KTH EI1120 Elkretsanalys (CENMI) KS 2 2019-02-13 kl. 08–10

Permitted material: Beyond writing-equipment, up to two pieces of paper up to A4 size can be brought, with free choice of content: handwritten, printed; small, large; text, diagram, image; one or both sides, etc. These papers do not need to be handed in with the exam.

Unless it is stated otherwise, the final answer to a question should be expressed in terms of the known quantities given in the question, and any clear simplifications should be done. Component values such as R for a resistor, U for an independent voltage source, or K for a dependent source, are assumed to be known quantities. Marked currents or voltages such as i_x are assumed to be definitions, not known quantities.

Clearly drawn and labelled diagrams are a good way to help yourself avoid mistakes, and to make clear to others what you are doing. By showing clearly your intermediate steps in a solution, you improve your chance of getting points even if the final result is wrong. You may write in Swedish or English; but we suggest that writing in either is seldom necessary if you make good use of diagrams and equations!

KS2 does not give any direct grade. Its points will be used to replace Section-B in the final exam or re-exam, if this would improve your points there. See therefore the rules for the exam to relate the points to grades: at least 40% is needed in Section-B alone, as well as 50% overall.

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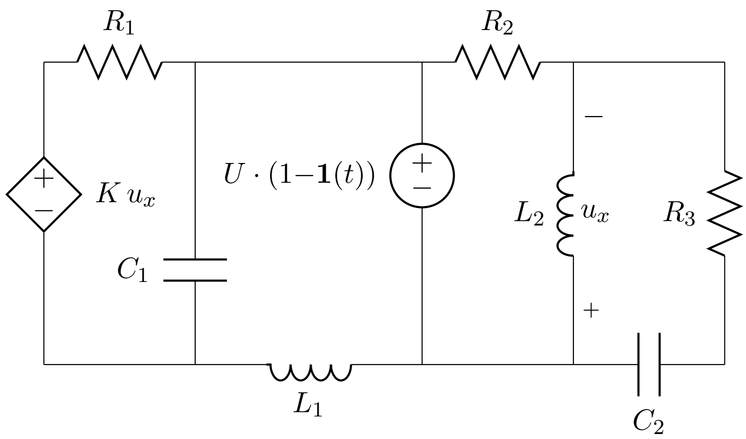
1) [5p] Find:

a) [1p] Power delivered from the dependent source at $t = 0^-$.

b) [1p] Energy stored in L_2 at $t = 0^+$.

c) [2p] Power absorbed by R_1 at $t = 0^+$.

d) [1p] Energy stored in L_1 as $t \to \infty$.



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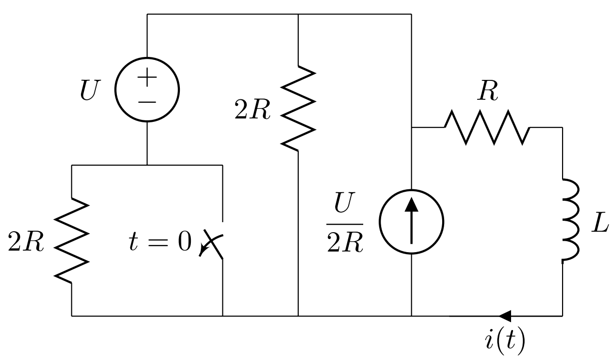
2) [5p]

The switch opens at $t = 0$.

a) [4p] Find the current $i(t)$, for $t > 0$.

b) [1p] Find the power delivered by the current source, as a function of time for $t > 0$.



Översättningar:

Hjälpmedel: Upp till två A4-ark (båda sidor) med studentens egna anteckningar på valfritt sätt: handskrivet eller datorutskrift; text, diagram, bild; stor eller liten textstorlek, o.s.v. Dessa måste inte lämnas in med skrivningarna.

Om inte annan information anges i ett tal ska: komponenter antas vara ideala; angivna värden av komponenter (t.ex. R för ett motstånd, U för en spänningskälla, K för en beroende källa) antas vara *kända* storheter; och andra markerade storheter (t.ex. strömmen markerad i ett motstånd eller spänningskälla) antas vara *okända* storheter. Lösningar ska uttryckas i kända storheter och förenklas.

