**KTH, Electric Circuit Analysis, EI1120**

**Topic 01: Basics** Practice Exercises (for individual or group study)

Exercises and answers are alternated on the following pages, so that you don't initially see the answer when solving the problem, but you can easily check it by moving to the next page.

**Caution:** the last two questions could be very challenging at this stage. Don't spend lots of time on them. Doing all the others is recommended. It's important to have these basics firmly trained.

In the Exercises for later topics, there will be links to relevant questions from past exams.

However, after just this first topic we aren't ready for any of the exam questions: concepts from later Topics are needed too. The old KTH compendium also doesn't have many questions that we're able to handle at this point.

If you want other exercises, you might like something from the old repetitionshäfte.

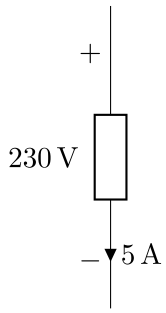
*This file was designed with the assumption of reading from a screen, where having lots of pages isn't wasteful.*

*More printer-friendly versions of the exercises can be found by removing the filename from this file's URL, i.e. going to **this list** of files, and selecting the file whose name ends in `_pr.pdf`.*

*If you like playing with computer things, you could instead download, edit and re-compile the Latex source-file (`.tex`).*

**Exercise 1**

Find the power consumed by the component:



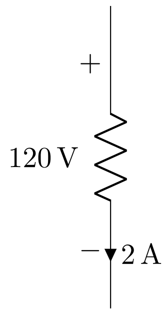

(You should assume the component is connected within a larger circuit, which we've not shown. Of course, it would violate KCL if the '5 A' just stopped at what looks like the end of the 'wire' [node] in this diagram.)

**Answer 1**

$P = 1150\,\mathrm{W}$

**Exercise 2**

Find the power consumed by the resistor:

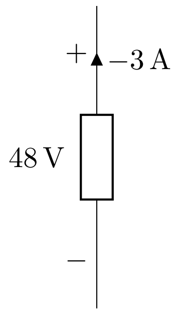


**Answer 2**

$P = 240\,\mathrm{W}$

**Exercise 3**

Find the power consumed by the component:

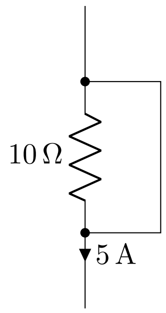


**Answer 3**

$P = 144\,\mathrm{W}$

**Exercise 4**

Find the power consumed by the resistor:



**Answer 4**

$P = 0\,\mathrm{W}$

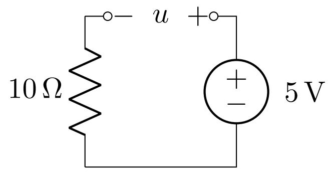
The resistor is "in parallel with a short-circuit (zero voltage)". KVL around the resistor and the short-circuit shows that the resistor must have zero voltage. Thus, no energy change happens to any current passing through the resistor, so no power goes in or out.

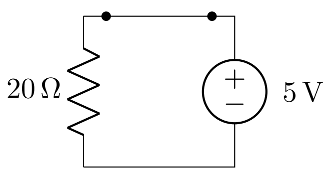
Indeed, Ohm's law tells us that (assuming $R$ is not the special case of a zero resistance), no current will pass in the resistor when there's zero voltage across it; so that gives a further reason that no energy can be moving in or out.

As another approach, one can think of the whole set of lines in the diagram forming a single node. There is therefore zero voltage on the resistor, since there is no difference in the potentials between its two sides.

**Exercise 5**

Find the marked voltage $u$, the power consumed by the $10\,\Omega$ resistor, and the power consumed by the $20\,\Omega$ resistor:



**Answer 5**

In the $10\,\Omega$ resistor, $P = 0\,\text{W}$.
The marked voltage is $u = 5\,\text{V}$.

The circuit contains a break (open-circuit), so no current can flow. We could express this formally by saying that an open-circuit demands zero current, then KCL demands zero current in the things that are series-connected with the open-circuit (which means both the other components!). By Ohm's law, a zero current in the resistor also means zero voltage across it: $u = iR = 0R = 0$. To find the voltage $u$ across the open-circuit, we can then apply KVL around the whole loop.

