KTH EI1120 Elkretsanalys (CENMI) TEN1 2020-03-10 kl. 14–19

Permitted material: Beyond writing-equipment, up to three pieces of paper up to A4 size can be brought, with free choice of content: handwritten, printed; small, large; text, diagram, image; one or both sides, etc. These papers do not need to be handed in with the exam.

Unless it is stated otherwise, the final answer to a question should be expressed in terms of the known quantities given in the question, and any clear simplifications should be done. Component values such as R for a resistor, U for an independent voltage source, or K for a dependent source, are assumed to be known quantities. Marked currents or voltages such as i_x are assumed to be definitions, not known quantities.

Clearly drawn and labelled diagrams are a good way to help yourself avoid mistakes, and to make clear to others what you are doing. By showing clearly your intermediate steps in a solution, you improve your chance of getting points even if the final result is wrong. You may write in Swedish or English; but we suggest that writing in either is seldom necessary if you make good use of diagrams and equations!

Determination of exam grade. Denote as A, B and C the available points from sections A, B and C of this exam: A=12, B=10, C=18. Denote as a, b and c the points actually obtained in the respective sections, and as a_k and b_k the points från KS1 and KS2, and as h the homework 'bonus'. The requirement for passing the exam (E or higher) is:

$$\frac{\max(a,a_\mathrm{k})}{A} \geq 40\% \quad \& \quad \frac{\max(b,b_\mathrm{k})}{B} \geq 40\% \quad \& \quad \frac{c}{C} \geq 40\% \quad \& \quad \frac{\max(a,a_\mathrm{k})+\max(b,b_\mathrm{k})+c+h}{A+B+C} \geq 50\%$$

The grade is then determined by the total including bonus, i.e. the last of the terms above: boundaries (%) are 50 (E), 60 (D), 70 (C), 80 (B), 90 (A). If the exam misses a pass by a small margin on just one criterion, a grade of Fx may be registered, with the possibility of completing to E by an extra task arranged later.

Special for the VT20 round: The optional project-task with up to 6 points substitutes for Question 9 in this exam if that gives an advantage. In selecting whether to use points from the exam or part-exams ('KS'), the selection will be done per question not just per section.

Nathaniel Taylor (08 790 6222)

Section A. Direct Current

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1) [4p]

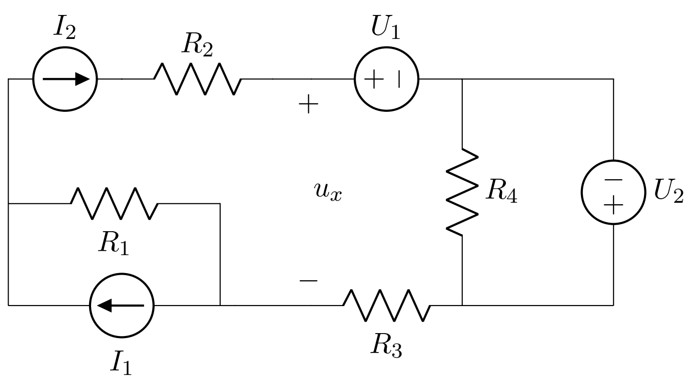
Find:

a) [1p] the power from source U_1

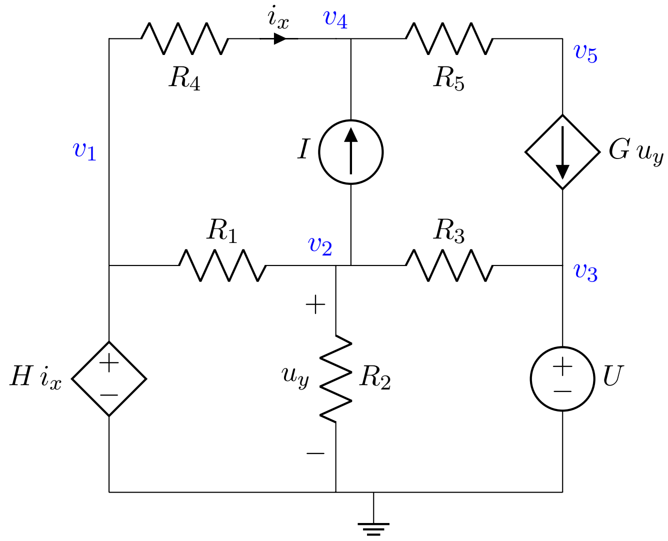
b) [1p] the power into resistor R_3

c) [1p] the marked voltage u_x

d) [1p] the power from source I_2

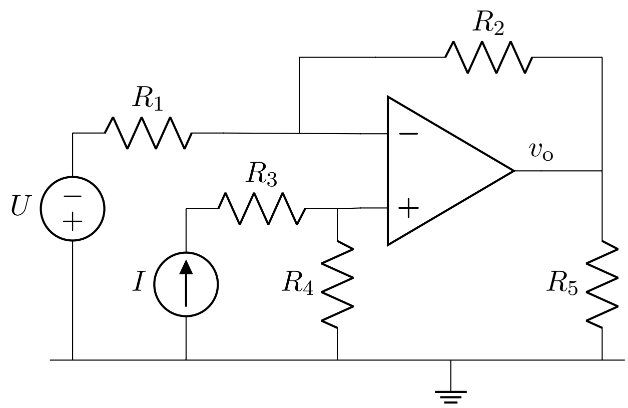


2) [4p]



Write equations that could be solved without further information to find the potentials v_1, v_2, v_3, v_4 and v_5 in this circuit in terms of the component values.

3) [4p]



Determine:

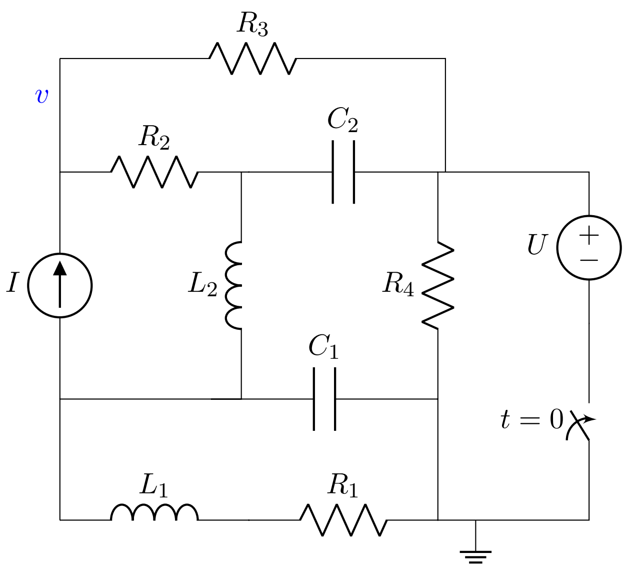
a) [3.5p] The opamp's output potential v_o.

b) [0.4p] The maximum power available to the resistor R_5 from the rest of the circuit.
Warning: 'conceptual' question.

c) [0.1p] Estimate the maximum output power (in watts) of a real opamp that is for example a small silicon chip made for driving headphones. Just a very crude order-of-magnitude estimate is wanted, for comparison with the theoretical ideal circuit in 'b'.

Section B. Transient Calculations

4) [5p] Find:



a) [1p] Energy stored in C_2 at $t = 0^-$

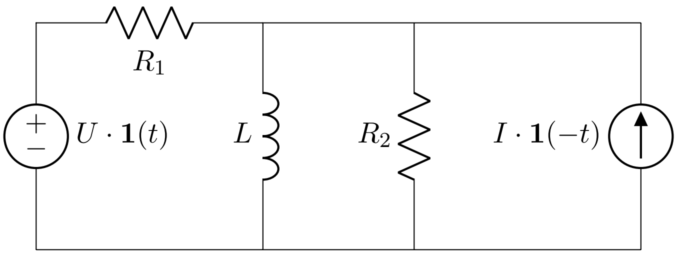
b) [1p] Power delivered by L_1 at $t = 0^+$

c) [2p] Potential v at $t = 0^+$

d) [1p] Power delivered by source I as $t \to \infty$

5) [5p]

Find the power delivered by the voltage source, for $t > 0$.

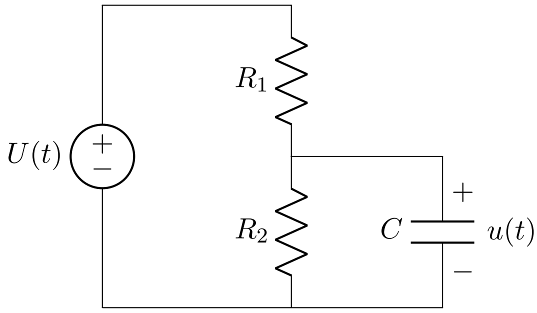


Section C. Alternating Current

6) [4p]

The source's voltage is $U(t) = \hat{U}\cos(\omega t)$.

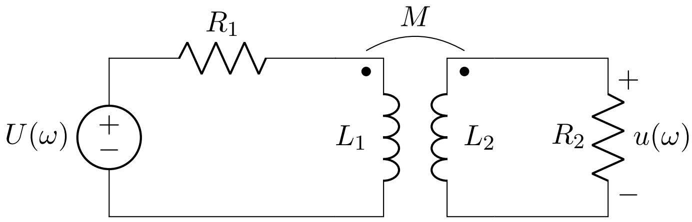
Determine $u(t)$.



