EI1110 Elkretsanalys, Kontrollskrivning KS1, 2015-09-29 kl 08–10

Hjälpmedel: Ett A4-ark (båda sidor) med studentens egna anteckningar på valfritt sätt.

Kontrollskrivningen har 3 tal, med totalt 12 poäng. Den omfattar ämnet 'Likström' och motsvarar sektion A i tentan. I tentan är kravet för godkänd minst 40% för sektion A, samt minst 50% över hela tentan.
Om inte annan information anges i ett tal, ska: komponenter antas vara idéala; angivna värden av komponenter (t.ex. R för ett motstånd, U för en spänningskälla, k för en beroende källa) antas vara *kända* storheter; och andra markerade storheter (t.ex. strömmen markerad i ett motstånd eller spänningskälla) antas vara *okända* storheter.
Lösningar ska uttryckas i kända storheter, och förenklas. Var tydlig med diagram och definitioner av variabler.
Dela tiden mellan talen — senare deltal brukar vara svårare att tjäna poäng på ... fastna inte på dessa.
Kontrollera svarens rimlighet genom t.ex. dimensionskoll eller alternativ lösningsmetod.

Använd återstående tid för att kontrollera svaren! Nathaniel Taylor (073 919 5883)

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1) [4p]

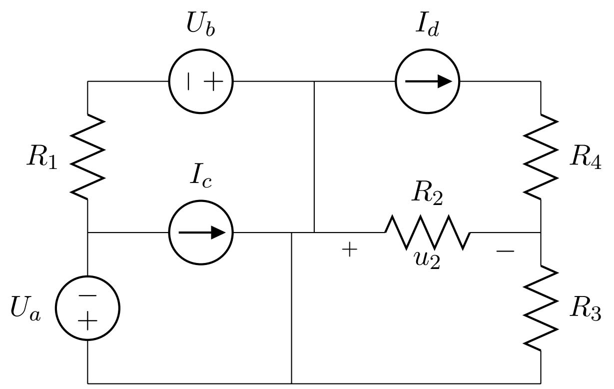
a) [0,5p] Vilken effekt absorberas av R_4?

b) [1p] Vilken effekt absorberas av R_1?

c) [0,5p] Vilken effekt levereras av källan I_c?

d) [1p] Bestäm spänningen u_2 över R_2.

e) [1p] Vilken effekt levereras av källan U_b?



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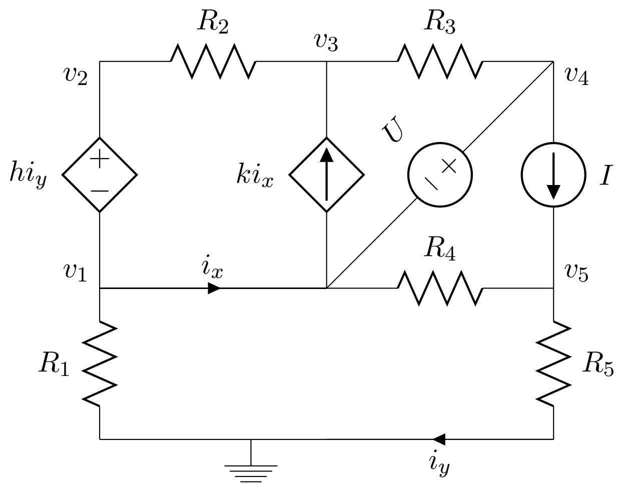
2) [4p]

Använd nodanalys för att skriva ekvationer som *skulle kunna lösas* för att få ut de markerade nodpotentialerna v_1, v_2, v_3, v_4 och v_5.

Du *måste inte* lösa eller förenkla ekvationerna: du behöver bara visa att du kan översätta från kretsen till ekvationerna.