In the $20\,\Omega$ resistor, $P = 1.25\,\text{W}$.

The current in the resistor is $i = 5\,\text{V}/20\,\Omega = 0.25\,\text{A}$. The power into the resistor, taking care about directions, is then $0.25\,\text{A} \cdot 5\,\text{V} = 1.25\,\text{W}$.

Ohm's law ensures the voltage across a resistor is always higher potential at the side where current is going in (there has to be a higher 'pressure' at the input to force current to keep flowing 'against the resistance'). Thus power will always go in to a resistor, not out.

We can simplify the above calculation of power in a resistor, to

$$P = ui = uu/R = u^2/R.$$

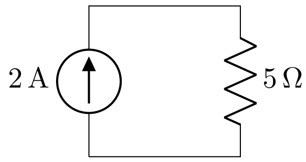
In cases where we know the current, a similar derivation gives,

$$P = ui = iRi = i^2R.$$

In these cases the squaring ($u^2$ or $i^2$) indicates that the direction of voltage or current won't affect the direction of the power flow, as discussed above.

**Exercise 6**

Find the power consumed by the resistor:



**Answer 6**

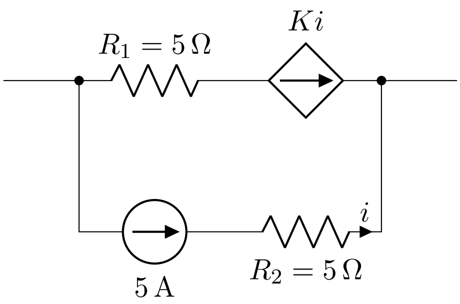
$P = 20\,\mathrm{W}$

Using the relation mentioned in the previous question, $P = i^2R$, gives $P = (2\,\mathrm{A})^2 5\,\Omega = 20\,\mathrm{W}$.

If you want to be very formal, you can invoke KCL to prove that the current in the resistor is $2\,\mathrm{A}$ (downwards). But we get used to doing such steps by familiarity, not mentioning the rules they're based on.

**Exercise 7**

Find the power consumed by the resistor $R_1$ ($P_{R_1}$) in the circuit for $K = 2$:

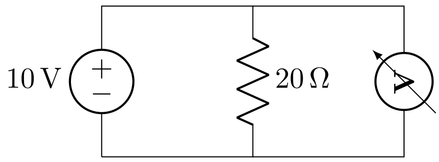


**Answer 7**

$P_{R_1} = 500\,\mathrm{W}$

**Exercise 8**

Find the power consumed by the resistor. What is the measurement shown on the screen of the voltmeter?

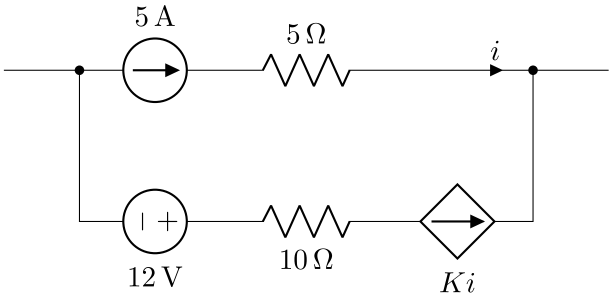


**Answer 8**

$P = 5\,\mathrm{W}$
The voltmeter displays $10\,\mathrm{V}$

**Exercise 9**

Find the power produced ($P$) by the voltage source in the circuit for $K = 5$:



**Answer 9**

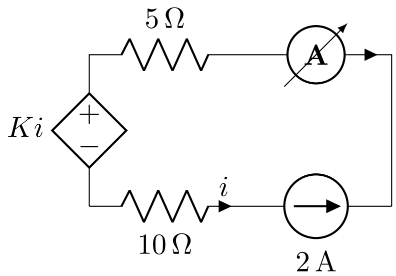
$P = 300\,\mathrm{W}$

**Exercise 10**

Is the controlled voltage source producing or consuming power, if $K = 1$?

How much power?

What will be the reading on the screen of the (ideal) ammeter? The ammeter's 'reference direction' is marked here by the arrow at the right of the ammeter symbol: a current actually flowing in that direction will be seen as positive.