7) [4p]

a) [2p] Show that this circuit's network function $u(\omega)/U(\omega)$ can be expressed as

$$\frac{u(\omega)}{U(\omega)} = \frac{\mathrm{j}\omega M/R_1}{(1+\mathrm{j}\omega L_1/R_1)\,(1+\mathrm{j}\omega L_2/R_2) + \frac{\omega^2 M^2}{R_1 R_2}}$$



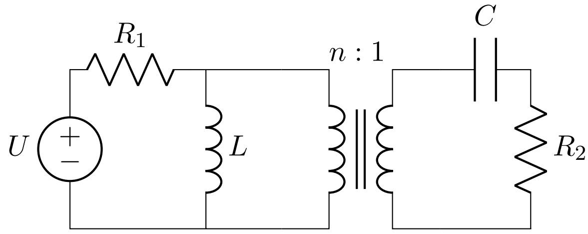
b) [2p] Sketch a Bode amplitude plot of the following network function:

$$H(\omega) = \frac{\mathrm{j}\omega/\omega_3}{(1+\mathrm{j}\omega/\omega_1)\,(1+\mathrm{j}\omega/\omega_2)}.$$

Assume $\omega_1 \ll \omega_2 \ll \omega_3$. Mark significant points and gradients.

8) [4p]

The source has angular frequency ω.
Component values C and R_2 can be chosen.
Other component values are fixed.



a) [3p] What values of C and R_2 will maximise the active power transferred from the left to the right side of the transformer? Express these values in terms of the other components' values.

b) [1p] What is the value of this maximum power when the condition in part 'a' is fulfilled?

9) [6p]

Consider a balanced three-phase system according to the description below:

> A three-phase voltage source provides line-voltage magnitude U at angular frequency ω.
> A three-phase load is formed from three resistors each of value R, connected in a delta.
> The source and load are *not* directly connected. They are connected through three single-phase transformers that together form a three-phase transformer. Each of the transformers has N_1 turns on its primary winding and N_2 on its secondary. The primaries are connected with each other in delta, and are supplied from the source. The secondaries are connected to each other in star, and supply the load.

Advice! Think calmly, and draw carefully. Use symmetry where you can. Write what you know. Simplify.

a) [1p] What is the voltage magnitude across each load resistor?

b) [2p] What complex power is supplied by the source?

c) [2p] What is the complex value of current in the primary of the transformer that connects between lines 2 and 3 of the source, in the direction from line-2 to line-3?
Assume that the source has phase-rotation 1,2,3 and that the voltage of line-2 relative to line-1 is taken as the angle-reference (zero).

d) [1p] The secondary of the transformer mentioned in 'c' becomes disconnected from the load: imagine, if you like, a fractured wire (open circuit) at one of the terminals of this transformer's secondary winding. This is no longer a balanced three-phase circuit.
What complex power does the source now supply? Think carefully!

The End. *Please don't waste remaining time … check your solutions!*

Översättningar:

Hjälpmedel: Upp till tre A4-ark (båda sidor) med studentens egna anteckningar på valfritt sätt: handskrivet eller datorutskrift; text, diagram, bild; stor eller liten textstorlek, o.s.v. Dessa måste inte lämnas in med skrivningarna.

Om inte annan information anges i ett tal ska: komponenter antas vara ideala; angivna värden av komponenter (t.ex. R för ett motstånd, U för en spänningskälla, K för en beroende källa) antas vara *kända* storheter; och andra markerade storheter (t.ex. strömmen markerad i ett motstånd eller spänningskälla) antas vara *okända* storheter. Lösningar ska uttryckas i kända storheter och förenklas.

Var tydlig med diagram och definitioner av variabler. Du får skriva på svenska eller engelska, men vi rekommenderar att diagram och ekvationer används i stället i de flesta fall.

1. [4p] Bestäm följande:
a) [1p] effekten från källan U_1
b) [1p] effekten in till motståndet R_3
c) [1p] spänningen u_x
d) [1p] effekten från källan I_2

2. [4p] Skriv ekvationer som skulle *kunna* lösas, utan vidare information, för att bestämma de fem potentialerna $v_{1,2,3,4,5}$ som funktioner av kretsens komponentvärden. Det rekommenderas inte att du försöker lösa dem!

3. [4p] Bestäm:
a) [3.5p] opampens utgångspotential v_o
b) [0.4p] den maximaleffekt som kan levereras till R_5 (om man kan fritt bestämma R_5)
c) [0.1p] uppskatta maximaleffekt som en praktisk opamp, exempelvis som är gjort för att driva hörlurar, kan leverera till en last.

4. [5p] Bestäm:
a) [1p] Energin lagrad i C_2 vid $t = 0^-$
b) [1p] Effekten levererad från L_1 vid $t = 0^+$
c) [2p] Potentialen v vid $t = 0^+$
d) [1p] Effekten levererad av källan I vid $t \to \infty$

5. [5p] Bestäm effekten levererad av spänningskällan för $t > 0$.

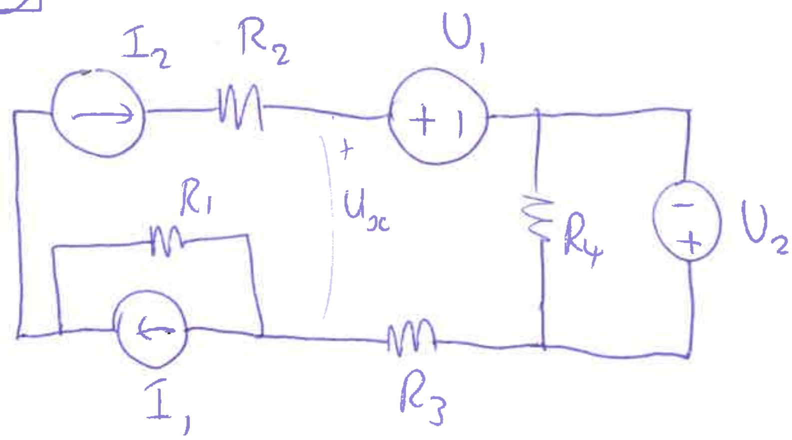
6. [4p] Källans spänning är $U(t) = \hat{U}\cos(\omega t)$. Bestäm $u(t)$.

7. [4p]
a) [2p] Visa att kretsen har den angivna nätverksfunktion (se ekvationen till vänster om diagrammet).
b) [2p] Skissa ett Bodeamplituddiagram av $H(\omega)$ som given i deltal 'b'. Antag $\omega_1 \ll \omega_2 \ll \omega_3$. Markera viktiga punkter och lutningar.

8. [4p] Källan har vinkelfrekvens ω. Komponentvärden R_2 och C kan väljas men andra komponentvärden är fasta.
a) [3p] Bestäm värden R_2 och C som ger maximal aktiveffekt levererad genom transformatorn.
b) [1p] Hur mycket aktiveffekt passar genom transformatorn när R_2 och C väljs enligt 'a'?

9. [6p] Betrakta följande balanserade trefassytemet: En trefas spänningskälla levererar huvudspänning U vid vinkelfrekvens ω. Tre enfasiga transformatorer som har N_1 varv på sina primäralindningar och N_2 varv på sina sekundäralindningar kopplas för att skapa en trefastransformator. Primäralindningarna deltakopplas, och matas från källan. Sekundäralindningarna stjärnkopplas och matar en last som består av tre mostånd R i delta.
a) [1p] Vad är det för spänningsmagnitud över varje fas i lasten (varje motstånd R).
b) [2p] Bestäm den komplexeffekt som källan levererar.
c) [2p] Bestäm, som magnitud och fasvinkel, strömmen i den primära lindningen som är kopplad mellan ledarna 2 och 3 från källan. Antag att källan har fasrotation 1,2,3, och att spänningen av ledare 2 från källan, relativ till ledare 1, används som referensvinkel.
d) [1p] Obalanserat fall: sekundärlindningen av *en* av de tre transformatorna blir en öppenkrets, för att ett brott har skett i anslutningen. Bestäm den aktiveffekt som källan nu levererar till lasten. Tänk försiktigt.

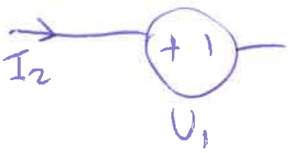
1



(a) power from source U_1 :

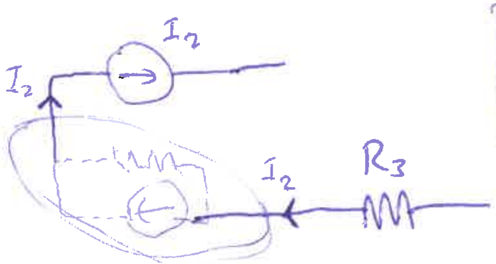
$$\boxed{P_{U_1 \text{(out)}} = -U_1 I_2}$$

By **KCL**, the current into this voltage source's '+' terminal is I_2



The product $I_2 U_1$ is then the power into this voltage source, so a negation is needed.

(b)



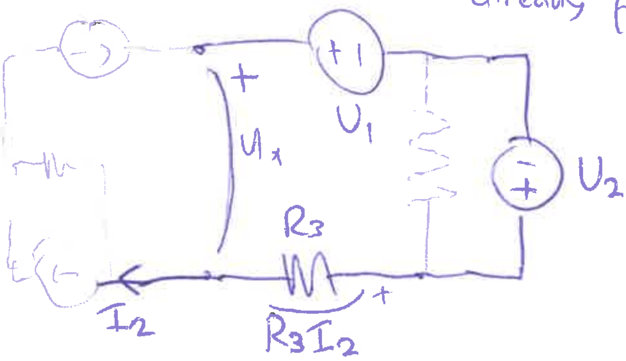
$$\boxed{P_{R3 \text{(in)}} = I_2^2 R_3}$$

By **KCL** all the current I_2 must also pass through R_3,

(c) To find U_x we could take KVL to the left or the right.

