EI1120 Elkretsanalys, Kontrollskrivning KS2, 2015-02-16 kl. 10–12

Hjälpmedel: Ett A4-ark med godtyckligt innehåll (handskriven, datorutskrift, diagram, m.m.).

Kontrollskrivningen har 2 tal, med totalt 10 poäng. Den omfattar ämnet 'Transient' och motsvarar del B i tentamen. Det högre av betygen från KS2 (den här) och från tentans del B kommer att användas vid betygsättning av tentan. Del B är godkänt vid ≥40%, men glöm inte att tentan kräver också minst 50% räknat över talen i alla tre delar.

Om inte annan information anges i ett tal, ska: komponenter antas vara idéala; angivna värden av komponenter (t.ex. R för ett motstånd, U för en spänningskälla, k för en beroende källa) antas vara kända storheter; och andra markerade storheter (t.ex. strömmen markerad i ett motstånd eller spänningskälla) antas vara okända storheter. Lösningar ska uttryckas i kända storheter, och förenklas. Var tydlig med diagram och definitioner av variabler. Kontrollera svarens rimlighet genom t.ex. dimensionskoll eller alternativ lösningsmetod.

Tips: Dela tiden mellan båda talen: satsa inte för mycket tid på den svårare sista biten av tal 1! Tänk på värdet av att rita om ett diagram för varje tillstånd (t.ex. innan och efter en brytare ändrar kretsen) och kanske att införa andra symboler (t.ex. kortslutningar och öppnakretsar kan ersätta några komponenter i jämviktsläge) samt borttagning av delarna som inte är relevanta till det sökta värdet. Då blir kretsen ofta mycket lättare att tänka på och lösa.

Använd återstående tid för att kolla igenom svaren! Examinator: Nathaniel Taylor

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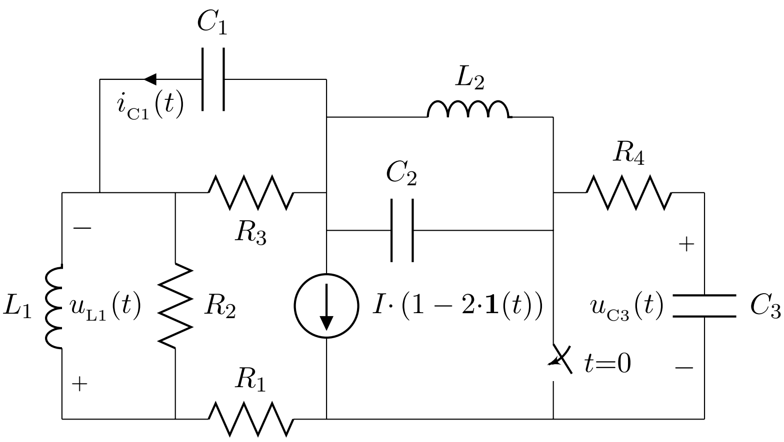
1) [5p]

Två ändringar sker i kretsen, båda vid tid $t = 0$: brytaren stängs, och strömkällan har en stegfunktion, $\mathbf{1}(t)$, i sitt värde.

a) [2p] Bestäm $i_{C1}(0^-)$ och $u_{C3}(0^-)$.

b) [1p] Hur mycket effekt absorberas av R_4 vid tiden $t = 0^+$.

c) [2p] Bestäm $i_{C1}(0^+)$ och $u_{L1}(0^+)$.