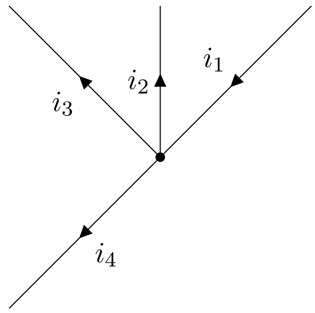
**Answer 10**

The controlled voltage source is consuming $P = 4\,\mathrm{W}$.
By KCL, the marked current $i$ is $2\,\mathrm{A}$. The dependent source therefore produces a voltage $Ki = i$ if $K = 1$. The current of $2\,\mathrm{A}$ passes all around this single-loop circuit, going down in potential through the marked voltage $Ki$. The power consumed by the dependent source is therefore $2\,\mathrm{A} \cdot 2\,\mathrm{V} = 4\,\mathrm{V}$.

The ammeter shows $-2\,\mathrm{A}$. The $2\,\mathrm{A}$ current source is pushing current in the direction opposite to the ammeter's reference direction, so it is seen as negative.

**Exercise 11**

Formulate the KCL equation for the marked currents within this node:



**Answer 11**

KCL for the given circuit can be expressed as:

$i_1 = i_2 + i_3 + i_4$ (by comparing "in versus out" ), or

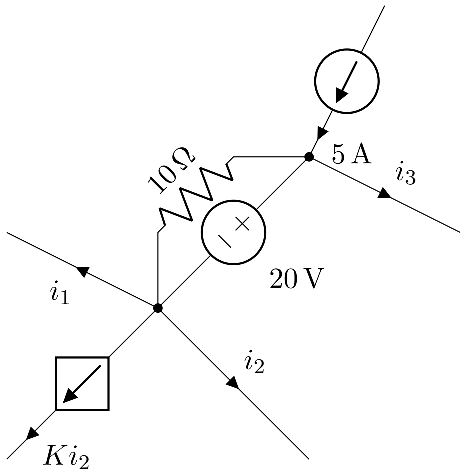
$i_2 + i_3 + i_4 - i_1 = 0$ (total current defined outward should be zero), or

$i_1 - i_2 - i_3 - i_4 = 0$ (total current defined inward should be zero).

These were written according to different choices of how we 'implement' KCL. They look different, but they are all in fact the same equation (they can be rearranged to be the same).

**Exercise 12**

Formulate the KCL for the circuit.
What must the value of $K$ be if $i_1 = i_2 = i_3 = 1\,\mathrm{A}$?
(You do not need to use all of the component values in your calculation.)



**Answer 12**

$K = 2$

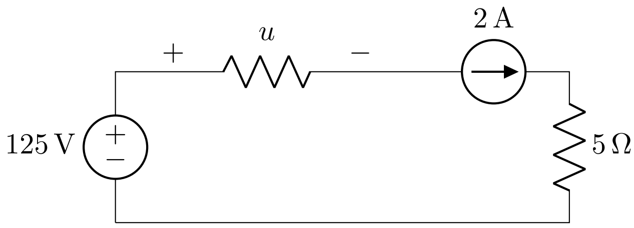
Taking KCL for all the currents entering or leaving the given circuit,

$$5\,\mathrm{A} = i_1 + i_2 + i_3 + Ki_2.$$

Then substitute the given information that $i_1 = i_2 = i_3 = 1\,\mathrm{A}$, to find $K$.

**Exercise 13**

Find the voltage $u$ if the power produced *by* the current source (i.e. given from the source to the circuit) is $P = 100\,\mathrm{W}$:



**Answer 13**

$u = 165\,\mathrm{V}$

KVL for the given circuit will be $125\,\mathrm{V} - u + 50\,\mathrm{V} - 10\,\mathrm{V} = 0$.

