat.
Var försiktig med att **inte satsa för mycket tid** på bara en uppgift om du fastnar: ta hänsyn till poängvärden av uppgifterna, och att man måste klara varje del av tentan. Det är ofta så att senare deltal är betydligt svårare än de första deltalen.

Godkänt vid ≥50% på del A, ≥25% på del B och på C, och ≥50% på delar B och C tillsammans.
Godkänd kontrollskrivning gör att man redan klarat del A här på tentan.
Betyget räknas utifrån resultatet på B och C delarna: det finns därför ingen fördel med att svara på A-delen om man har godkänd KS, eller att försöka få mycket hög poäng i A-delen.
Eventuella bonuspoäng från KS och hemuppgifter tillkommer enligt KursPM. Se också PM:et angående rättningsnormer och överklagande. Instruktionerna ovan tar prioritet över PM vid skillnad (t.ex. hjälpmedel).

Examinator: Nathaniel Taylor

In English

Part A. DC (static solutions). NOT NEEDED IF KS PASSED: SEE NOTES ABOVE.

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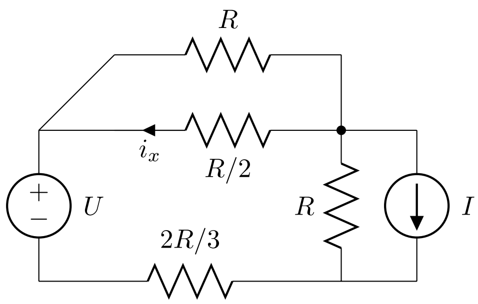
1) [5p]

The current source I has value $I = U/R$.
Solve for the current marked i_x, in the resistor of value $R/2$.
Answer in terms of the known quantities U and R.



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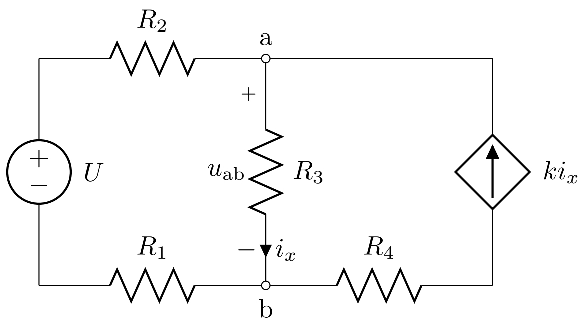
2) [5p]

In this circuit the unknown current i_x is marked; it controls a dependent current source with gain k.

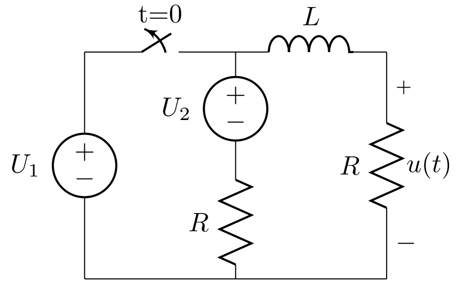
a) [3p] Use nodal analysis to determine the voltage u_{ab} between points 'a' and 'b'.
There is no single 'right' way of doing the analysis: for example, you can choose which node is defined as zero potential (ground), and whether you simplify the diagram first or instead do the simplifications in the algebra.



b) [2p] Find the Thevenin equivalent between nodes 'a' and 'b'.
(This should the equivalent for the entire circuit shown above, not just for the parts to the left or the right of nodes 'a' and 'b'.)

Part B. Transient analysis

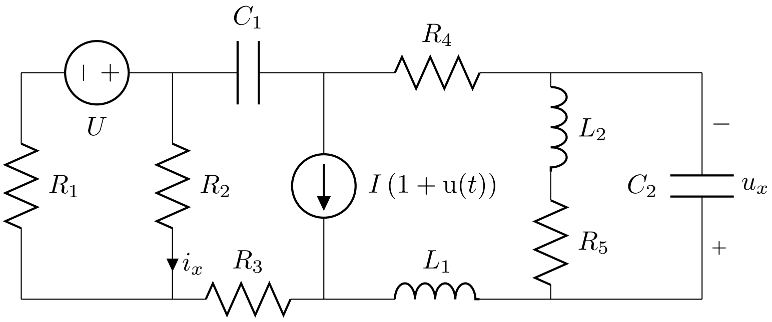
3) [6p]



Find $u(t)$ for all time $t > 0$.
Note that the switch turns *off* (becomes an open-circuit) at $t = 0$.