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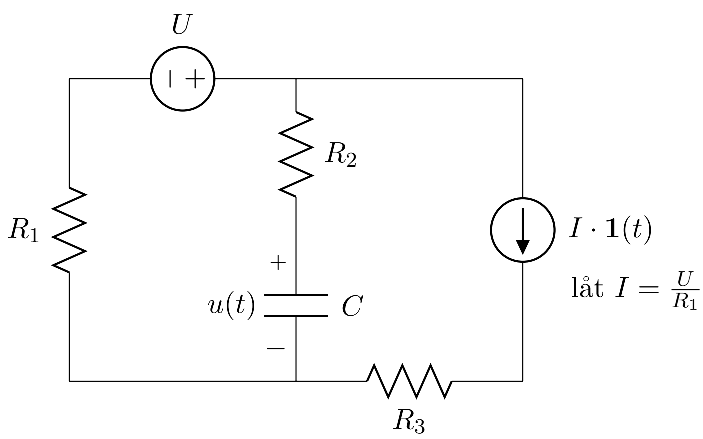
2) [5p]

Bestäm $u(t)$, för perioden $t > 0$.

Strömkällens värde I kan betraktas som känd, men det finns vidare information att $I = U/R_1$, vilken möjliggör ett förenklande av uttrycken.

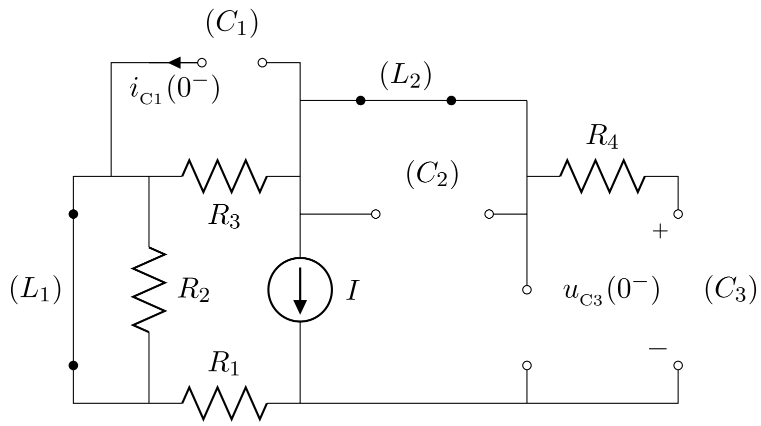


Solutions, EI1120 KS2 2015-02-16

1) Equilibrium & Continuity

a) Equilibrium calculation, at $t = 0^-$.

For the equilibrium solution, it is generally a good idea to draw the whole circuit with the short-circuit and open-circuit replacements, and with switches and time-dependent sources set to their actual values.



$i_{C1}(0^-) = 0$

$u_{C3}(0^-) = -I\,(R_1 + R_3)$

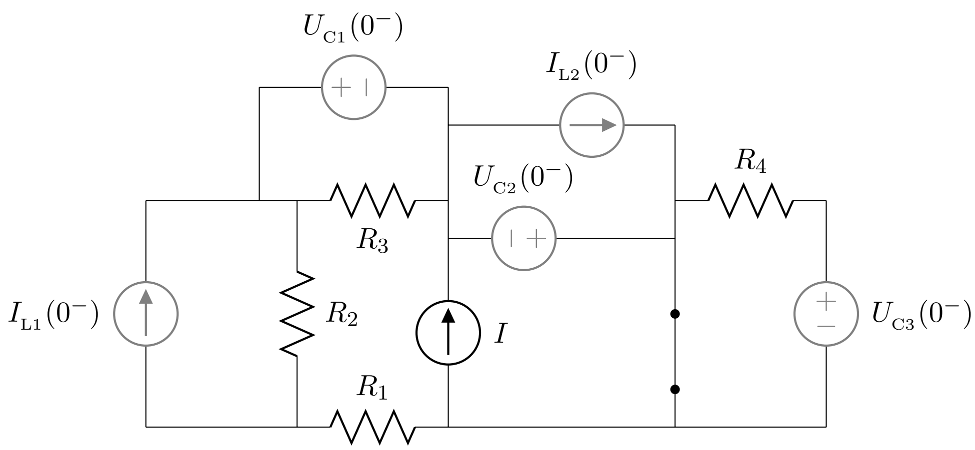
b) Continuity: immediately after disturbance of equilibrium.