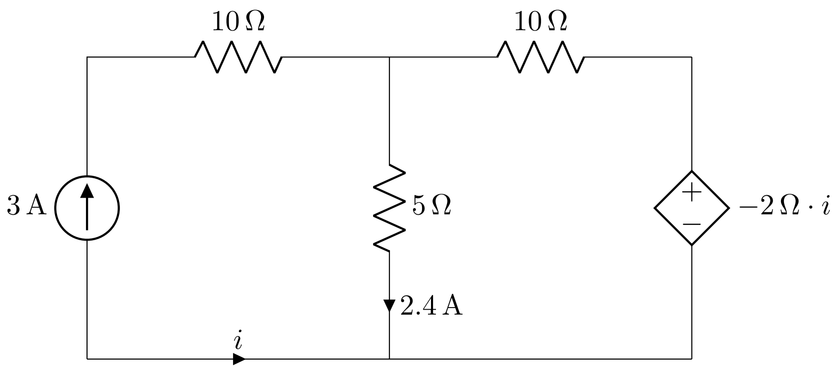
The $-10\,\mathrm{V}$ is due to the current of $2\,\mathrm{A}$ through the $5\,\Omega$ resistor.

The $50\,\mathrm{V}$ is due to the size and direction of voltage that there must be on the current source, if this source is providing $100\,\mathrm{W}$ to the circuit. In order to do that, a $2\,\mathrm{A}$ source must give the current an *increase* of potential of $50\,\mathrm{V}$ (i.e. $100\,\mathrm{W}\ /\ 2\,\mathrm{A}$).

**Exercise 14**

Find the power produced by the controlled voltage source (CCVS).

The marked current value (2.4 A) will help you.
(This value can be worked out from the rest of the circuit, as all component values and connections are defined; but that would be a rather hard calculation for this stage in the course. Perhaps you'd like to try?)




A little point:

In this case we have specified the CCVS's *value* as a number-and-unit pair ($-2\,\Omega$), instead of as a symbol such as $h$ or $K$ etc.

It might look strange that it's in ohms, but this simply reflects that the value of the source is a ratio between a voltage (its output: it's a voltage source!) and a current (its 'input': the controlling-variable $i$).

In contrast, a CCCS or VCVS has a dimensionless value, as these components' values relate quantities that have the same dimension: voltage/voltage or current/current. And a VCCS has a value that could be expressed in siemens ($\frac{1}{\Omega}$).

**Answer 14**

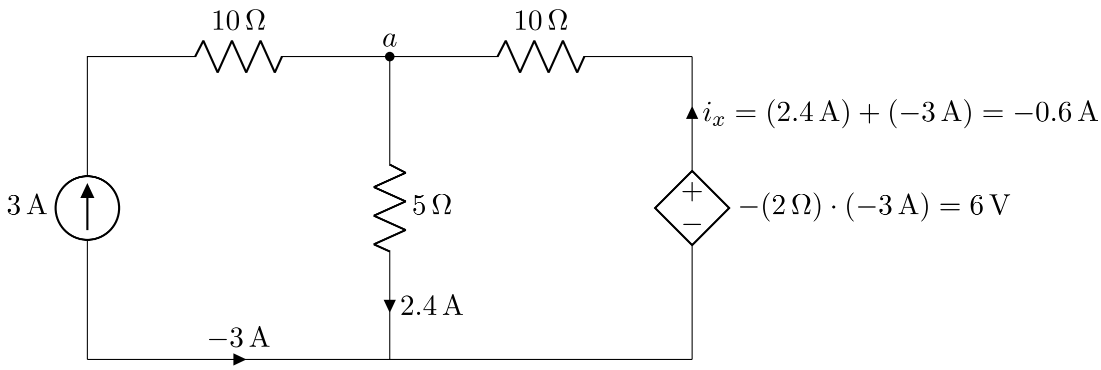
Power out of the controlled source.

As with any two-terminal component, this power can be found by defining the component's current and voltage, finding what these are, then multiplying them *being careful to use the appropriate + or − sign, depending on the directions in which the current and voltage were defined!*.

If we want the product of current and voltage to give the power *out* of the component (the power delivered to the circuit by the component), then we need to define the current to be going "up" through the defined voltage, so that it is gaining the energy that this voltage describes. This relative direction of the markings of voltage and current is called the *active convention*, as it's convenient for situations where we expect power to be coming out: i.e. active components such as sources. If instead the *passive convention* is used, with one of the directions reversed, then we need a negative sign when calculating power output.