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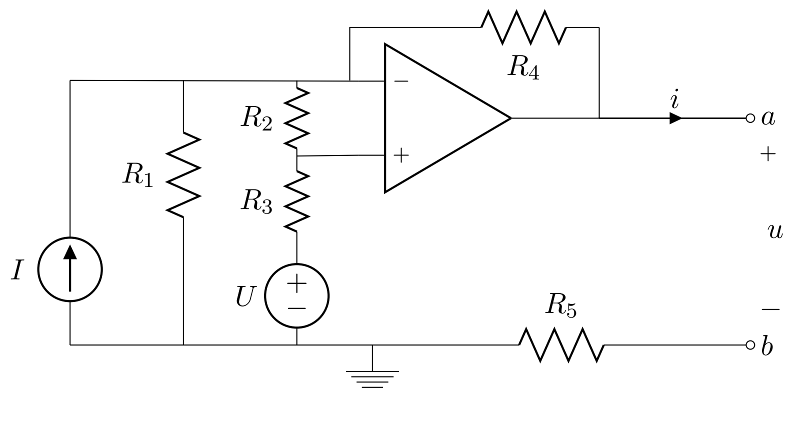
3) [4p]

a) [3p] Vad är spänningen u när polerna a och b är öppenkrets ($i = 0$)?

b) [1p] Bestäm Theveninekvivalenten med avseende på polerna a och b.



Solutions, EI1110 KS1 2015-09-29

1)

a) Power absorbed by R_4:
$I_d^2 R_4$ series connection with current-source ensures current of I_d.

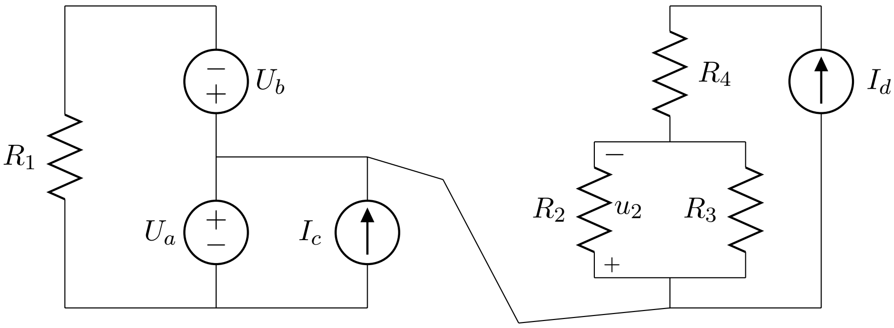
b) Power absorbed by R_1:
$(U_a - U_b)^2 / R_1$ KVL around R_1 and the two voltage-sources.

c) Power delivered *from* source I_c:
$U_a I_c$ U_a is the voltage across source I_c, as these sources are parallel-connected.

d) Voltage u_2 across R_2:
$\frac{-I_d R_2 R_3}{R_2 + R_3}$ resistors R_2 and R_3 are in parallel, with current I_d passing through.

e) Power delivered *from* source U_b:
$U_b \frac{U_b - U_a}{R_1}$ current in U_b is same as in R_1; see part 'b'.

The circuit can be analysed in the way that it was shown, or it could be re-drawn. As usual, re-drawing has the *disadvantage* of taking time and risking errors. The *advantage* of easier analysis may, however, outweigh the disadvantage. See what you think of the following way of drawing the circuit diagram:



This circuit can be seen as two separate circuits, joined only in one node. In the above re-drawing, the two parts have therefore been treated separately, each with its own choice of which node goes at the bottom. A connection is included between the two parts, to make the circuit exactly the same as the original one. Note, however, that this single connection cannot have any current, and thus does not affect the voltages and currents (and therefore the powers) in the components. The connection would only be relevant if the questions had been about potentials, with an earth node somewhere in the circuit.

2) Two examples will be shown. Many variations are possible. The first example is the one that we suggest is probably easiest to do for this type of question.

Extended nodal analysis ("the simple way")

Let's define the unknown currents in the voltage-sources to be going into the source's + terminal. We'll call them i_α in the independent source U, and i_β in the dependent source hi_y.

Now we can write KCL at all nodes except ground:

$$\begin{aligned}
\mathrm{KCL}(1)_{(\mathrm{out})}: \quad 0 &= -i_\beta + \frac{v_1}{R_1} + \frac{v_1 - v_5}{R_4} + ki_x - i_\alpha && (1)\\
\mathrm{KCL}(2)_{(\mathrm{out})}: \quad 0 &= i_\beta + \frac{v_2 - v_3}{R_2} && (2)\\
\mathrm{KCL}(3)_{(\mathrm{out})}: \quad 0 &= \frac{v_3 - v_2}{R_2} + \frac{v_3 - v_4}{R_3} - ki_x && (3)\\
\mathrm{KCL}(4)_{(\mathrm{out})}: \quad 0 &= \frac{v_4 - v_3}{R_3} + i_\alpha + I && (4)\\
\mathrm{KCL}(5)_{(\mathrm{out})}: \quad 0 &= -I + \frac{v_5 - v_1}{R_4} + \frac{v_5}{R_5} && (5)
\end{aligned}$$

These are only 5 equations so far, but with 9 unknowns: v_1, v_2, v_3, v_4, v_5, i_α, i_β, i_x, i_y.