(Assume the circuit has reached an equilibrium before $t = 0$.)

4) [6p]



All these components have known values. All are constant *except* the current source, which has a step-change in output at $t = 0$.
Note: $\mathrm{u}(t)$ is the unit step.
The quantities u_x and i_x are unknown.

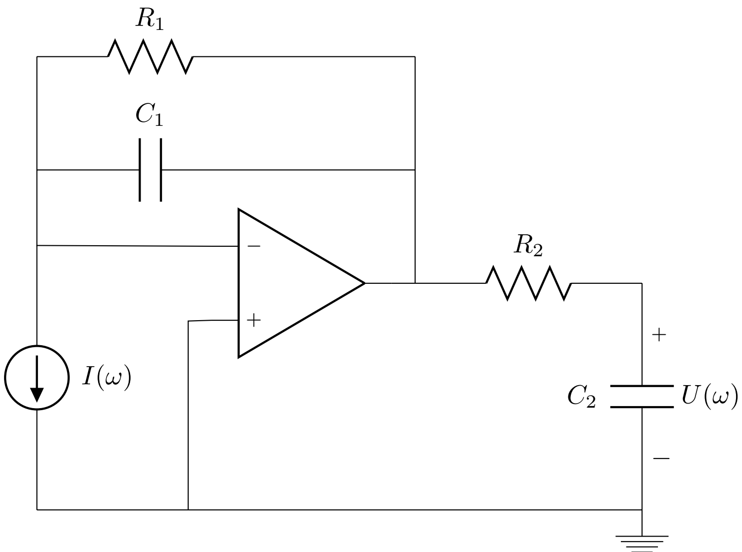
a) [2p] Find u_x and i_x just *before* time $t = 0$, i.e. the equilibrium when the current source has a value of just I.

b) [3p] Find u_x and i_x just *after* time $t = 0$, i.e. "$t = 0^+$" when the current source has just changed its current but the stored energies in reactive components (L and C) have not changed.

c) [1p] Find u_x and i_x as $t \to \infty$.

Part C. AC (sinusoidal steady state)

5) [6p]

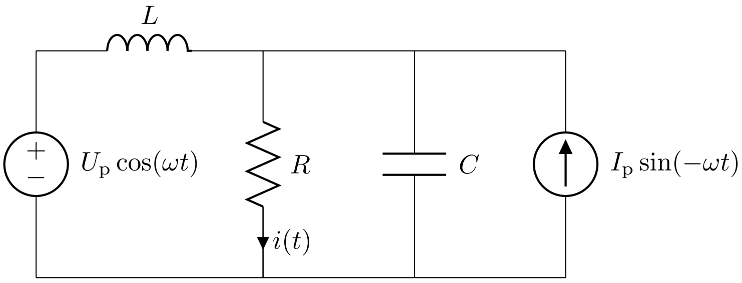


a) [3p] Derive the network function $H(\omega) = U(\omega)/I(\omega)$, for the circuit shown on the right, expressed in known quantities R_1, C_1, R_2, C_2, I. Assume the opamp is ideal.

b) [1p] Perform a dimensional check of the solution to part a).

c) [2p] Sketch a Bode amplitude plot of $H(\omega)$. This should include the dc (0 Hz) gain, the gradients (dB/decade), and the corner frequencies; you can assume $R_1C_1 \ll R_2C_2$.

6) [6p]



a) [3p] In the circuit to the right, use steady-state sinusoidal analysis ("jω-method") to find $i(t)$.

b) [2p] Determine the power into resistor R from the rest of the circuit, as a function of time. The answer to part a) may be useful.

c) [1p] Calculate the "complex power" into R (i.e. the frequency-domain way of describing power as a complex function of angular frequency ω, in contrast to the time-domain way used in part b)).