$$\boxed{U_x = I_2 R_3 - U_2 + U_1}$$

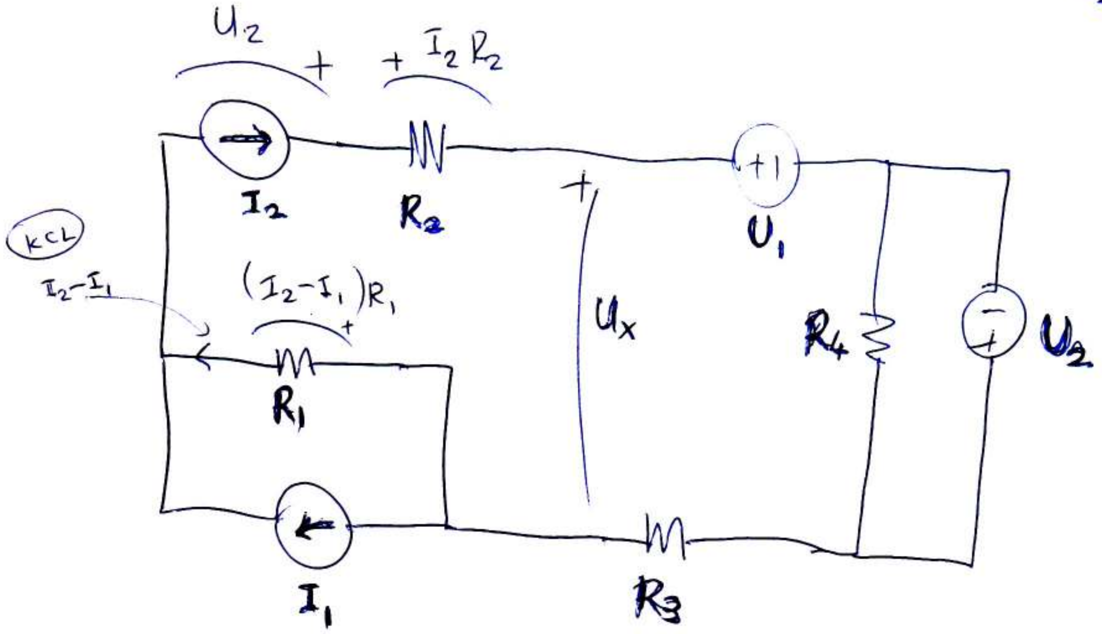
At the left are current sources with voltages that we don't immediately know.
At the right we can choose a path through two voltage sources and the R_3 where we already found the current in (1b).



KVL

$$U_x - U_1 + U_2 - I_2 R_3 = 0$$

(d) find first the voltage U_2 across current source I_2.



By KVL around (I_2, R_2, U_x, R_1),

$$U_2 = I_2 R_2 + U_x + (I_2 - I_1) R_1$$

taking U_x from part c), or KVL around (U_1, U_2, R_3)

$$U_2 = I_2 R_2 + U_1 + I_2 R_3 - U_2 + (I_2 - I_1) R_1$$

$$= I_2 (R_1 + R_2 + R_3) + U_1 - U_2 - I_1 R_1$$

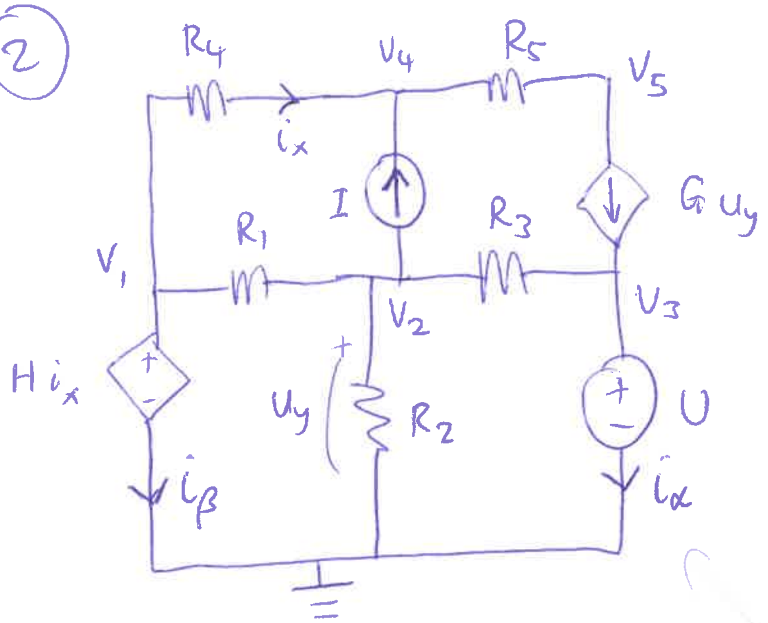
U_2 is defined in the direction such that $P_{(\text{out of } I_2)} = U_2 I_2$

Therefore:

$$\boxed{P_{I_2} = I_2 \left(I_2 (R_1 + R_2 + R_3) + U_1 - U_2 - I_1 R_1 \right)}$$

Superposition is another good way to this result.

2



9 unknowns:

$V_1 \;\; V_2 \;\; V_3 \;\; V_4 \;\; V_5$

$i_x \;\; u_y \;\; i_\alpha \;\; i_\beta$

9 equations (below), expected to be independent because of the rules followed to make them.

'Extended' nodal analysis.

KCL all nodes except reference (⏚).

Define currents in the voltage sources: i_α i_β

KCL(1) (out): $$i_\beta + \frac{V_1 - V_2}{R_1} + \frac{V_1 - V_4}{R_4} = 0$$

KCL(2) $$\frac{V_2}{R_2} + \frac{V_2 - V_1}{R_1} + \frac{V_2 - V_3}{R_3} + I = 0$$

KCL(3) $$i_\alpha + \frac{V_3 - V_2}{R_3} - G u_y = 0$$

KCL(4) $$-I + \frac{V_4 - V_1}{R_4} + \frac{V_4 - V_5}{R_5} = 0$$

KCL(5) $$\frac{V_5 - V_4}{R_5} + G u_y = 0$$

Voltage sources

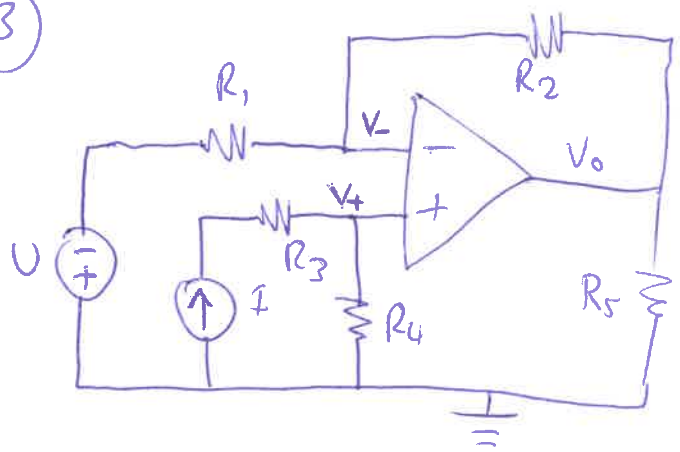
(6) $$V_3 - 0 = U$$

(7) $$V_1 - 0 = H i_x$$

Controlling variables (8) $$i_x = \frac{V_1 - V_4}{R_4}$$

(9) $$u_y = V_2$$

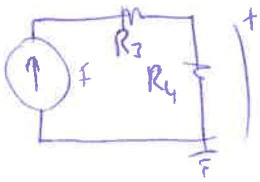
③



ⓐ

$$V_o = \frac{UR_2}{R_1} + I\frac{R_4(R_1+R_2)}{R_1}$$

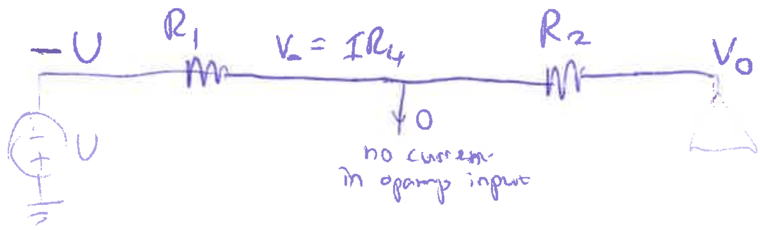
ⓐ In this circuit the potential V_+ can be found from just a subpart of the circuit:



$IR_4 = V_+$

Due to the negative feedback and ideal opamp, we also know $V_- = V_+ = IR_4$.

Now, by KCL in the upper part of the circuit,



we have $$\frac{IR_4 - (-U)}{R_1} + \frac{IR_4 - V_o}{R_2} = 0$$

So $$V_o = R_2\left(IR_4\left(\frac{1}{R_1}+\frac{1}{R_2}\right) + \frac{U}{R_1}\right)$$

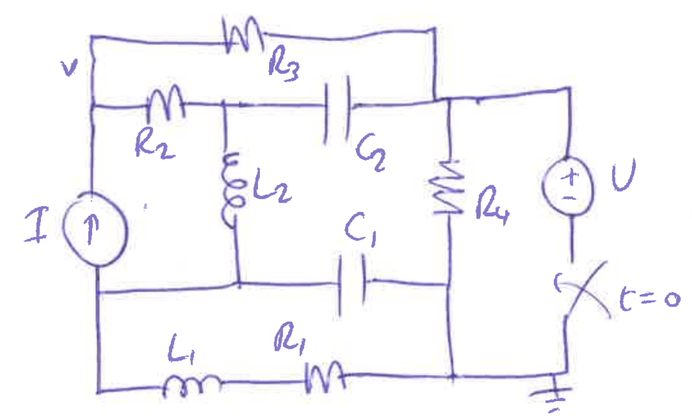
$$= \frac{UR_2 + IR_4(R_1+R_2)}{R_1}$$

(b) In the case of an "ideal" circuit, there is no limit, as the resistor is connected to a voltage that does not depend on the current that R_5 carries. In other words, the terminals "opamp out" and ground node have a Thevenin equivalent with $R_T = 0$ (and $U_T = V_0$). Making R_5 closer and closer to zero results in higher and higher powers.

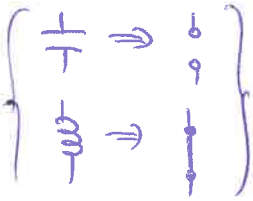
(c) "order of 1 W". Probably closer to 0.2 W.
Anything from 0.01 W to 10 W is not unreasonable.
Estimates could be made from thinking of the battery powering the opamp (few volts, up to a few amps in short circuit) or a few times the headphone power; or about specifications that can be found in the table at the end of the "chapter" document of topic "opamps" (typically tens of milliamps in short circuit, but hundreds of milliamps for the "power" opamps).

