KTH, Electric Circuit Analysis, EI1120

Topic 02: Simplifications Practice Exercises

The following pages have exercises that are intended to start quite simply, then to build up in level, covering the main points of this Topic.

As usual, our assumption/recommendation is that your main material is the website's:
NOTES introduce new ideas,
EXERCISES practice for yourself,
TUTORIAL QUESTIONS practice with help,
HOMEWORK finish your practice of the topic, before the next topic starts, and
PAST EXAM QUESTIONS a realistic check of how it's going.

The above probably provides more than enough to fill your available study time. You need to prioritise. Do not spend too much time on e.g. a single question where you find can't make the algebra work! More important is to learn and practice the main circuit principles of the topic: you'll practice the maths in other courses too. But if you finish it all and want more – or if you just prefer other sources – then that's fine too. The sequence of new concepts is usually different in each book or website: be careful to focus on what you actually need for this course.

If you want **more exercises**, there are some good ones about equivalent resistors and division in the old compendium: `övningsexempel`. See Ex-2.1 up to Ex-2.21.
Notice that some of these have more detailed solutions later in the file (e.g. L-2.5 on page 46).

Most **past exam questions** use skills from later Topics, so they are not suitable for trying now. However, you could try some of the first subquestions (e.g. 'a' and 'b' and perhaps 'c') from the following exam-questions on the [exams-webpage]:

2015-01˙E˙omtenta1 Q3

2015-06˙IT˙omtenta Q1

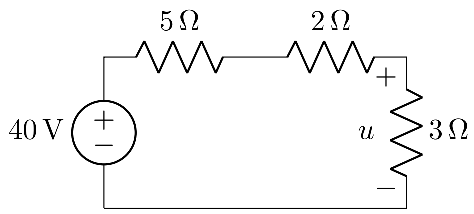
2015-03˙EM˙tenta Q1

These are mainly based on applying KCL and KVL (and Ohm's law) in suitable ways. Sometimes it can be simpler than it looks, as long as you choose the most suitable loop or node(s) for applying Kirchhoff's laws: in the first subquestions it often happens that most of the shown components are irrelevant to the answer.

Most other old exams from 'EI11xx' circuits courses in the past two or three years have similar questions, too. Look for those questions that have several subtasks (i, ii, etc) that require finding a current, voltage or power. Such questions commonly appear as Q1.

Exercise 1

Find voltage u.



Answer 1

$u = 12\,\mathrm{V}$

Combine the other two resistors (to $7\,\Omega$) then use voltage division.

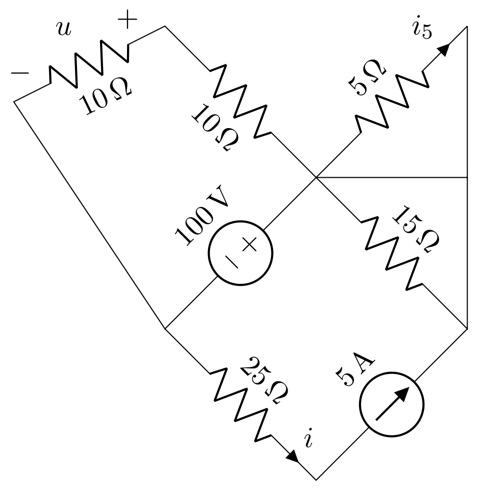
Or, think of the resistors as being equivalent to $10\,\Omega$, which can be made up of ten $1\,\Omega$ resistors in series, each of which gets $\frac{1}{10}$ of the total voltage; the voltage u is then the voltage across three of these $1\,\Omega$ resistors. Hence, $\frac{1}{10} \cdot 40\,\mathrm{V} \cdot 3$.

Or, calculate the current (which must pass in all components, due to KCL in the series connection) then the voltage across this resistor can be found by Ohm's law.

If you want to think formally about each step, you might want to use KVL to show that the voltage across all the resistors together is $40\,\mathrm{V}$; but we get used to just seeing this directly: we'd say that the combination of 3 resistors is "connected directly across a $40\,\mathrm{V}$ source".

Exercise 2

Find voltage u and currents i and i_5.



Answer 2

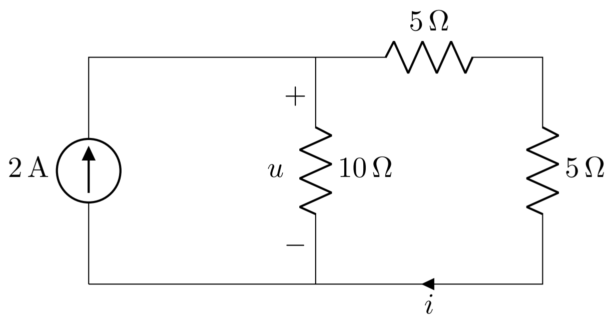
$u = 50\,\mathrm{V}$
The two $10\,\Omega$ resistors are connected in series, and this combination is in parallel with the $100\,\mathrm{V}$ source (as well as with other components). The source sets the voltage across the two resistors, regardless of the rest of the circuit. Voltage division can then be used between the resistors.