We now add the further information given by the voltage sources:

$$\begin{aligned}
v_4 - v_1 &= U && (6)\\
v_2 - v_1 &= hi_y && (7)
\end{aligned}$$

It is also necessary to define the marked (but unknown) quantities that are the controlling variables of the dependent sources:

$$\begin{aligned}
i_x &= i_\beta - \frac{v_1}{R_1} && (8)\\
i_y &= \frac{v_5}{R_5} && (9)
\end{aligned}$$

End. That's it. The above 9 equations in 9 unknowns should be solvable. The node potentials will thus be found, along with the currents i_α and i_β in the voltage-sources and the marked current i_x and i_y.

Note that these definitions could easily have been expressed in a very different way. For example, it is also true that $i_y = \frac{-v_1}{R_1}$ in this circuit.[1] A similar situation exists for i_x. It doesn't matter which of the possible definitions is written, because the KCL equations contain the information that shows the different ways to be identical! If we had written KCL for the earth node, it would tell us that $\frac{v_1}{R_1} + \frac{v_5}{R_5} = 0$, and therefore that the two alternative equations we've shown for i_y are equivalent. We *didn't* actually write KCL for the earth node: we don't do that, as we know that this KCL must be the sum of the KCLs for all the other nodes, so it tells us nothing new.

A familiar warning from earlier exams is repeated here! *The systematic way in which this was done is important! There are many ways to write a sufficient set of equations, but there are also many ways to write insufficient equations: it is dangerously easy to write some linearly dependent equations and assume that "n unknowns, n equations, therefore it's all ok". The procedure used above is a simple way to ensure the equations are sufficient.*

[1] Notice that it would *not* have to be true if the earth symbol could have any current flowing in it. This could happen if another node had an earth symbol too. One example is if there were an opamp in the circuit, getting its output current from the earth node but without showing this explicitly.

Alternative: supernode

There are two voltage sources, both connected to v_1. There is thus one supernode, consisting of the nodes with potentials v_1, v_2 and v_4. Just one of these potentials is needed as an unknown in KCL: let's choose v_1. Then the other potentials in the supernode are defined in terms of this: $v_4 = v_1 + U$, and $v_2 = v_1 + hi_y$. Only the potentials v_1, v_3 and v_5 should be used in the KCL equations.

In order to avoid introducing i_y as a further unknown, we immediately define it in terms of existing variables: $i_y = \frac{v_5}{R_5}$. Then the expressions for the supernode potentials can be written in terms of just the node potentials and the known values of components.

$$v_4 = v_1 + U \tag{1}$$
$$v_2 = v_1 + \frac{h}{R_5}v_5 \tag{2}$$

These two equations are needed as part of the answer, in order to find the potentials v_2 and v_4 after v_1 has been solved.

The marked current i_x can be defined as $i_x = \frac{v_3 - \left(v_1 + \frac{h}{R_5}v_5\right)}{R_2} - \frac{v_1}{R_1}$, and this expression will be substituted when the current in the dependent current source is needed in KCL. In this way, we avoid having a further unknown in the equations.[2]

KCL at the supernode and other nodes (except earth node), using outgoing currents, gives

$$\text{KCL}(124): \quad 0 = \frac{v_1}{R_1} + \frac{v_1 - v_5}{R_4} + \frac{v_1 + \frac{h}{R_5}v_5 - v_3}{R_2} + \frac{v_1 + U - v_3}{R_3} + I + k\left(\frac{v_3 - \left(v_1 + \frac{h}{R_5}v_5\right)}{R_2} - \frac{v_1}{R_1}\right) \tag{3}$$
$$\text{KCL}(3): \quad 0 = \frac{v_3 - (v_1 + U)}{R_3} + \frac{v_3 - \left(v_1 + \frac{h}{R_5}v_5\right)}{R_2} - k\left(\frac{v_3 - \left(v_1 + \frac{h}{R_5}v_5\right)}{R_2} - \frac{v_1}{R_1}\right) \tag{4}$$
$$\text{KCL}(5): \quad 0 = -I + \frac{v_5 - v_1}{R_4} + \frac{v_5}{R_5} \tag{5}$$

These equations could be rearranged to separate the terms for v_1, v_3 and v_5. There is no requirement to do so for this question.