MAIN POINTS:
- Very limited, totally different from "ideal" circuit.
- the tiny 0.1p was to encourage an answer to be given; fairly reasonable answers will get points
- extra point.... there are often many ways of estimating things you know little about, or at least of putting some reasonable bounds on them.... try making a habit of this and checking your results.

(4) Original circuit.

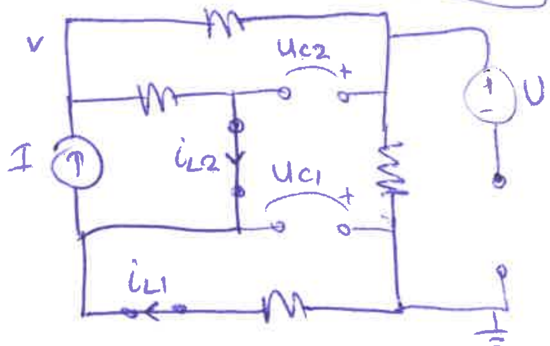


At $t=0^-$, Switch is open

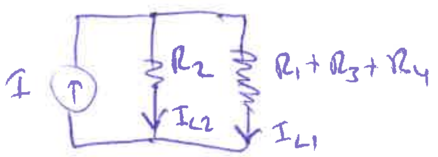


(equilibrium)

Redraw the circuit for $\boxed{t=0^-}$



To find i_{L1} & i_{L2}:



current division

$$i_{L1} = \frac{I R_2}{R_1 + R_2 + R_3 + R_4}$$

$$i_{L2} = \frac{I(R_1 + R_3 + R_4)}{R_1 + R_2 + R_3 + R_4}$$

KVL gives u_{C1} & u_{C2}:

$$u_{C1} = i_{L1} R_1 = \frac{I R_1 R_2}{R_1 + R_2 + R_3 + R_4}$$

$$u_{C2} = i_{L1}(R_1 + R_4) = \frac{I R_2 (R_1 + R_4)}{R_1 + R_2 + R_3 + R_4}$$

(a) $$W_{C2}(0^-) = \frac{1}{2} C_2 U_{C2}^2$$

$$= \frac{C_2}{2}\left(\frac{I R_2 (R_1 + R_4)}{R_1 + R_2 + R_3 + R_4}\right)^2$$

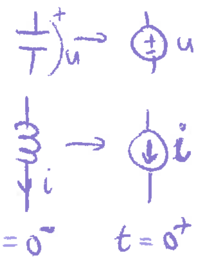
We didn't need to find the other continuous quantities in order to answer the questions about $t=0^-$, but we'll find they may be needed in later questions about $t=0^+$.

4 cont.d

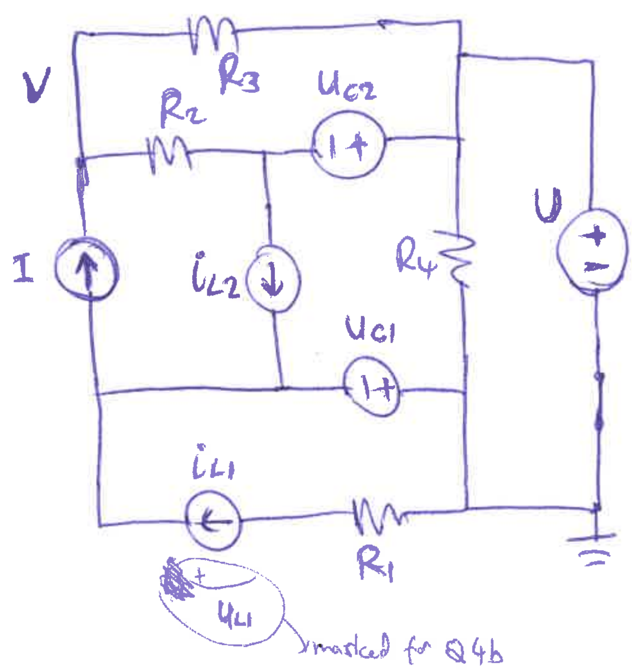
At $t=0^+$,

Switch is closed:

continuity:



Re-draw the circuit for $t=0^+$:



(b) $P_{L1}(0^+) = 0$

C1 forces the voltage across R_1 & L_1 to be the same as at $t=0^-$. And L_1 forces the current in R_1 to be the same. So KVL in the loop $R_1\ C_1\ L_1$ at $t=0^+$ is the same as at $t=0^-$, when the inductor was acting as a short circuit and thus had no power.

By algebra: $P_{L1} = u_{L1}\cdot i_{L1} = \underbrace{(R_1 i_{L1} - u_{C1})}_{KVL} i_{L1}$

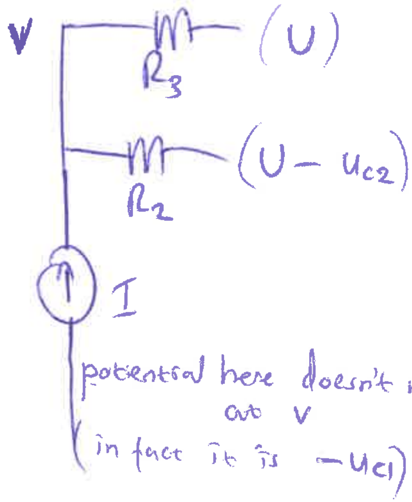
$$= R_1 \frac{IR_2}{R_1+R_2+R_3+R_4} - \frac{IR_1R_2}{R_1+R_2+R_3+R_4} = 0$$

(c) $V(0^+) = U + \dfrac{IR_2R_3}{R_1+R_2+R_3+R_4}$

At $t=0^+$, with a voltage source and two capacitors holding fixed voltage between nodes, there are several nodes with fixed potentials.

In other words, there is a big "grand supernode" including three nodes other than the ground node.

The region around V can be shown like this for writing KCL:



KCL $\quad -I + \dfrac{V-U}{R_3} + \dfrac{V-(U-u_{C2})}{R_2} = 0$

$$(V-U)\left(\frac{1}{R_2}+\frac{1}{R_3}\right) = I - \frac{u_{C2}}{R_2}$$

$$(V-U)\left(\frac{R_2+R_3}{R_2R_3}\right) = I - \frac{\cancel{R_2}\,I(R_1+R_4)}{\cancel{R_2}(R_1+R_2+R_3+R_4)}$$

$$V-U = \frac{I\,\cancel{(R_2+R_3)}}{(R_1+R_2+R_3+R_4)\cancel{(R_2+R_3)}}\,R_2R_3$$

$$V = U + \frac{IR_2R_3}{R_1+R_2+R_3+R_4}$$

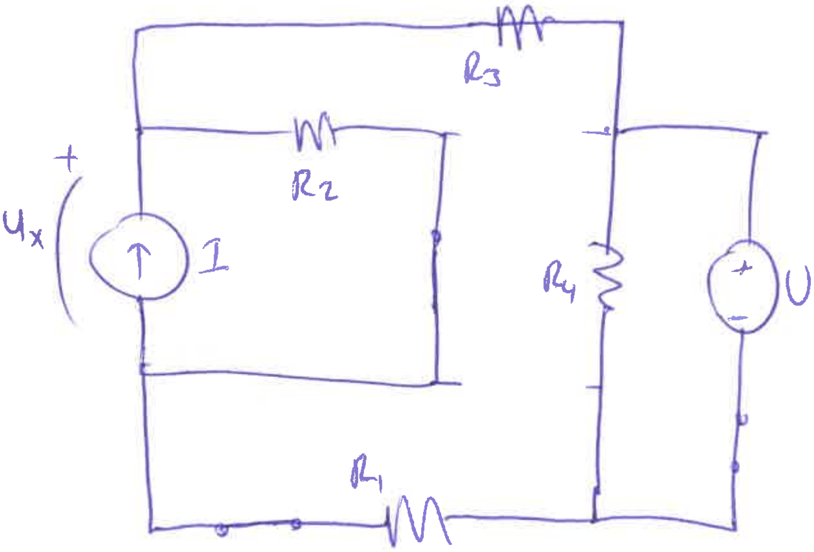
4 cont'd.

$t \to \infty$

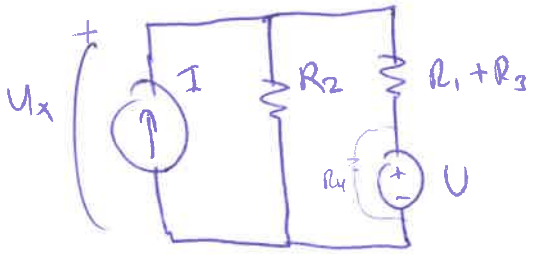
Equilibrium again.

Different from $t = 0^-$ in that the switch is closed so source U is present.

Now we just need to find the voltage across source I -- no need to find capacitor voltages or inductor currents for later use.