$i = 5\,\mathrm{A}$
This is a classic easy case, as long as you notice (KCL) that this current must be the same as the current in the $5\,\mathrm{A}$ current source.

$i_5 = 0\,\mathrm{A}$
This resistor is connected at both sides to the same node (note the line below it); so both sides have the same potential, meaning that the voltage across the resistor is zero, which by Ohm's law means the current is zero.
Alternatively, you can say that it is in parallel with a zero voltage-source (short-circuit), so the other components become irrelevant.
Or, just take KVL around the loop of the resistor and the node that passes below it.

Exercise 3

Find current i and voltage u.



Answer 3

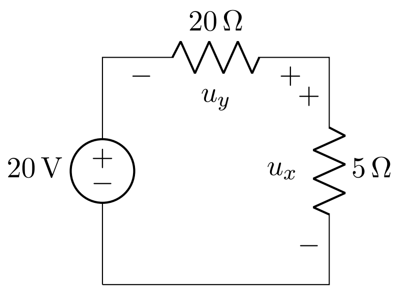
$i = 1\,\mathrm{A}$
The two $5\,\Omega$ resistors can be combined into an equivalent, then current division can be used between the two parallel branches. (In these questions with artificially simple numbers, we should get used to seeing that half the source current goes in each of the equal (total $10\,\Omega$) branches of the circuit, without needing to calculate it formally. It's good to get used to estimating the answer, even in situations where the numbers demand a calculation.)

$u = 10\,\mathrm{V}$
The two parallel branches can be combined into a single equivalent resistor of $5\,\Omega$. The voltage across this equivalent is $5\,\Omega \cdot 2\,\mathrm{A} = 10\,\mathrm{V}$. This must be the same as the voltage across the current source (KVL), which is the same as u.
Alternatively, one could use the result from calculating i, to note that $1\,\mathrm{A}$ must flow in the $10\,\Omega$ resistor, and thus get the answer by Ohm's law.

We mustn't get careless about signs! These questions have had answers that are positive; but we need to be careful about the directions of the marked currents and voltages, in case another circuit actually has a negative answer.

Exercise 4

Find voltages u_x and u_y.



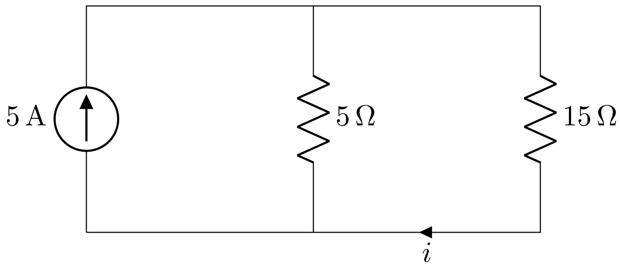
Answer 4

$u_x = 4\,\mathrm{V}$ – a straightforward voltage division.

$u_y = -16\,\mathrm{V}$ – similarly, but careful about the direction (sign)!

Exercise 5

Find current i.



Answer 5

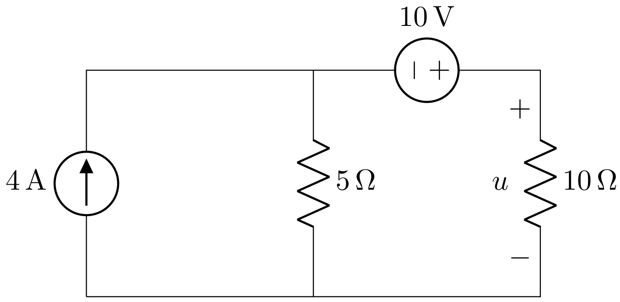
$i = 1.25\,\mathrm{A}$
The quickest method is current division.

Exercise 6

Find voltage u.

Can you (directly) use current-division to find the current in the $10\,\Omega$ resistor?

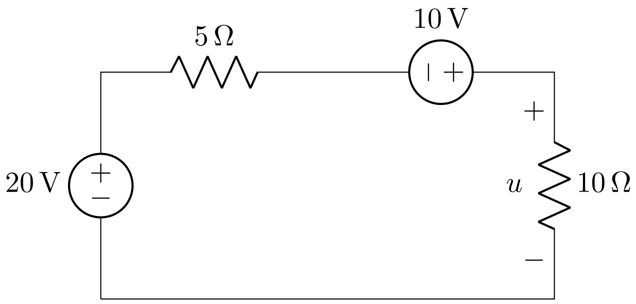
Can you apply another method from Topic 2, to simplify the circuit?