Var tydlig med diagram och definitioner av variabler. Du får skriva på svenska eller engelska, men vi rekommenderar att diagram och ekvationer används i stället i de flesta fall.

KS2 ger inte direkt betyg, utan poäng som kan ersätta poängen i sektion-B i tentan (TEN1, mars) om KS:en gav mer. Se därför reglerna för TEN1 angående gränser.

1. Bestäm följande storheter:

a) [1p] Effekten levererad av den beroende källan vid $t = 0^-$.

b) [1p] Energin lagrad i L_2 vid $t = 0^+$.

c) [2p] Effekten absorberad av R_1 vid $t = 0^+$.

d) [1p] Energin lagrad i L_1 när $t \to \infty$.

2. Brytaren öppnas vid tid $t = 0$. Bestäm följande:

a) [4p] Strömmen $i(t)$, för $t > 0$.

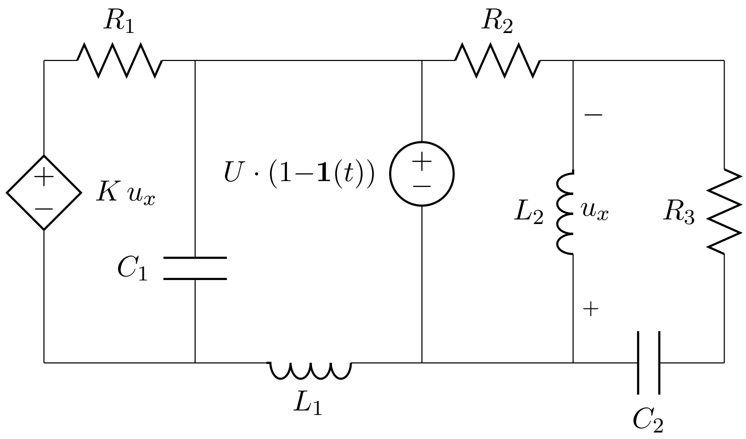
b) [1p] Effekten levererad av strömkällan, som tidsfunktion för $t > 0$.

The End. *Don't waste remaining time … check your solutions!*

Solutions (EI1120 KS 2 VT19, 2019-02-13)

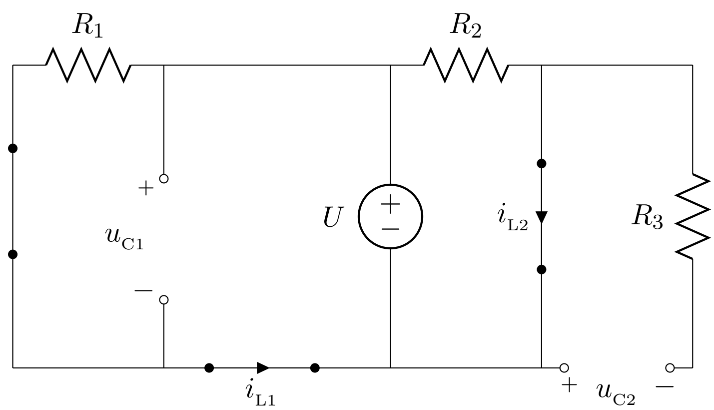
Q1.

The original circuit is this:



We'll start by drawing the situation at $t = 0^-$, and finding the inductor currents and capacitor voltages. (It might turn out that we don't need all of these to answer the questions, but it's easy to do it fully anyway. In fact, in this case, we will need all these values in the end.)

At time $t = 0^-$, no change has happened 'for a long time', so we assume equilibrium. Thus, capacitors have zero current and inductors have zero voltage, allowing the circuit to be simplified to the following. The dependent source has been replaced with a short-circuit, because its controlling voltage u_x is defined across an inductor and therefore $u_x(0^-) = 0$. The independent source has been given its specific value for this time, which is before the step function changes.