$\Downarrow$



d

$$P_I(\infty) = \frac{UIR_2 + I^2R_2(R_1+R_3)}{R_1 + R_2 + R_3}$$

This is from

$$P_I(\infty) = I \cdot U_x$$

where U_x is as found below

find U_x

<u>Nodal analysis</u>

$$\frac{U_x}{R_2} + \frac{U_x - U}{R_1 + R_3} - I = 0$$

$$U_x\left(\frac{1}{R_2} + \frac{1}{R_1+R_3}\right) = \frac{U}{R_1+R_3} + I$$

$$U_x = \left(\frac{U}{R_1+R_3} + I\right)\frac{(R_1+R_3)R_2}{R_1+R_2+R_3} = \frac{UR_2 + IR_2(R_1+R_3)}{R_1+R_2+R_3}$$

Superposition:

$$U_x = I\,\frac{R_2(R_1+R_3)}{R_1+R_2+R_3} + \frac{UR_2}{R_1+R_2+R_3} = \checkmark$$

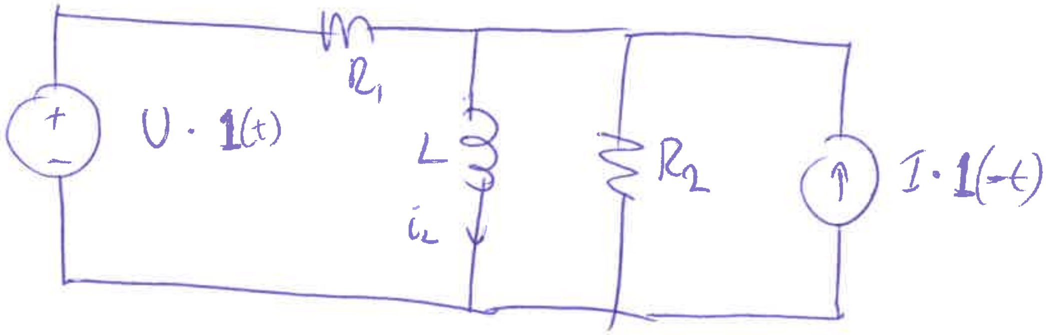
pretty easy! compared to KCL

$I \cdot R_{equivalent}$

voltage division

5

$\mathbb{1}()$ is "unit step function"



find $p(t)$ for $t > 0$, where p is the power from the voltage source.

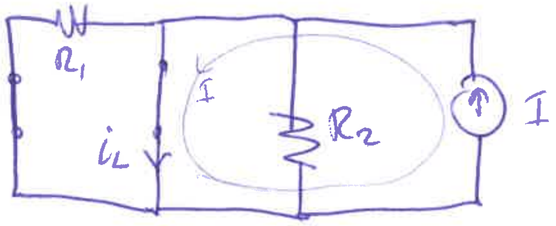
Standard procedure: find first the "continuous variables" which in the case here is just the inductor's current i_L

First stage: initial value, $i_L(0^+)$ which is $i_L(0^-)$ due to continuity, and can be found at the equilibrium before the change (steps).

$t = 0^-$

Voltage source $= 0$

and ~~so~~ inductor behaves as short circuit.



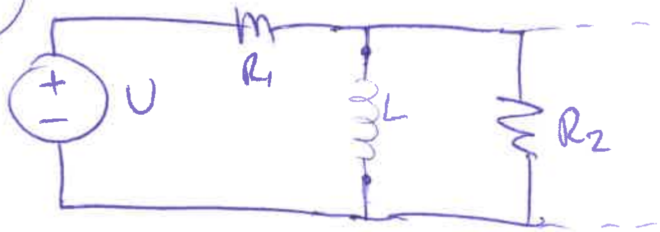
INITIAL CONDITION

$$\boxed{i_L(0^+) = i_L(0^-) = I}$$

No current in R_1 or R_2 as the inductor "short circuits" them (zero voltage).

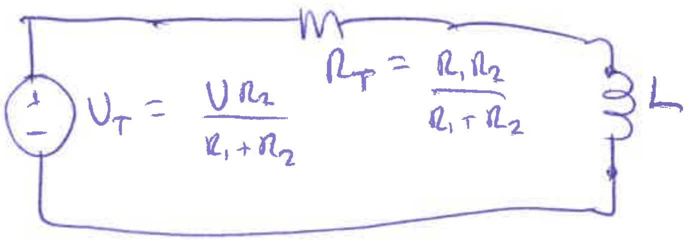
After the step functions change, the circuit is like this:

$t = 0^+$



Current source is zero

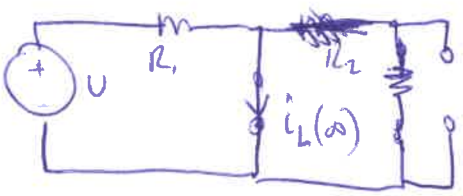
'seen' by the inductor, the rest of the circuit is a Thevenin equivalent of:



We already know $i_L(0^+) = I$

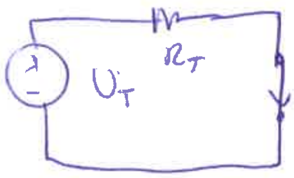
The final state as $t \to \infty$ is an equilibrium, so we again can replace capacitors and inductors, (The difference is just which sources are active, compared to $t = 0^-$)

Altered circuit in equilibrium:



$i_L(\infty) = \frac{U}{R_1}$

Or Thevenin form:



$i_L(\infty) = \frac{U_T}{R_T}$

$i_L(\infty) = \frac{U R_2 / (R_1+R_2)}{R_1 R_2 / (R_1+R_2)} = \frac{U}{R_1}$

So $i_L(\infty) = \frac{U}{R_1}$

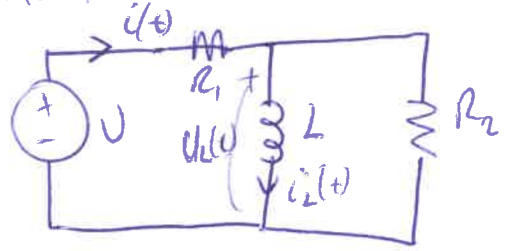
The time constant is $\tau = L/R_T$. As this circuit has just the one inductor (no capacitors either), we can write its time function as

$$i_L(t) = i_L(\infty) + \left(i_L(0^+) - i_L(\infty)\right) e^{-t/\tau}$$

$$= \frac{U}{R_1} + \left(I - \frac{U}{R_1}\right) e^{-t / \left(\frac{L(R_1+R_2)}{R_1 R_2}\right)}$$

$$\tau = \frac{L}{R_T} = \frac{L}{\left(\frac{R_1 R_2}{R_1+R_2}\right)}$$

We want to find the power from the voltage source. This power is $U \cdot i(t)$ as shown below:



By KVL,

$$i(t) = \frac{U - u_L(t)}{R_1}$$

$u_L(t)$ can be found from $i_L(t)$ using $u_L(t) = L\frac{di_L(t)}{dt}$ (inductor equation)

$$u_L(t) = L\frac{di_L(t)}{dt} = L\frac{d}{dt}\left(\frac{U}{R_1} + \left(I - \frac{U}{R_1}\right)e^{-t/\tau}\right)$$

$$= L\left(I - \frac{U}{R_1}\right)\left(\frac{-1}{\tau}\right)e^{-t/\tau}$$

$$\cancel{= L\left(I - \frac{U}{R_1}\right)\left(\frac{-R_1 R_2}{(R_1+R_2)L}\right)e^{-t\frac{R_1+R_2}{L R_1 R_2}}}$$

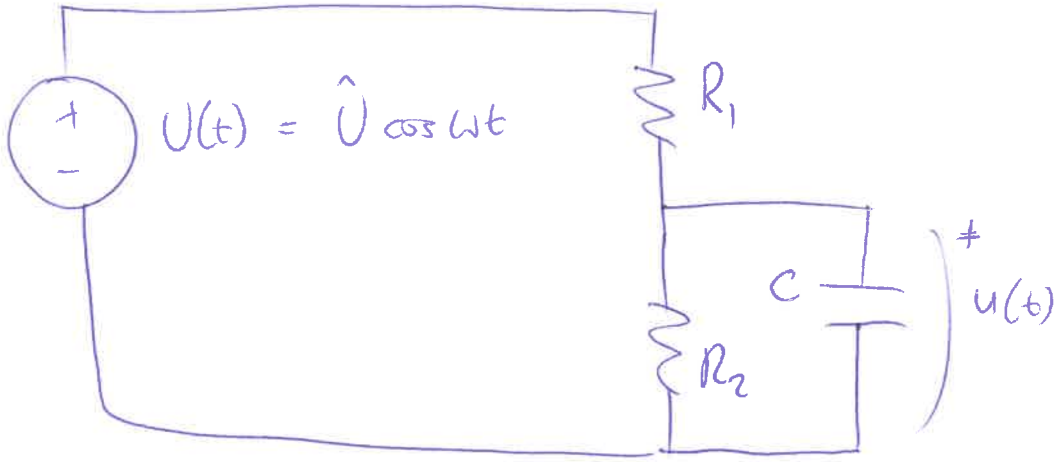
$$= L\left(I - \frac{U}{R_1}\right)\left(\frac{-1}{\frac{L(R_1+R_2)}{R_1 R_2}}\right)e^{-t\frac{R_1 R_2}{L(R_1+R_2)}}$$

$$i(t) = \left(\frac{U}{R_1} - I R_1\right)\frac{R_2}{R_1+R_2} e^{-t\frac{R_1 R_2}{L(R_1+R_2)}}$$

So $P = U \cdot i(t)$

$$\boxed{P = \left(U^2 - U I R_1\right)\frac{R_2}{R_1+R_2} e^{-t\frac{R_1 R_2}{L(R_1+R_2)}}}$$

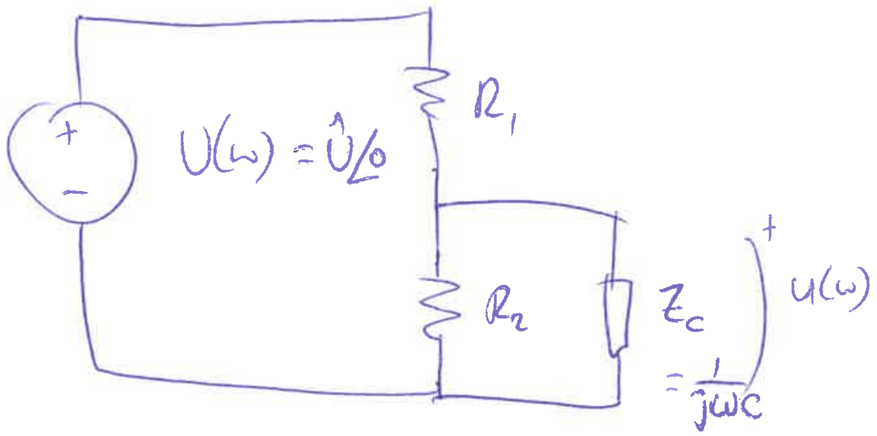
6



Sinusoidal steady-state solution ("ac") is assumed to be wanted, as we are in the ac part of the exam...