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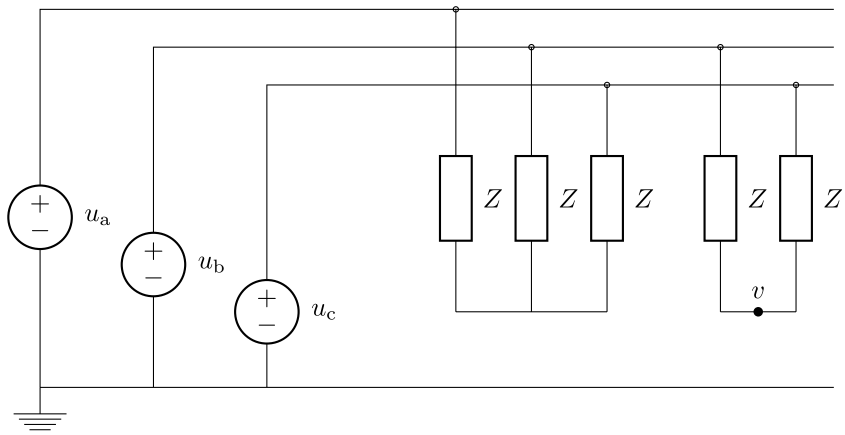
7) [6p]



The diagram above shows an ideal three-phase voltage-source supplying a balanced three-phase load and an unbalanced load. The source has angular frequency ω, rms phase-voltage magnitude U_p, and phase-sequence *abc*.

We will work entirely in the frequency domain, taking u_a as the reference phase (0°) and using rms values. This means for example that the phasor for the c-phase voltage is $u_c = U_p\underline{/-4\pi/3}$ (or equivalently, $U_p\underline{/2\pi/3}$).

The balanced load consists three impedances $Z = R + j\omega L$ in a 'Y'-connection. The unbalanced load consists of two impedances Z (the same value as in the balanced load) connected in series between two phases.

a) [1p] What is the *magnitude* of the voltage across each impedance Z in the *balanced* load? (You are not required to show a derivation for this.)

b) [1p] What is the total complex power into the balanced load? Express the answer in U_p, R, L and ω.

c) [1p] What is the *magnitude* of the voltage across each impedance Z in the *unbalanced* load?

d) [2p] What is the total *active* power into the unbalanced load? Express the answer in U_p, R, L and ω.

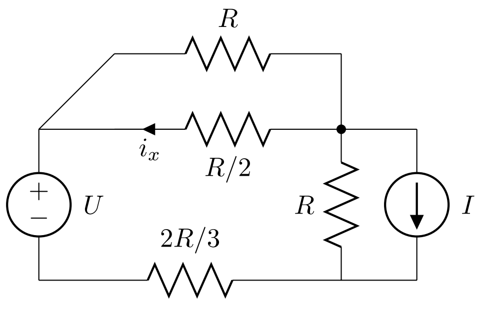
e) [1p] What is the potential v (as a polar complex number: magnitude and angle) of the node between the two impedances in the unbalanced load?

END of exam. Don't forget to use remaining time to check all your answers in every imaginable way!

På svenska (samma problem)

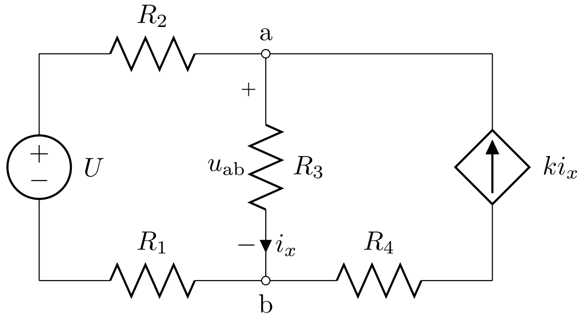
Del A. Likström. Behövs INTE vid godkänd KS: Se sida 1.

1) [5p]



Strömkällan I har värdet $I = U/R$.
Bestäm strömmen i_x, som går i resistorn $R/2$.
Ge svaret i de kända storheterna U och R.

2) [5p]



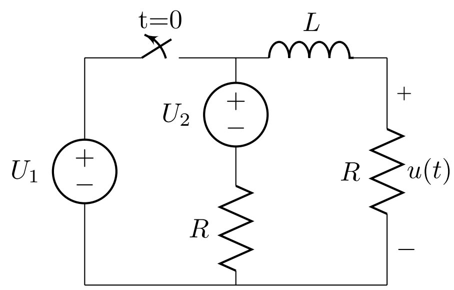
I denna krets är den okända strömmen i_x markerad; den styr en beroende strömkälla med känd förstärkning k.

a) [3p] Använd nodanalys för att bestämma spänningen u_{ab} mellan noderna 'a' och 'b'.
Det finns inte bara ett rätt sätt att göra analysen: man kan, t.ex. fritt välja vilken nod som definieras som jordnoden, och kan förenkla kretsen i diagrammet eller senare inom ekvationerna.

b) [2p] Bestäm Theveninekvivalenten mellan noderna 'a' och 'b'. (Denna är en ekvivalent till hela kretsen som står i diagrammet, inte bara till den vänstra eller högre sidan av 'a' och 'b'.)