In the above, the four marked quantities linked to stored energies are:

$u_{\text{C1}} = U$ from KVL,

$i_{\text{L1}} = U/R_1$ from KVL, Ohm's law, KCL,

$i_{\text{L2}} = U/R_2$ ditto,

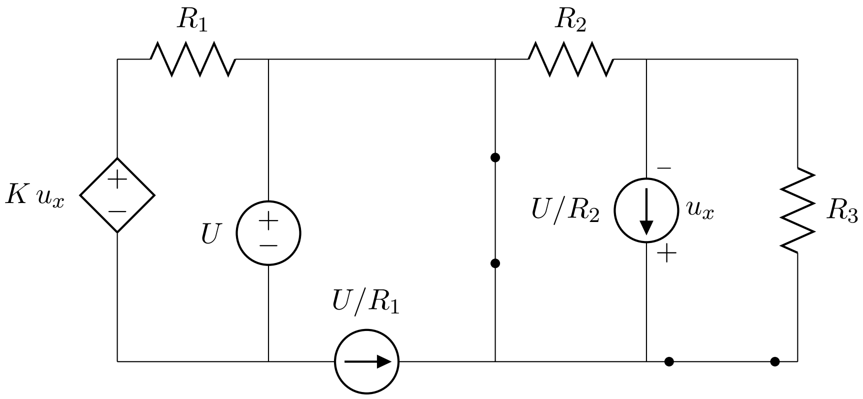
$u_{\text{C2}} = 0$ KCL gives no current in R_3, then KVL in the rightmost loop.

a) $P_{\text{K}} = 0$.
Easy answer, when we look at the $t = 0^-$ diagram or consider that $u_x(0^-) = 0$.
With its voltage set to zero, this source cannot deliver or receive power.

For the following two questions we need to consider the circuit at $t = 0^+$.

From the equilibrium, we've found solutions for the continuous quantities. By continuity, their values are the same immediately after the step change in the circuit.

In the diagram below, the capacitors and inductors are modelled as voltage- and current-sources, respectively, with the values determined from $t = 0^-$. The voltage on C_2 was zero, so it could be represented as a voltage source with zero value, but instead it has been represented by a short-circuit, as that is even simpler. The independent voltage source has value zero at $t = 0^+$, so it is represented as a short-circuit.



b) $W_{\mathrm{L2}}(0^+) = \dfrac{1}{2}\left(\dfrac{U}{R_2}\right)^2 L_2.$

This is the same as at $t = 0^-$, as it's the energies in the capacitors and inductors that don't immediately change. We know from the above and earlier diagram that the current in L_2 is U/R_2. An inductor L with current i has stored energy $Li^2/2$.

c) $P_{\mathrm{R1}}(0^+) = U^2\left(1 - \dfrac{KR_3}{R_2+R_3}\right)^2 /R_1.$

By KVL the resistor R_1 has a voltage $U - Ku_x$. Its power is therefore $(U - Ku_x)^2/R_1$.
The direction of the voltage doesn't matter, as it is squared to find the power.

The value of u_x needs to be expressed in terms of known quantities. It is defined in the original circuit across L_2, i.e. across the current source U/R_2 in the diagram above.

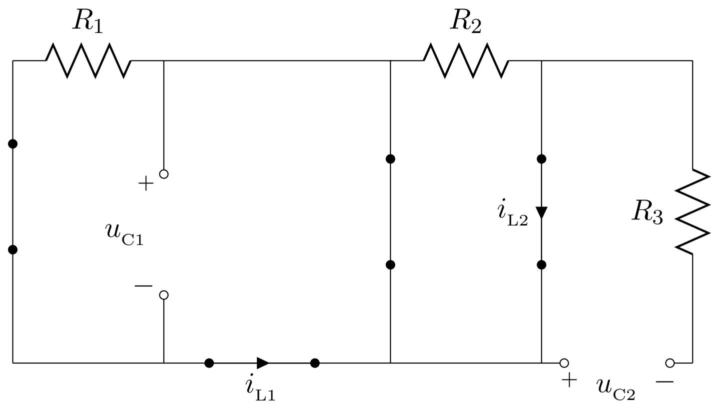
If we re-drew the circuit, making the short-circuited independent voltage source become a single point, it might be clearer that the current U/R_2 passes purely through two parallel resistors, so that

$$u_x = \frac{U}{R_2}\cdot\frac{R_2R_3}{R_2+R_3}.$$

An alternative way to see this is to use KVL in two loops to show the each of R_2 and R_3 each have voltage u_x, and then use Ohm's law to find their currents in terms of u_x, and KCL at the node above the current source.

d) $W_{L1}(\infty) = 0$.