Convert to phasors and impedances.

for phasors let's choose cosine reference for angle, and peak value for magnitude.



By voltage division between the bottom part $\left(R_2 \overset{\text{parallel with)}}{\|} Z_c\right)$ and top,

$$u(\omega) = U(\omega) \cdot \frac{\frac{R_2 Z_c}{R_2 + Z_c}}{\frac{R_2 Z_c}{R_2 + Z_c} + R_1} = U(\omega)\left(\frac{R_2 Z_c}{R_2 Z_c + R_1 (R_2 + Z_c)}\right)$$

Put in the value of Z_c,

$$u(\omega) = U(\omega) \frac{R_2 \frac{1}{j\omega C}}{R_2 \frac{1}{j\omega C} + R_1 \left(R_2 + \frac{1}{j\omega C}\right)} = \frac{R_2}{(R_1 + R_2) + j\omega C \cdot R_1 R_2}$$

So $$U(\omega) = \frac{\hat{U}\angle 0 \cdot R_2}{R_1 + R_2 + j\omega C R_1 R_2}$$

This has magnitude $$|U(\omega)| = \frac{\hat{U} R_2}{\sqrt{(R_1+R_2)^2 + (\omega C R_1 R_2)^2}}$$

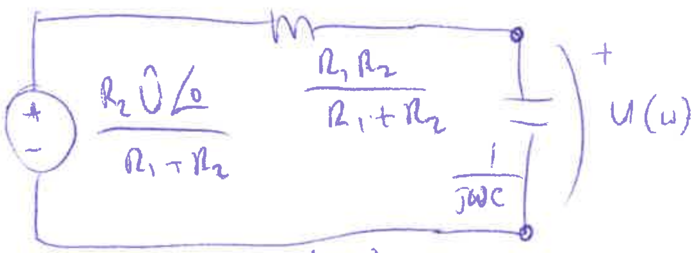
and angle $$\angle U(\omega) = \frac{\angle 0}{\angle \operatorname{atan} \frac{\omega C R_1 R_2}{R_1+R_2}} = -\operatorname{atan}\left(\frac{\omega C R_1 R_2}{R_1 + R_2}\right)$$

Using the same cosine reference and peak magnitudes,

$$u(t) = |U(\omega)| \cos\left(\omega t + \angle U(\omega)\right)$$

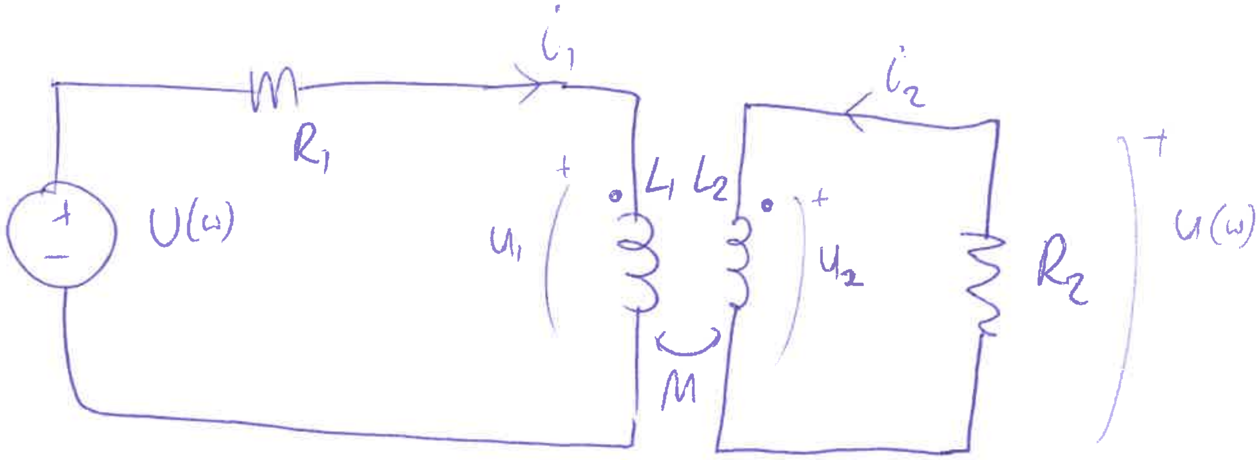
$$\boxed{u(t) = \frac{\hat{U} R_2}{\sqrt{(R_1+R_2)^2 + (\omega C R_1 R_2)^2}} \cos\left(\omega t - \operatorname{atan} \frac{\omega C R_1 R_2}{R_1 + R_2}\right)}$$

Another way would be to start from a Thevenin equivalent of everything outside the capacitor:



$$U(\omega) = \frac{\frac{1}{j\omega C}\left(\frac{R_2 \hat{U}\angle 0}{R_1+R_2}\right)}{\frac{1}{j\omega C} + \frac{R_1 R_2}{R_1+R_2}} = \frac{R_2 \hat{U}\angle 0}{R_1+R_2 + j\omega C R_1 R_2} \quad \text{etc.}$$

7a



Heres the original circuit with u_1, u_2, i_1, i_2 added.

We want to find $\frac{u(\omega)}{U(\omega)}$

What we know:

Coupled inductors:

$$u_1 = j\omega L_1 i_1 + j\omega M i_2 \qquad (1)$$

$$u_2 = j\omega L_2 i_2 + j\omega M i_1 \qquad (2)$$

KVL left $\quad U = u_1 + i_1 R_1 \qquad (3)$

right $\quad 0 = u_2 + i_2 R_2 \qquad (4)$

$(1) \rightarrow (3)$: $U = (R_1 + j\omega L_1) i_1 + j\omega M i_2 \qquad (5)$

$(2) \rightarrow (4)$: $0 = (R_2 + j\omega L_2) i_2 + j\omega M i_1 \qquad (6)$

subs. (6) for i_1 in (5) $\quad \left(i_1 = \frac{-(R_2 + j\omega L_2)}{j\omega M} i_2 \quad (6) \right)$

$$U = \left((R_1 + j\omega L_1) \cdot \frac{-(R_2 + j\omega L_2)}{j\omega M} + j\omega M \right) i_2$$

Now from the diagram

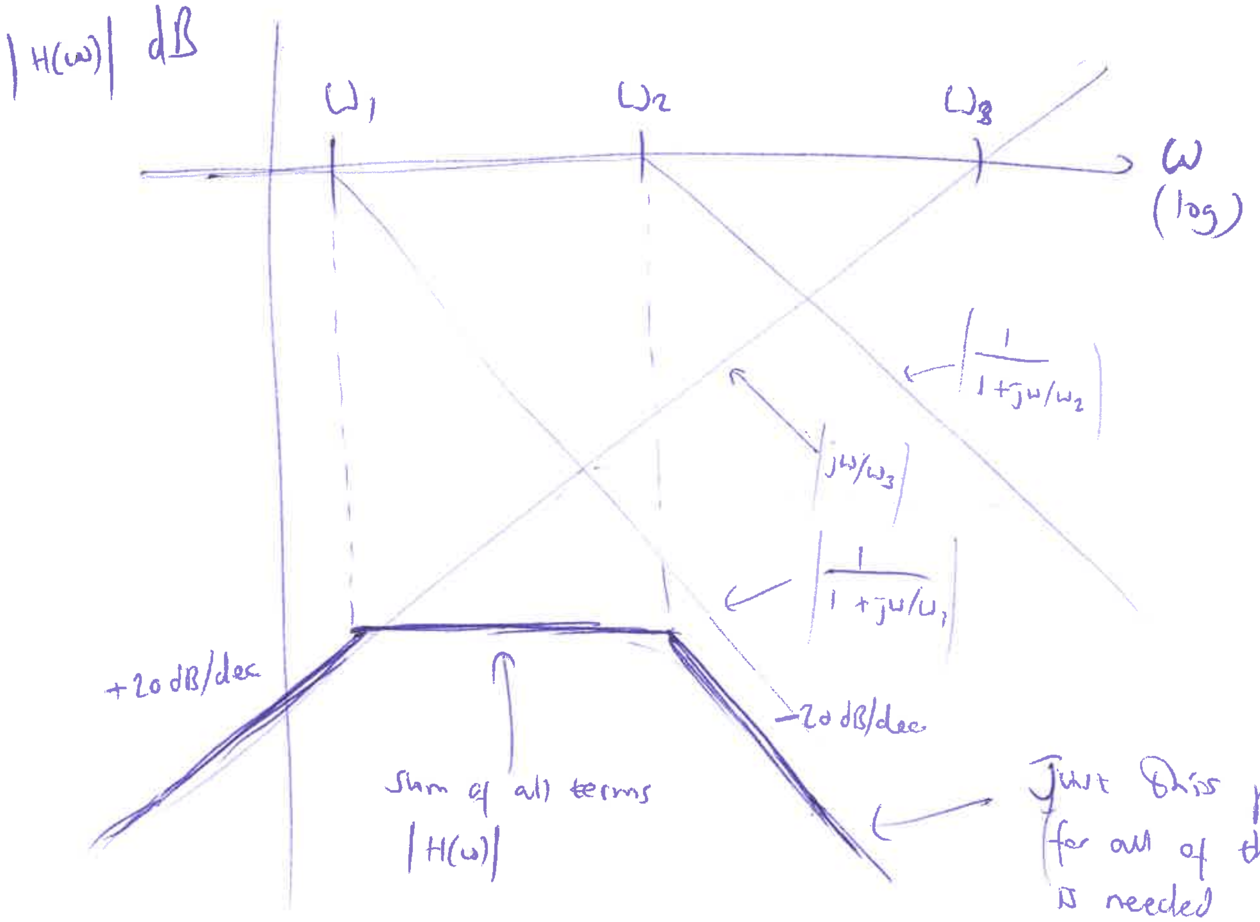
$$u(\omega) = u_2 = -i_2 R_2 = \frac{-R_2 U}{-(R_1 + j\omega L_1) \cdot \left(\frac{R_2 + j\omega L_2}{j\omega M} \right) + j\omega M}$$

$$\frac{u(\omega)}{U(\omega)} = \frac{R_2 j\omega M}{(R_1 + j\omega L_1)(R_2 + j\omega L_2) - j\omega M j\omega M} = \frac{j\omega M R_2}{R_1 R_2 (1 + j\omega L_1/R_1)(1 + j\omega L_2/R_2) + \omega^2 M^2}$$

$$\frac{u(\omega)}{U(\omega)} = \frac{j\omega M / R_1}{(1 + j\omega L_1/R_1)(1 + j\omega L_2/R_2) + \frac{\omega^2 M^2}{R_1 R_2}}$$

(7b) $$H(\omega) = \frac{j\omega/\omega_3}{(1+j\omega/\omega_1)(1+j\omega/\omega_2)}$$

Sketch a Bode amplitude plot of $H(\omega)$, given $\omega_1 \ll \omega_2 \ll \omega_3$.