Del B. Transientanalys

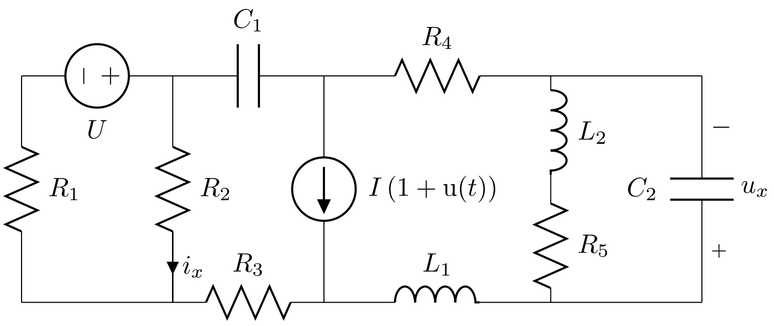
3) [6p]



Bestäm $u(t)$ som funktion av tid, för tider $t > 0$.
Observera att brytaren *öppnas* (blir oppenkrets) vid tiden $t = 0$.

(Anta att kretsen har kommit till jämviktsläge innan $t = 0$.)

4) [6p]



Alla dessa komponenter har kända värde. Alla är konstanta *förutom* strömkällan, vilken har en diskret ändring i sitt värde vid $t = 0$.
Obs: $\mathrm{u}(t)$ är enhetsstegfunktionen.
Kvantiteterna u_x och i_x är okända.

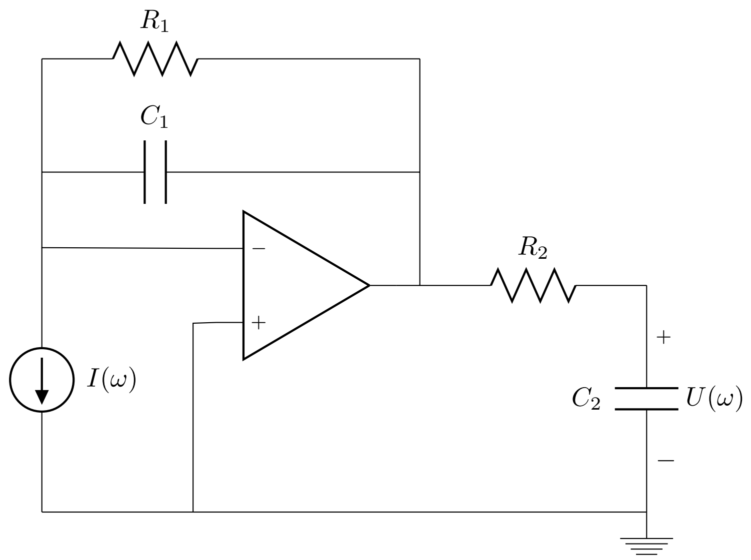
a) [2p] Bestäm u_x och i_x just *innan* tiden $t = 0$, d.v.s. jämviktsläget när strömkällan har värdet I.

b) [3p] Bestäm u_x och i_x precis *efter* tiden $t = 0$, d.v.s. "$t = 0^+$" när strömkällan har ett nytt värde av ström, men de lagrade energierna i de reaktiva komponenterna (L och C) har inte hunnit ändras.

c) [1p] Bestäm u_x och i_x när $t \to \infty$.

Del C. Stationärväxelström

5) [6p]

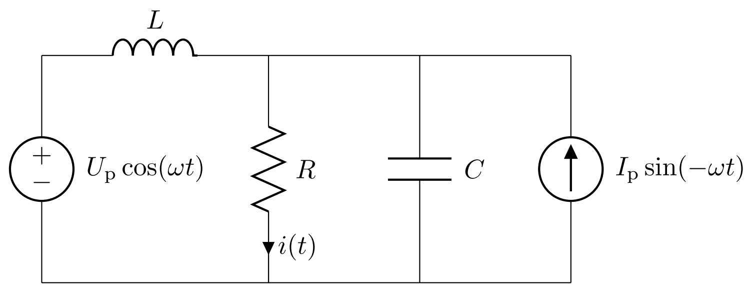


a) [3p] Bestäm nätverksfunktionen $H(\omega) = U(\omega)/I(\omega)$, for denna krets, uttryckt i kända variabler R_1, C_1, R_2, C_2, I. Anta att operationsförstärkaren är idéal.

b) [1p] Gör dimensionskoll på lösningen till del a).

c) [2p] Skissa ett Bode amplitud diagram av $H(\omega)$. Diagrammet ska även visa brytpunkter, lutningar (dB/dekad), och förstärkning vid 0 Hz; anta $R_1C_1 \ll R_2C_2$.