The equilibrium as $t \to \infty$ has no independent source to drive the circuit. Our equilibrium calculation will find that all currents and voltages are zero, including the current in L_1 which determines its stored energy.



This is not a general result – that is, one cannot assume all voltages and currents to be zero in an equilibrium just because all independent sources are zero or are not present. A counterexample is a circuit where a capacitor has some charge at $t = 0$, and a switch in series with it is opened. Then the equilibrium calculation for $t \to \infty$ would have two open-circuits in series – the switch and the capacitor – and one would have to argue that the capacitor's voltage must be the same at $t \to \infty$ as at $t = 0$, since the open switch guaranteed no change of the charge.

However, in the specific case of KS2 2019, there are resistive paths that will gradually remove all energy from the circuit in the absence of an independent source: the equilibrium diagram clearly gives solutions with zero values, as it has no ambiguities for KCL (parallel short-circuits) or KVL (series open-circuits).

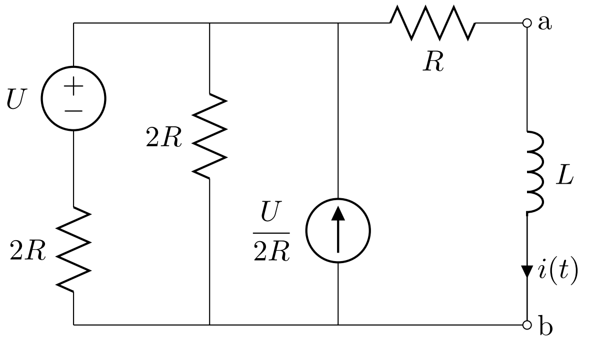
Q2.

As usual (consider the homework 8, VT19!) there are many possible solutions to this type of question. One could derive and solve the differential equation, or find initial and final values and time-constant. Within these possibilities one could use superposition or equivalent sources.

a) $i(t) = \frac{U}{2R}\left(1 + \mathrm{e}^{-t\,2R/L}\right)$.

This circuit – other than the inductor – is rather begging to be turned into a Thevenin (or Norton) equivalent, don't you think?

For $t > 0$, the circuit is:



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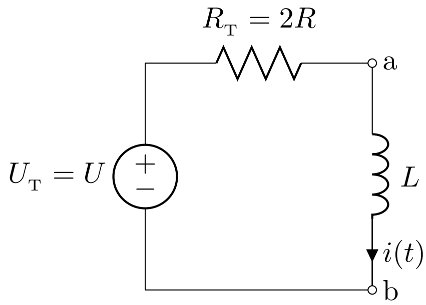


The above can be found by source transformation. Replace the Thevenin branch { U, $2R$ } with a Norton source, then join the two parallel current sources into a single source of U/R, and the two parallel resistors into R. Transform this Norton source to a Thevenin source of U and R, and add the further series resistance R at the right.

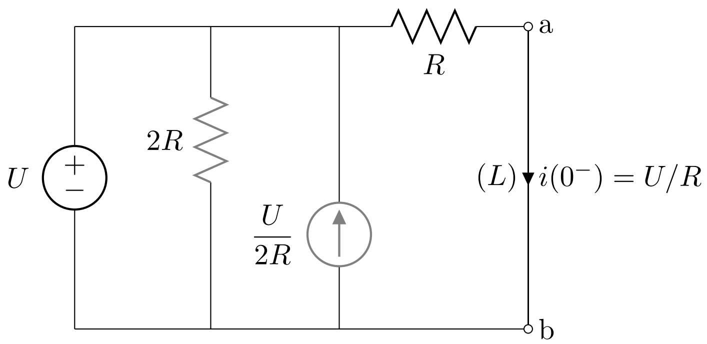
Based on the above Thevenin equivalent, the final value of $i(t)$ can be found. This is at the equilibrium when $t \to \infty$, when the inductor has zero voltage (short circuit).