The CCVS's voltage is already marked, and can be found using the information of this component's value ($-2\,\Omega$) and its controlling variable ($i = -3\,\mathrm{A}$).

Let's define the CCVS's current as $i_x$, which follows the active convention.




KCL at node $a$ or at the bottom node gives a current of $i_x = (2.4\,\mathrm{A}) + (-3\,\mathrm{A}) = -0.6\,\mathrm{A}$.

The power delivered by the CCVS is therefore

$$P = (6\,\mathrm{V}) \cdot (-0.6\,\mathrm{A}) = -3.6\,\mathrm{W}.$$

This is a negative value: that means that the controlled source is actually recieving power *from* the circuit.

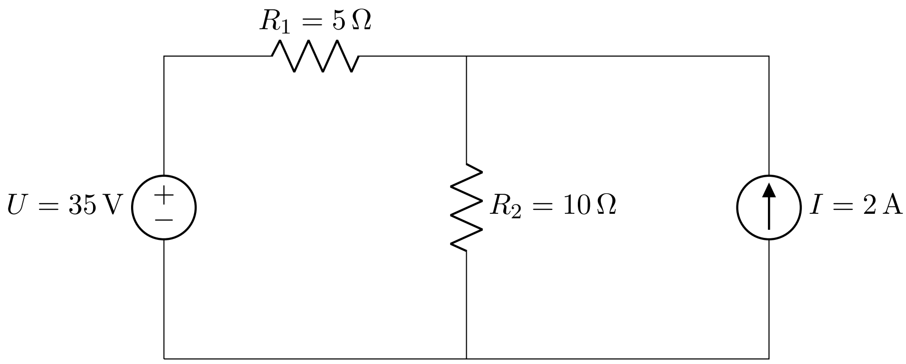
It's nice that as long as we make clear markings, write the correct expressions, and do the algebra correctly, we don't have to think about whether the power is actually going in or out: we choose to calculate one way or the other, and the algebra gives us a negative value if the power flow is actually the other way.

**Exercise 15**

Find the power consumed by the resistor $R_2$.

This problem is hard to solve by just taking small steps in sequence (why?).

You will probably need to write down two equations, then solve them together. Don't waste lots of time on this. It's just a challenge for people who want to get into the algebra already.



**Answer 15**

$P_{\mathrm{R2}} = 90\,\mathrm{W}$

$i_{\mathrm{R2}} = 3\,\mathrm{A}$

Methods from later topics, of source-transformation or nodal analysis, would provide good standard ways of solving this circuit.

The main trouble is that no one thing can be immediately solved. We don't know the voltage across $R_2$. We can't assume that $I$ passes through $R_2$, as KCL tells us that the currents from the right *and* left branch are combined in $R_2$. Without knowing the voltage across $R_2$ we can't work out the current through $R_1$.

It's this 'interaction' between the sides that makes it complicated. We *can* of course solve it using KCL, KVL, Ohm's law and the source definitions. But we have to start with an equation where the unknown appears several times, or else with solving simultaneous equations.

For example:

Let's *define* the unknown voltage across $R_2$ as $u$, with its +-reference upward.
When stuck, it's often a good idea to define something[s] and continue! Then we can use that definition in later equations for KVL, etc., being careful to remember that it's an unknown quantity and that we will need to find as many independent equations as the number of unknowns.

The current down through $R_2$ is then $u/R_2$.
By KCL at the node above $R_2$, this current is the sum of $I$ (from the right) and $(U-u)/R_1$ (from the left, found by KVL and Ohm's law).

Putting these together, $\frac{u}{R_2} = I + \frac{U-u}{R_1}$, which becomes $u\left(\frac{1}{R_1} + \frac{1}{R_2}\right) = \frac{U}{R_1} + I$.

Rearranging to give $u$, we get $u = \frac{UR_2 + IR_1R_2}{R_1+R_2} = 30\,\mathrm{V}$.

By the relation $P_{\mathrm{R2}} = \frac{u^2}{R_2}$ the requested power can be found to be $90\,\mathrm{W}$.

By Ohm's law, $i_{\mathrm{R2}} = \frac{u}{R_2} = 3\,\mathrm{A}$.

As usual, many other approaches can also be used!