6) [6p]

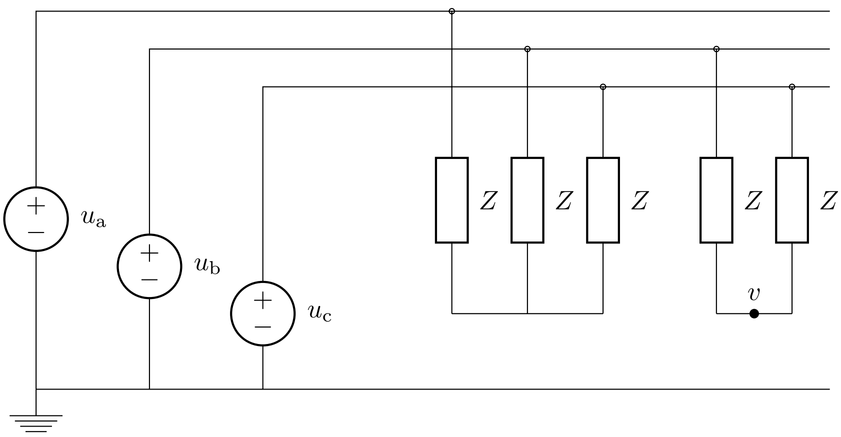


a) [3p] Använd "jω-metoden" för att bestämma $i(t)$.

b) [2p] Bestäm tidsfunktionen som beskriver effekten in till R från resten av kretsen. Svaret till del a) kan vara användbart här.

c) [1p] Bestäm komplexeffekten som går in i R (d.v.s. beräkna nu i det vanliga sättet för frekvensdomänen, till skillnad från del 'b)' som var gjort i tidsdomänen).

7) [6p]



Diagrammet ovan visar en ideal trefas spänningskälla vilken försörjer en symmetrisk last och en asymmetrisk last. Källan har vinkelfrekvensen ω; absolutbeloppet av fasspänningen är U_p (effektivvärde), och fasföljden är *abc*.

Vi räknar helt i frekvensdomänen, med u_a som referensfas (0°), och i effektivvärdeskalan. Därför är till exempel visaren för spänningen mellan fas-c och noll-ledaren $u_c(\omega) = U_p\underline{/-4\pi/3}$ (eller, ekvivalent, $U_p\underline{/2\pi/3}$).

Den symmetriska lasten har tre Y-kopplade impedanser $Z = R + j\omega L$. Den asymmetriska lasten har två impedanser, också av det samma värdet Z, serieanslutna med varandra och kopplade mellan faser b och c.

a) [1p] Vad är *absolutbeloppet* av spänningen över varje impedans Z i den *balanserade* lasten? (Du behöver inte visa härledningen av svaret.)

b) [1p] Vilken total komplexeffekt går in till den balanserade lasten?
Uttryck lösningen i kvantiteterna U_p, R, L och ω.

c) [1p] Vad är *absolutbeloppet* av spänningen över varje impedans Z i den *obalanserade* lasten?

d) [2p] Vilken total *aktiv*effekt går in till den obalanserade lasten?
Uttryck lösningen i kvantiteterna U_p, R, L och ω.

e) [1p] Vad är potentialen v (som ett komplextal i polärform: absolutbelopp och vinkel) av noden mellan de två impedanserna i den obalanserade lasten?

Slutet av tentamen. Glöm ej att använda återstående tid för att dubbelkolla svaren!