After $t = 0$, the rightmost branch of the circuit, with R_4 and C_3, is separated from the rest by the short-circuit of the closed switch. KVL in this loop shows that the voltage across the resistor is the voltage remaining on the capacitor from the equilibrium state: $u_{C3}(0^-)$, as found above.

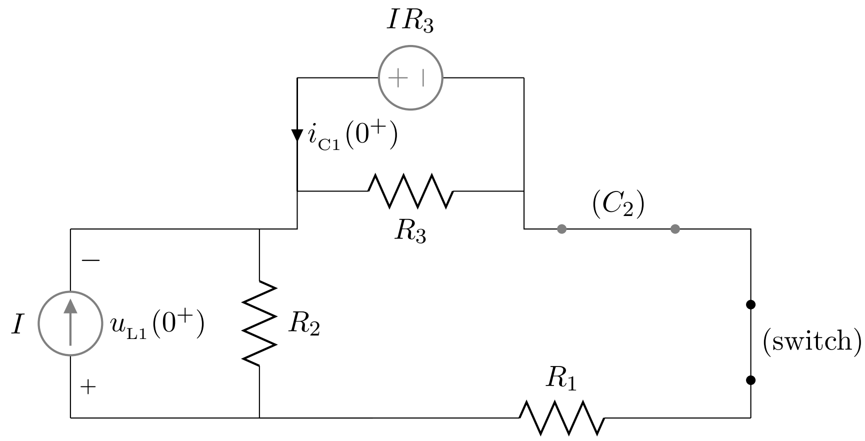
Therefore, $P_{R4}(0^+) = \frac{u_{R4}^2(0^+)}{R_4} = \frac{I^2(R_1+R_3)^2}{R_4}$.

c) Harder continuity questions.

The circuit can be re-drawn for the state $t = 0^+$. The current source now has the opposite direction of current, due to the step function. The switch is closed. All continuous variables have the same values as at $t = 0^-$. In the following diagram we represent the capacitors and inductors by sources that enforce the continous variables (another way to draw is just to show the capacitor or inductor, but mark its continuous variable: some people find it easier to think of the sources).



The above diagram may become easier to handle if we replace our 'labels' such as U_{C3} with the quantities that can be calculated from the equilibrium state at $t = 0^-$. (Note how all the 'artificial' sources' — current sources representing inductors, or voltage sources representing capacitors, at $t = 0^+$ — are inevitably representing continuous quantities, which are easily found from the equilibrium case.) If a source is found to have zero value, it can be further simplified by a short or open circuit: this can be done for C_2. The following diagram shows the circuit after these replacements.



Notice that at $t = 0^+$, the current source I in the original circuit is short-circuited by the capacitor C_2 and the closed switch. It is therefore irrelevant to the requested variables. (However, the quantity 'I' is still relevant, as this is included in the expressions for the current in inductor L_1 and the voltage on capacitor C_1, still shown above.)

The relevant part of the circuit is just R_1, R_2, R_3, and the sources representing L_1 and C_1. From this, the solutions are found as:

$i_{\mathrm{C1}}(0^+) = I\frac{R_1+R_3}{R_1+R_2}$

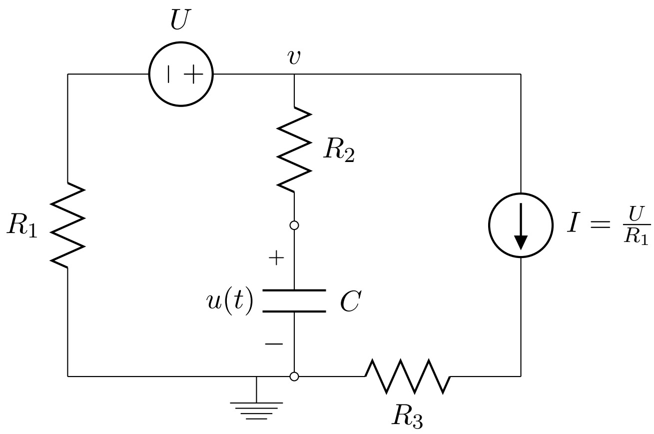
$u_{\mathrm{L1}}(0^+) = -IR_2\frac{R_1+R_3}{R_1+R_2}$