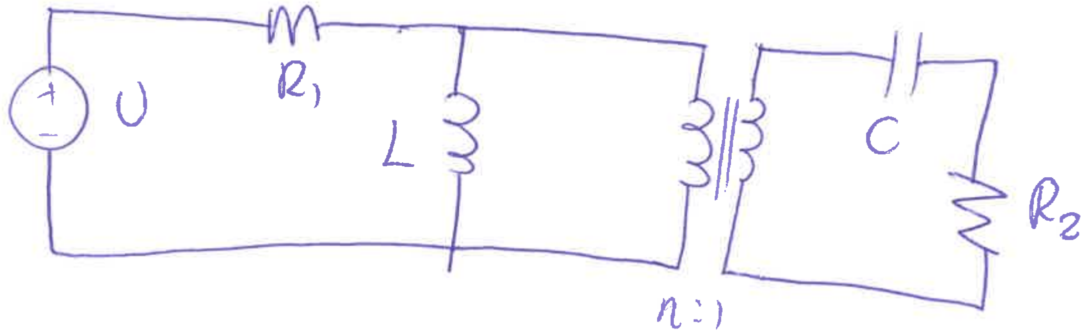


Just this part (for all of the terms) is needed

It should have the nonzero gradients and the change-points ω_1 ω_2 marked.

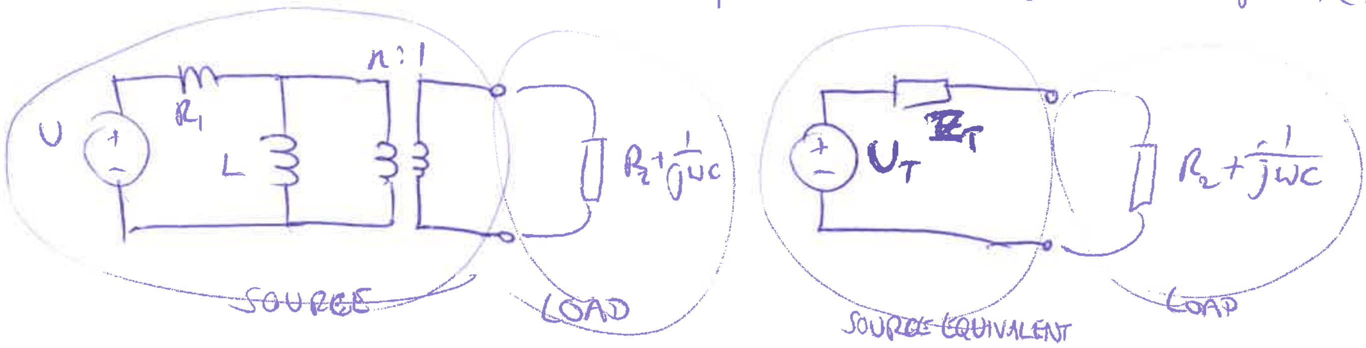
The whole curve of $|H(\omega)|$ should be well below 0 dB since $\omega_3 \gg \omega_1$ & ω_2 so "the down-slope starts before the up slope reaches 0 dB"

8

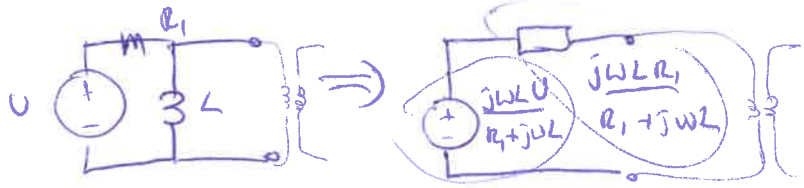


This is a very classic "ac maximum power" situation, where a clear "load" can be freely chosen to obtain maximum power from a fixed source. As the transformer doesn't ~~affect~~ consume or produce power (it just transfers it) we could include it in the "load" or "source". We'll choose to include it in the source this time.

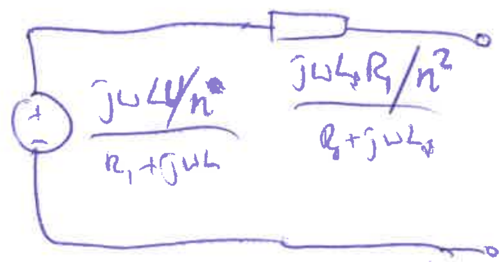
For ac maximum power, load and source impedances must be complex conjugates. Let's find the Thevenin equivalent of the source, to find its impedance.



Without the transformer the other source components are



With the transformer included by scaling the values, this gives:



So the source impedance is

$$Z_S = \frac{j\omega L R_1 / n^2}{R_1 + j\omega L}$$

We want $\boxed{Z_L = Z_s^*}$ for maximum power transfer.

$$Z_L = R_2 + \frac{1}{j\omega C} = R_2 - j\frac{1}{\omega C}$$

$$Z_s = \frac{j\omega L R_1/n^2}{R_1 + j\omega L} = \frac{j\omega L R_1 (R_1 - j\omega L)}{n^2 (R_1 + j\omega L)(R_1 - j\omega L)} = \frac{\omega^2 L^2 R_1 + j\omega L R_1^2}{n^2 (R_1^2 + \omega^2 L^2)}$$

Equating real and imaginary parts after the complex conjugate,

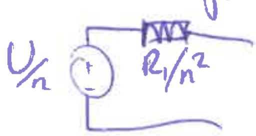
$$Z_L = Z_s^* \implies R_2 - j\frac{1}{\omega C} = \frac{\omega^2 L^2 R_1 - j\omega L R_1^2}{n^2(R_1^2 + \omega^2 L^2)}$$

$$\therefore \boxed{R_2 = \frac{\omega^2 L^2 R_1}{n^2 (R_1^2 + \omega^2 L^2)} \qquad C = \frac{n^2 (R_1^2 + \omega^2 L^2)}{\omega^2 L R_1^2}}$$

(b) We could find the maximum power by putting in the above values of R_2 and C then calculating the current and voltage at one or other side of the transformer, and using these to find the power transferred.
It would be a bit heavy to simplify (perhaps?).

Simpler is to notice that L does not affect maximum active power, as the load is chosen to resonate against this component (if we took a parallel equivalent of our load, its capacitor would have the same impedance magnitude as the inductor after scaling by the transformer ratio (squared))

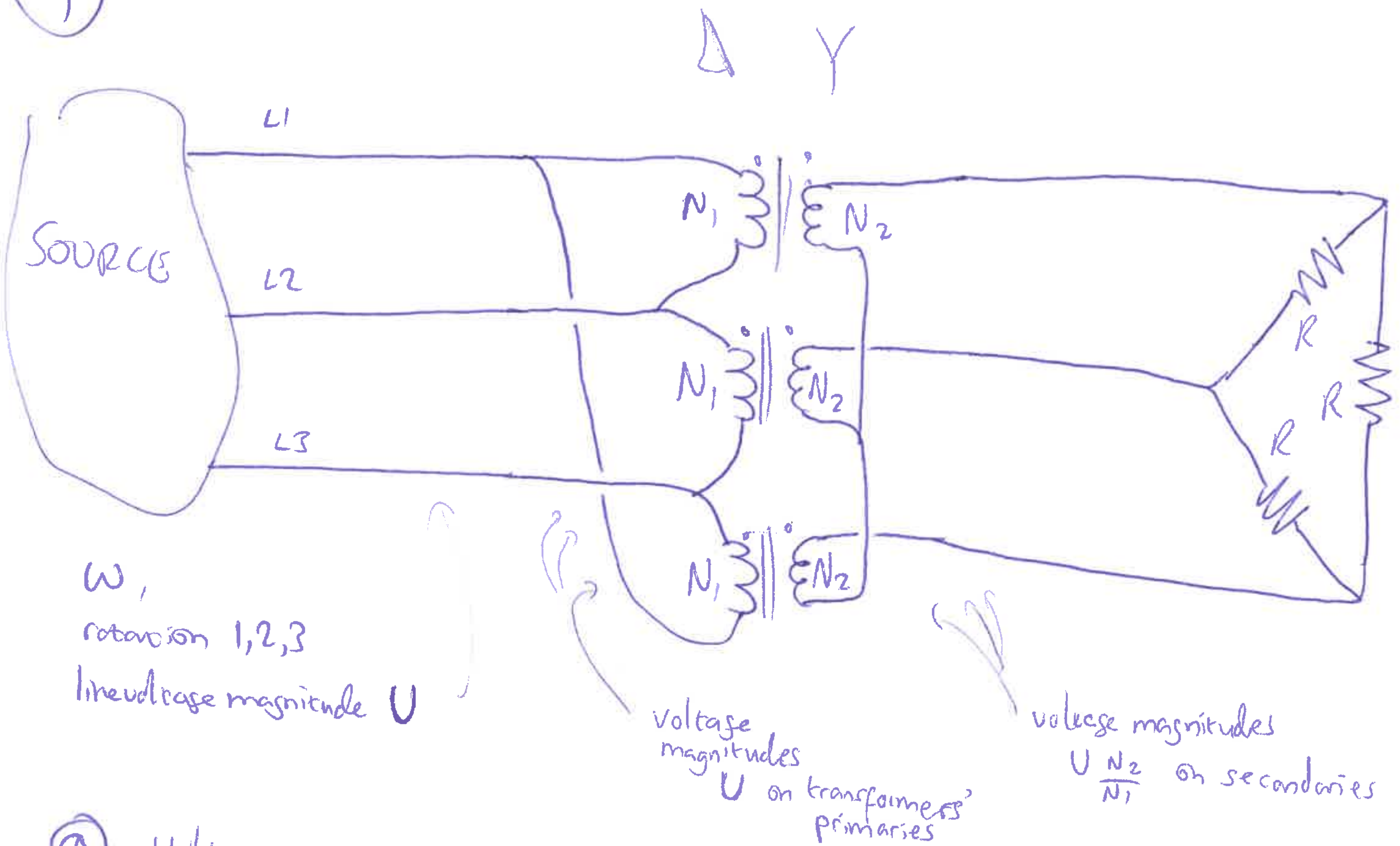
So our maximum power is the maximum power that:



can provide, which is $\frac{(U/n)^2}{4 R_1/n^2} = \boxed{\frac{U^2}{4R_1}}$ P_{max}

This is what passes across the transformer.

9



(a) Voltage across each resistor R.

Each transformer primary has a voltage U, (magnitude) as it connects between two lines, and line-voltage magnitude is U.

Each transformer secondary therefore has voltage $\frac{N_2}{N_1}U$ (magnitude).

The secondaries are connected in Y (star).
The line voltage out of the transformer to the load is therefore $\sqrt{3}\frac{N_2}{N_1}U$.

The resistors are connected line-to-line (Δ) so this is the voltage magnitude that they have.

$$\rightarrow |U_R| = \sqrt{3}\frac{N_2}{N_1}U$$

for any of the resistors

(b) The source supplies the load through lines with no mentioned impedance, and transformers that were not described as being anything except ideal (see exam instructions: everything is ideal by default!)

The complex power from the source is therefore the complex power to the load.

We know from (a) that each of the three resistors R has a voltage of $\sqrt{3}\,\frac{N_2}{N_1}U$ across it.

The total complex power is therefore $S = 3\frac{|U|^2}{R} = \frac{3\left(\sqrt{3}\,\frac{N_2}{N_1}U\right)^2}{R}$

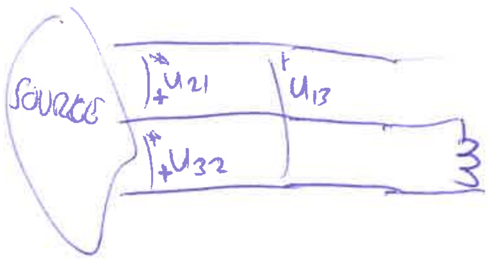
$$\boxed{S = 9\left(\frac{N_2}{N_1}\right)^2\frac{U^2}{R},}$$

from source to load.

(purely real)

(c) Source voltage u_{21} is the angle reference, $\angle u_{21} = 0$

Phase rotation 1,2,3 means $\angle u_{32} = -120°$



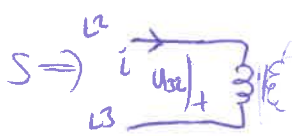
The situation is balanced three phase, so $\frac{1}{3}$ of the total power from Q(b) passes through each transformer.

So, for this transformer primary, the power in from the source is

$$\boxed{\frac{9\left(\frac{N_2}{N_1}\right)^2\frac{U^2}{R}}{3} + j0}$$

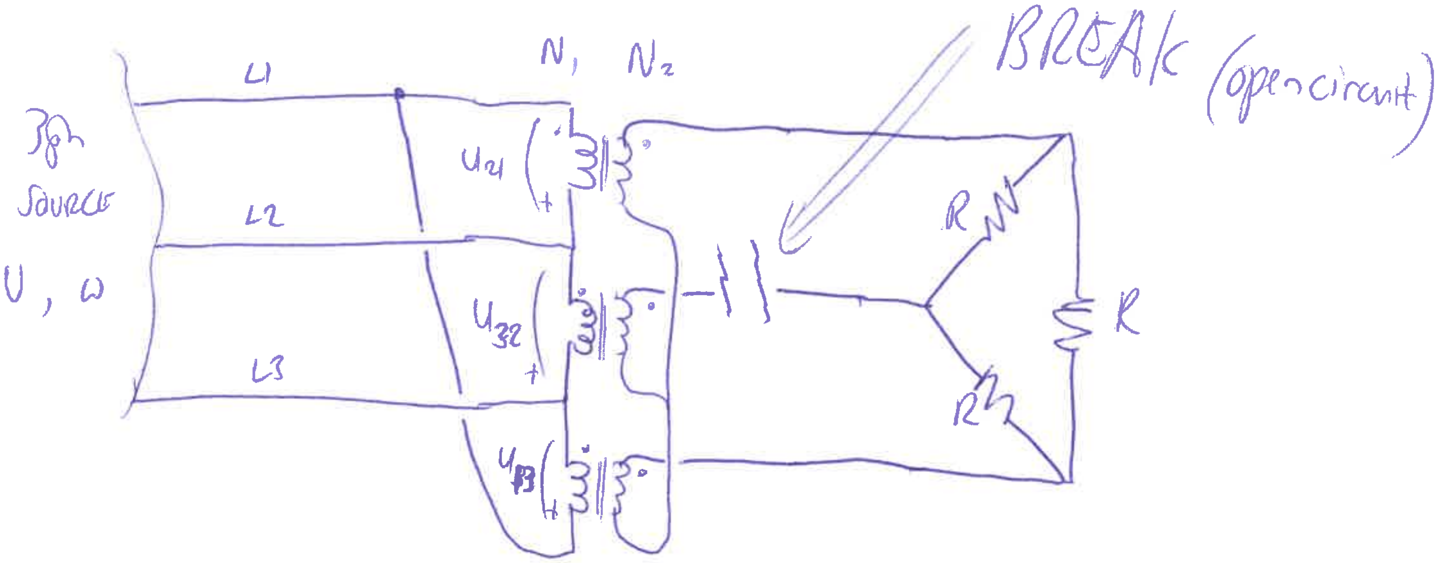
from $s = ui^*$, $i = \left(\frac{s}{u}\right)^*$

In our case we want current from line 2 to line 3,

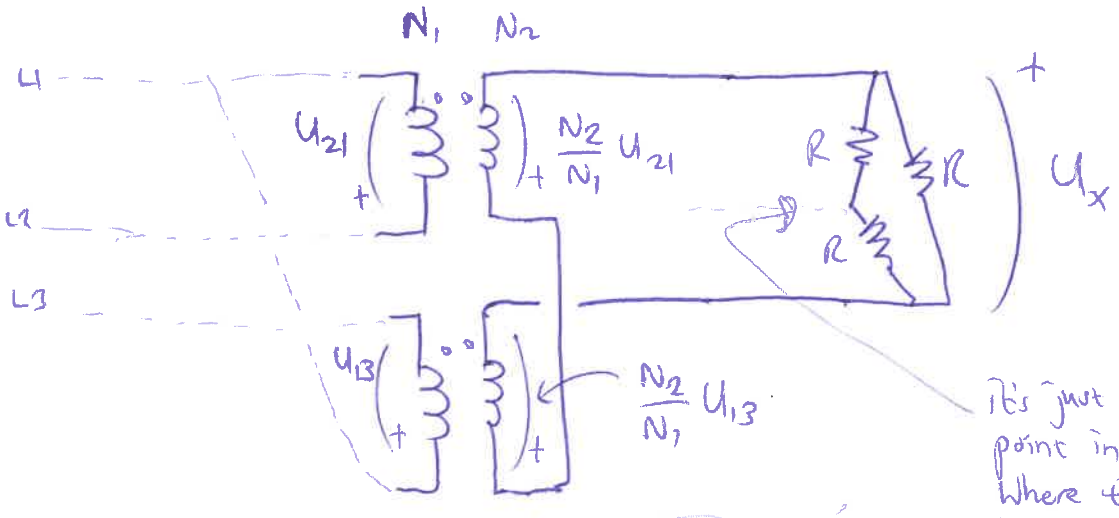


so we need $i = \left(\frac{-S}{u}\right)^* = \left(\frac{-\left(\frac{N_2}{N_1}\right)^2\frac{U^2}{R}\,3}{U\angle -120°}\right)^* = \boxed{\frac{3U}{R}\left(\frac{N_2}{N_1}\right)^2 \angle 60°}$

(d)



The break prevents all current in the middle transformer, since its secondary current is 0 so its primary current is also 0.
This transformer can be removed to simplify the circuit.



It's just this point in the Δ where the potential has now changed because it is disconnected from its source

The magnitude of U_x is still $\boxed{\sqrt{3}\,\frac{N_2}{N_1}U}$.
This could be worked out from $U_{21} = U\angle 0$, $U_{13} = U\angle{-240^\circ}$
and $U_x = \frac{N_2}{N_1}\left(U_{13} - U_{21}\right)$ by KVL.

But it's also reasonable to infer this from just removing the middle line from the transformer to the load, which leaves the other lines as before.

This voltage magnitude is applied to one resistor R and to two in series.
So the total load power is $\left(\sqrt{3}\,\frac{N_2}{N_1}U\right)^2\left(\frac{1}{R}+\frac{1}{2R}\right) = \boxed{\frac{9}{2}\left(\frac{N_2}{N_1}\right)^2\frac{U^2}{R}}$

(This is still the same as the source power.) (half as much as before the break)