Solutions

1)
There are several reasonable methods for solving this. Nodal analysis can be used without simplifying the circuit: there are 3 nodes plus a ground. The current i_x can then be found from the voltage across the resistor $2R/3$. However, note that the only potentials actually needed to solve the problem are those at the two nodes at the ends of the resistor $R/2$; this guides us about what circuit-simplifications might be profitable before doing the nodal analysis. A source conversion of the current source ($I = U/R$) in parallel with resistor R gives us a voltage source of U and a series resistor R. Then the four original components I, R, $2R/3$ and U can be reduced to a single branch of a voltage source of $2U$ and series resistor of $5R/3$. Let's define as ground the node into which i_x flows, and define the dotted node above R as an unknown potential, V. Using the simplified branch derived above, KCL in node V gives $0 = \frac{V+2U}{5R/3} + \frac{V}{R} + \frac{V}{R/2}$, whence $V = -U/3$. The current i_x is therefore $i_x = (-U/3)/(R/2) = -\frac{2}{3}\frac{U}{R}$.
Another method is to reduce the entire circuit to a loop, using source conversion (as above) converting parallel resistors $R/2$ and R to an equivalent of $R/3$. The loop simplifies to an equivalent voltage source of $2U$ and a total resistance of $2R$, so the current (passing clockwise through U and $2R/3$ in the original circuit) is U/R. The current i_x is then found by current division, giving the above solution of $i_x = -\frac{2}{3}\frac{U}{R}$.

2)
a) Let node 'b' be ground. Let the potential of node 'a' be V. KCL into 'a' gives: $0 = \frac{U-V}{R_1+R_2} - \frac{V}{R_3} + ki_x$.
Express i_x in node potentials to give $0 = \frac{U-V}{R_1+R_2} - \frac{V}{R_3} + \frac{kV}{R_3}$.
Due to our definition of node V and the ground node, $V = u_{\rm ab}$. Thus, $u_{\rm ab} = V = U/\left(\frac{R_1+R_2}{R_3}(1-k)+1\right)$.
b) We already have the open-circuit voltage of the Thevenin equivalent, from part 'a)'. Now we need the Thevenin impedance, or else the short-circuit current from which we can calculate this impedance. When there is a dependent source in the circuit, it's more reliable to calculate short-circuit current than to look directly for the "impedance of the circuit seen from its terminals with all independent sources set to zero". In our case the short-circuit current is easily found: a short-circuit between 'a' and 'b' forces voltage $u_{\rm ab}$ to zero, and therefore the current i_x and the current in the dependent current-source are both zero. The only contribution to short-circuit current is therefore the left branch: $i_{\rm sc} = U/\left(R_1+R_2\right)$. The Thevenin impedance is $u_{\rm ab}/i_{\rm sc}$, giving $R_{\rm t} = 1/\left(\frac{1-k}{R_3} + \frac{1}{R_1+R_2}\right)$.
A diagram should be drawn to show the series voltage source and resistor with these values.

3)
The only reactive component is L, so we expect a simple first-order response. After the switch opens, the relevant part of the circuit is just the loop of R, U_2, L, R. Define current i clockwise in this loop: KVL gives that $L\frac{{\rm d}i}{{\rm d}t} + 2Ri = U_2$. We also need to find the initial condition. At $t = 0^-$ the switch is still on, so the voltage source U_1 fixes the voltage across the series branch of L and R. The equilibrium current in L is therefore $i_{\rm L}(t=0^-) = U_1/R$, as the inductor behaves as a short-circuit in the equilibrium; this must also be the current at $t = 0^+$. Given this differential equation and the initial condition, and noting that $u(t) = Ri(t)$, the full solution for $t > 0$ is $u(t) = U_1 + (U_2/2 - U_1)(1 - \exp(-2Rt/L))$, or in shorter form, $u(t) = U_2/2 + (U_1 - U_2/2)\exp(-2Rt/L)$.

4)
a) Case $t = 0^-$. Equilibrium means C_1 is not carrying current; therefore R_3 cannot either carry current (there is not a full circuit for the current to flow). So the parts of the circuit at the left and right of C_1 and R_3 can be treated separately. In this case, clearly $i_x = \frac{U}{R_1+R_2}$.
If replacing the inductors and capacitors on the right with the short- and open-circuits, it is clear that a current of I has to pass through R_5 and therefore that $u_x = IR_5$.
b) Case $t = 0^+$. Now we are no longer in steady state: the current source has just changed its current to $2I$. We cannot therefore assume that inductor voltages and capacitor currents are zero. We must use the fact that inductor *currents* and capacitor *voltages* are the same as in the previous case, since they have not had time to change. Thus, the current in L_1 is still I (left to right). The current in R_3 must therefore be the new source-current minus this: i.e. a current I flows right to left in R_3, and back through C_1. The change in i_x is probably most simply found by superposition: it is the original value plus a contribution $-I\frac{R_1}{R_1+R_2}$ found by current-division of the extra current I, i.e. $i_x = \frac{U}{R_1+R_2} - \frac{IR_1}{R_1+R_2}$.
As capacitor voltages cannot change instantaneously, u_x is unchanged: $u_x = IR_5$.
c) Case $t \to \infty$. This is easy because as it's the same situation as in part 'a)', but with a different value of the current source ($2I$ instead of I). Thus $i_x = \frac{U}{R_1+R_2}$ and $u_x = 2IR_5$.