The calculation requires consideration of all those components. Superposition of the effects of the two sources (L_1 and C_1) is one quite efficient approach. Nodal analysis using the bottom of L_1 as earth and expressing the potential above this as $-u_{\mathrm{L1}}$ is also quite efficient, and the component R_3 is irrelevant to the solution of u_{L1} although it is needed for finding u_{C1}. Source-transformation of R_2 and the L_1 source is possible, but didn't feel as quick: it carries the risk that you find the voltage across the 'new' R_2 in the transformed source, which is *not* necessarily the same as the voltage across R_2 in the actual circuit (remember: the equivalence of the Thevenin and Norton sources is only when seen from outside the terminals ... the internal details of voltage and current in the source resistance do not have to be the same). Several other sensible methods of solution can also be found.

2) Time-functions

During the studied period, $t > 0$, the current source has a current I, which we have been told is equal to U/R_1.

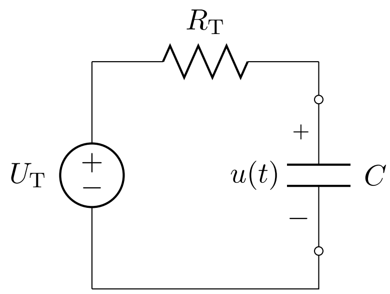
The two-terminal equivalent, seen if we take away the capacitor and look at the two terminals where it was connected, can be found by e.g. nodal analysis or superposition.



With an open circuit instead of the capacitor, the voltage u is zero! We can see that by following the current I, which must all go around the outer loop if the capacitor is removed. The resistor R_3 is irrelevant to our question. By KVL, we see $v = -IR_1 + U$. Substituting the information that $I = U/R_1$, this becomes $v = 0$. Because there is no current in R_2 in the open-circuit case, we can say $u_{(\mathrm{oc})} = 0$. That is the Thevenin equivalent's voltage, at $t > 0$: $U_\mathrm{T} = 0$.

To find the Thevenin resistance, we can use the short-circuit current method, or directly find the resistance by setting the sources to zero. The latter is easier here. The result is $R_\mathrm{T} = R_1 + R_2$.

Now the capacitor can be brought back, but connected to the simple equivalent-circuit to make the solution of $u(t)$ simpler.



valid $t > 0$

By equilibrium calculation, the final value, $u(\infty)$, is seen to be U_T, i.e. $u(\infty) = 0$.

The circuit's time-constant is $\tau = CR_\mathrm{T}$, which is $\tau = C\,(R_1 + R_2)$.

The missing detail is the initial value $u(0^+)$. This is a continuous variable (capacitor voltage), so it can is the same as $u(0^-)$. We cannot get any help from the equivalent circuit, as our equivalent was developed for the time $t > 0$: it assumes a particular value of the current source. We have to look back to the original circuit. The current source at $t < 0$ is zero, so the right-hand branch of the circuit can be ignored. In that case, we see $u(0^-) = U$.

Putting these three things together — initial and final values, and time-constant — the equation describing $u(t)$ for $t > 0$ is

$$u(t) = U \exp\left(\frac{-t}{C\,(R_1 + R_2)}\right).$$

If some other quantity had been sought, such as the current through the capacitor or the voltage across the current-source, we would now calculate this from the known function of $u(t)$. But fortunately, it was $u(t)$ that was sought in this question: so that's the end.

Other approaches more directly based on forming and solving a differential equation could also have been used here.