The supernode method has given 5 equations in 5 unknowns, which are the node potentials. Only the 3 KCL equations need simultaneous solution. The earlier 2 equations for the potentials in the supernode can then be used to find v_2 and v_4 after v_1, v_3 and v_5 have been found.

[2] This immediate substitution is done to reduce the number of unknowns and thus the number of equations. It is the 'standard' way that we've seen in the supernode method for nodal analysis. However, in this particular case the expression for i_x is rather long: it might actually be easier to use the short name i_x during part of the solution, then to substitute the above expression later.

3)

a) In the open-circuit condition, no current flows in R_5: the node b is therefore at zero potential, and so the marked voltage u is equal to the opamp's output potential.

This potential can be found by a step-by-step approach or by formal nodal analysis.

The nodal analysis could be done in the following way. Define the opamp's output potential as v_o. Define the potential of the opamp inputs[3] as v_i.

KCL can be written for the nodes at the two opamp inputs. The opamp output and the point above source U can be treated as fixed potentials where we don't care about the current in the voltage sources (the supernode type of approach). Remember: we ultimately only want to find v_o.

$$\mathrm{KCL}(+)_{(\mathrm{out})}: \quad 0 = \frac{v_\mathrm{i}-v_\mathrm{i}}{R_2} + \frac{v_\mathrm{i}-U}{R_3} \tag{6}$$

$$\mathrm{KCL}(-)_{(\mathrm{out})}: \quad 0 = \frac{v_\mathrm{i}-v_\mathrm{i}}{R_2} + \frac{v_\mathrm{i}}{R_1} - I + \frac{v_\mathrm{i}-v_\mathrm{o}}{R_4} \tag{7}$$

After solving for v_o, which was shown above to the equal to u for the open-circuit case, the result is

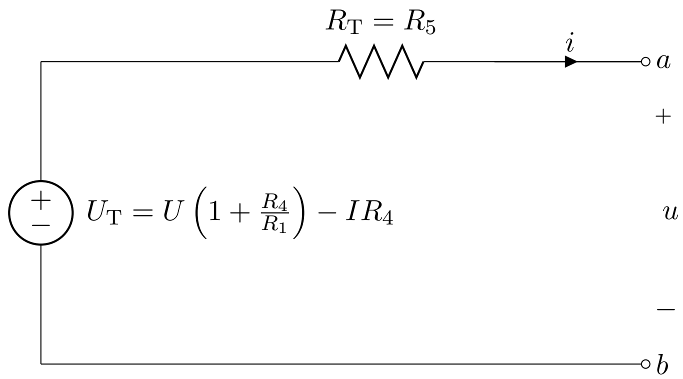
$$u = v_\mathrm{o} = U\left(1+\frac{R_4}{R_1}\right) - IR_4$$

The less formal, step-by-step method is to notice that no current can flow in R_2 if the opamp inputs have equal potential, and thus that no current can flow in R_3 either, as the opamp input has no current. Thus, both inputs are at potential U. KCL can then be written for the inverting input, using potential U; the only unknown is the sought potential v_o.

b) The ideal opamp's output is an ideal voltage-source, with its other side connected to earth. The Thevenin resistance between the opamp's output and earth is therefore zero. In the shown circuit, the terminals a-b are connected *'almost'* between opamp output and earth ... the difference is that there is a resistance R_5 in series. Thus the Thevenin resistance between a-b is $R_\mathrm{T} = R_5$. The Thevenin voltage is the open-circuit voltage of the circuit, which was found in part 'a'.

There are several other ways of determining the Thevenin resistance. One could derive an expression for the relation of i and u, then check the gradient that this gives in the u-i plane. Or one could solve for the current i with the terminals a-b short-circuited: the ratio of open-circuit voltage to short-circuit current is the Thevenin resistance.

An equivalent circuit should be shown as a diagram, to make clear the direction of the source, and that the voltage-source and resistance are in series in a Thevenin source.



[3] The opamp inputs are expected to have the same potential, so we only need to define one symbol v_i. An alternative would be to define v_+ and v_-, then to include an equation stating $v_+ = v_-$.