5)
a) One good method here is nodal analysis, as long as we are careful *not* to attempt KCL on the node of the opamp output! (Remember, the opamp output is really like a controlled voltage source, where the other side is connected to ground but is hidden from the diagram for simplicity.)
The opamp is ideal and has negative feedback, so we assume the the potentials of both of its inputs are the same: thus, the node at the top of the current source is at zero potential (virtual ground). The node above C_2 has unknown

potential that is defined in the diagram as U. Let us define the opamp's output node as potential V. At the node of the opamp's inverting input, KCL gives $-I + V(\frac{1}{R_1} + \mathrm{j}\omega C_1) = 0$. In the (trivial) node of potential U, KCL gives $(V-U)/R_2 - \mathrm{j}\omega C_2 U = 0$. Therefore, $V = I\frac{R_1}{1+\mathrm{j}\omega C_1 R_1}$, and $U = V\frac{1}{1+\mathrm{j}\omega C_2 R_2}$. Putting these together to eliminate V (which isn't needed in the solution), $H = \frac{U}{I} = \frac{R_1}{(1+\mathrm{j}\omega C_1 R_1)(1+\mathrm{j}\omega C_2 R_2)}$.

b) Dimensions! The left-hand side (lhs) of the above solution, U/I, is an impedance. On the right-hand side, the numerator is an impedance, and the denominator is dimensionless: that fits with the lhs. Check: expect ωCR to be dimensionless (as it's added to a pure number, 1): CR is a time-constant, and ω is an [angular-]frequency, so the product is indeed dimensionless. In written solution to the exam, this analysis is best done by writing the equation and showing that the terms on either side of each plus, minus or equality symbol have the same dimension.

c) Both stages of this circuit form single-pole low-pass filters. As $\omega \to 0$ the output voltage tends to IR_1, so the dc gain $|H(0)|$ is R_1, which in dB is $20\log_{10} R_1$. The pole [angular-]frequencies are $1/(R_1C_1)$ and $1/(R_2C_2)$: when the frequency passes from below to above each of these, the gain starts changing by a further $-20\,\mathrm{dB/decade}$. So the plot should have three slopes: $0\,\mathrm{dB/dec}$ at low frequency, then $-20\,\mathrm{dB/dec}$ in the range $\frac{1}{R_2C_2} < \omega < \frac{1}{R_1C_1}$, and $-40\,\mathrm{dB/dec}$ at frequencies above this range. (This is clearly a low-pass filter.)

6)

a) Let's be conventional by using a cosine reference. This time we'll follow the convention of communications people and use the peak of the sinusoid as the magnitude of the phasor (instead of the common power-engineering practice where phasors' magnitudes are rms values). These choices makes no difference to the solution, as long as we are consistent. The voltage source is then $U_\mathrm{p}\underline{/0}$, and the current source is $I_\mathrm{p}\underline{/\pi/2}$ or equivalently $\mathrm{j}I_\mathrm{p}$.

Define the node above R to have potential V, and the node below R to be ground; the trivial node between the voltage source and inductor can be ignored by treating both of these components together as a single branch. By KCL in the upper node, $\frac{U_\mathrm{p}-V}{\mathrm{j}\omega L} - \frac{V}{R} - \mathrm{j}\omega CV + \mathrm{j}I_\mathrm{p} = 0$. Thus, $V = \mathrm{j}\frac{I_\mathrm{p}-U_\mathrm{p}/(\omega L)}{1/(\mathrm{j}\omega L)+1/R+\mathrm{j}\omega C}$, and dividing by R gives $i(\omega) = \mathrm{j}\frac{I_\mathrm{p}-U_\mathrm{p}/(\omega L)}{1+\mathrm{j}(\omega CR - R/(\omega L))}$. Converting back to the time-domain with the same conventions (peak value, and cosine reference) the magnitude and angle of $i(\omega)$ give $i(t) = \frac{I_\mathrm{p}-U_\mathrm{p}/(\omega L)}{\sqrt{1+(\omega CR - R/(\omega L))^2}} \cos\left(\omega t + \pi/2 - \tan^{-1}\left(\omega CR - R/(\omega L)\right)\right)$.