$$i(\infty) = \frac{U}{2R}.$$

The time-constant is found from the Thevenin resistance and the inductance,

$$\tau = \frac{L}{R_{\mathrm{T}}} = \frac{L}{2R}.$$

The initial value of $i(t)$ has to be found from the equilibrium state before the switch opened. At this time, $t = 0^-$, the circuit simplifies to the following.



The result $i(0^-) = U/R$ comes from KVL around the outer loop. The current source and parallel resistor do not affect the result, as they are in parallel with the voltage source when seen from the inductor's position.

Putting the above together,

$$i(t) = i(\infty) + \left(i(0^+) - i(\infty)\right) \mathrm{e}^{-t/\frac{L}{2R}},$$

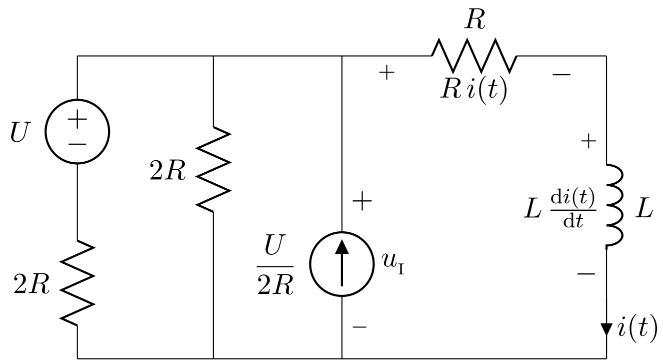
$$i(t) = \frac{U}{2R} + \left(\frac{U}{R} - \frac{U}{2R}\right) \mathrm{e}^{-t/\frac{L}{2R}}$$

$$i(t) = \frac{U}{2R}\left(1 + \mathrm{e}^{-t\,2R/L}\right) \qquad (t > 0).$$

b) $P_{\mathrm{I}} = \frac{U^2}{4R}\left(1 - \mathrm{e}^{-t\,2R/L}\right)$.

We need to find the power delivered by the current source. This sought quantity is not the inductor's continuous quantity $i(t)$ that we found before, but we can express it in terms of $i(t)$ and other components. The principle here is that as we have found the inductor's current for all times $t > 0$, we can treat the inductor as a known current in the circuit at all these times.

The power delivered by the current source is the product of its current and voltage, with correct choice of sign. From the circuit diagram for $t > 0$, this delivered power is $u_{\mathrm{I}} \frac{U}{2R}$.



It appears that the easiest way to find the voltage across the current source is to take KVL in the rightmost loop,

$$u_{\mathrm{I}} = R\,i(t) + L\frac{\mathrm{d}i(t)}{\mathrm{d}t}.$$

Putting in the values shown in the diagram, from Ohm's law and the inductor's equation,

$$u_{\mathrm{I}} = R\frac{U}{2R}\left(1 + \mathrm{e}^{-t\,2R/L}\right) + L \cdot \frac{U}{2R} \cdot \frac{-2R}{L} \cdot \mathrm{e}^{-t\,2R/L},$$

$$u_{\mathrm{I}} = \frac{U}{2}\left(1 - \mathrm{e}^{-t\,2R/L}\right).$$

The sought power is the product of the source's current and the voltage across it,

$$P_{\mathrm{I}} = \frac{U}{2R} \cdot u_{\mathrm{I}} = \frac{U}{2R} \cdot \frac{U}{2}\left(1 - \mathrm{e}^{-t\,2R/L}\right),$$

simplifying to

$$P_{\mathrm{I}} = \frac{U^2}{4R}\left(1 - \mathrm{e}^{-t\,2R/L}\right).$$