b) Power in a resistor R is, at every instant, given by the product of resistance and the square of the current: $P(t) = i^2(t)R$. From the relation that $A^2\cos^2(\theta) = \frac{A^2}{2}(1+\cos(2\theta))$, the solution to part 'a)' can be adapted, $P(t) = \frac{R}{2}\frac{\left(I_\mathrm{p}-U_\mathrm{p}/(\omega L)\right)^2}{1+(\omega CR - R/(\omega L))^2}\left[1 - \cos\left(2\omega t - 2\tan^{-1}\left(\omega CR - R/(\omega L)\right)\right)\right]$.

c) Complex power is defined in terms of complex rms voltage and current as $S = UI^*$. Using the relation $U = IR$, this gives $S = RII^*$, or $S = R|I|^2$. We know $|I|$ from part 'a)', but we've used peak values so we should divide I by $\sqrt{2}$ (or divide I^2 by 2), to give $S = \frac{R}{2}\frac{\left(I_\mathrm{p}-U_\mathrm{p}/(\omega L)\right)^2}{1+(\omega CR - R/(\omega L))^2}$. The result is purely real: this is to be expected, as a linear resistor has no reactive power consumption.

7)

a) Answer: U_p. Explanation (not required): The common node of the balanced load (the "star point" or "neutral point") will have zero potential. This can be seen by symmetry, or by using KCL on that node and solving for its potential. The voltage across each impedance is therefore the voltage of the voltage-source that it is connected to, which means a magnitude of U_p. Another way of thinking is that the source and load are both Y-connected.

b) Complex power into an impedance $R+\mathrm{j}\omega L$ due to a voltage of rms magnitude U_p is $S = \frac{U_\mathrm{p}^2}{R^2+\omega^2L^2}(R+\mathrm{j}\omega L)$. This can be seen by starting from $S = UI^*$, as in question 6c), but substituting out the I instead of the U, then multiplying top and bottom by Z^*. There are three impedances, each with this complex power. Therefore, $S_\mathrm{balanced} = \frac{3U_\mathrm{p}^2}{R^2+\omega^2L^2}(R+\mathrm{j}\omega L)$.

c) The unbalanced load is two impedances of Z connected in series from phase to phase: it is therefore exposed to the line voltage, which has magnitude $\sqrt{3}U_\mathrm{p}$, so each of the two impedances has $\frac{\sqrt{3}}{2}U_\mathrm{p}$ by voltage division.

d) Using the general result from part 'b', for a single impedance, we know the complex power into the unbalanced load will be $S = \frac{3U_\mathrm{p}^2}{2\left(R^2+\omega^2L^2\right)}(R+\mathrm{j}\omega L)$. Note the extra '2' in the denominator, because of the impedance being $2Z$, and the '3' in the numerator because of the impedances being connected across the line voltage. We want, however, only the real part (the *active* power). That's easy, because the above formula is already arranged to separate the real and imaginary parts: we just take one term, $P = \frac{3U_\mathrm{p}^2}{2\left(R^2+\omega^2L^2\right)}R$.

Note: an alternative way to express the apparent power would be $\frac{3U_\mathrm{p}^2}{2(R-\mathrm{j}\omega L)}$; one should then be careful to avoid the fundamental but common mistake of claiming that "$P = \frac{3U_\mathrm{p}^2}{2R}$" [false!].

e) One way to calculate this is nodal-style analysis: KCL in point v gives $\frac{u_\mathrm{b}-v}{Z} + \frac{u_\mathrm{c}-v}{Z} = 0$. Thus $v = \frac{u_\mathrm{b}+u_\mathrm{c}}{2}$ which is $\frac{U_\mathrm{p}}{2}\left(1\underline{/2\pi/3} + 1\underline{/-2\pi/3}\right)$ and can be simplified to $-U_\mathrm{p}$, i.e. $v = \frac{U_\mathrm{p}}{2}\underline{/\pi}$.