Answer 6

$u = 20\,\mathrm{V}$

The most obvious simplification (from Topic 2) is source-transformation of the two parallel components on the left, to produce a single loop.



Then, KVL around the loop allows the current to be found, and thereby (Ohm's law) the sought voltage u.

Note the danger of trying source-transformation on the components on the *right*, to make a totally parallel circuit (two current sources, two resistors) instead of the totally series circuit above. That is bad because the sought quantity u is on one of the transformed components. We don't want to do lose this: when a Thevenin source of series U and R is transformed to an equivalent Norton source of parallel $I = U/R$ and R, it behaves identically *at its pair of terminals* but the current (and thus the voltage) in the resistor R can be different in the two cases. It it only the quantities *outside* the transformed region that are guaranteed to stay the same after the transformation.

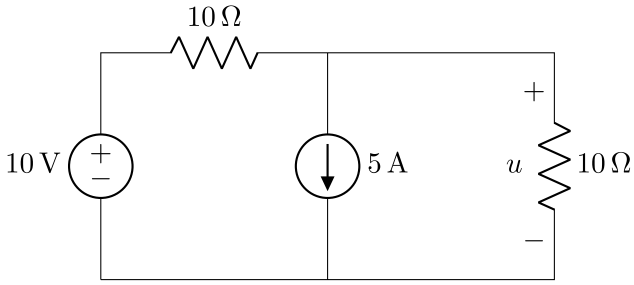
Another way to solve this would be by using KCL and/or KVL on the original circuit to find simultaneous equations that can be solved for u. This avoids using any of the methods from Topic 2. It's probably slower.

The need for solving two equations together is that both sources in this circuit affect the quantities such as u. If instead the sources were swapped, then it would be simpler ... try!

Later, we'll learn about node analysis and superposition, which are further good ways to deal with this sort of circuit.

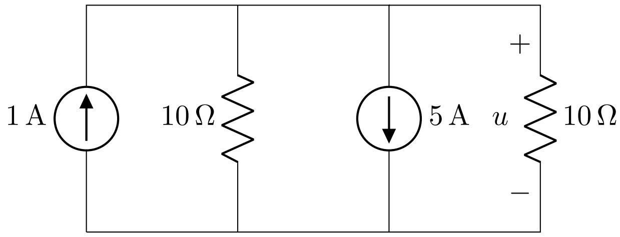
Exercise 7

Find voltage u.



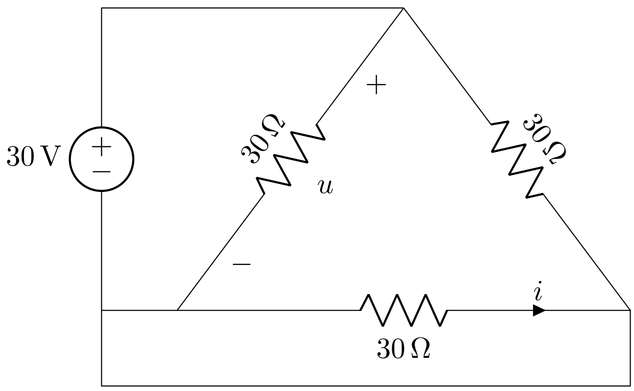
Answer 7

$u = -20\,\mathrm{V}$
The sources can be made into one equivalent of $4\,\mathrm{A}$, downwards.
The resistors can be made into a parallel equivalent; alternatively, current-division and Ohm's law can be used.



Exercise 8

Find voltage u and current i.



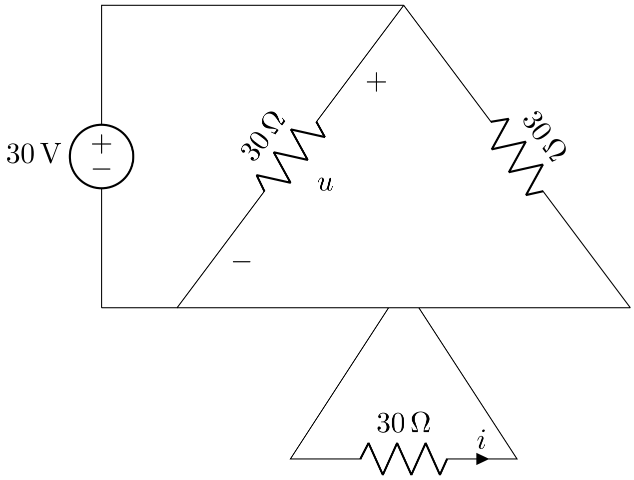
Hint: it doesn't really require calculation or much time.

Answer 8

$u = 30\,\mathrm{V} \quad i = 0\,\mathrm{A}$

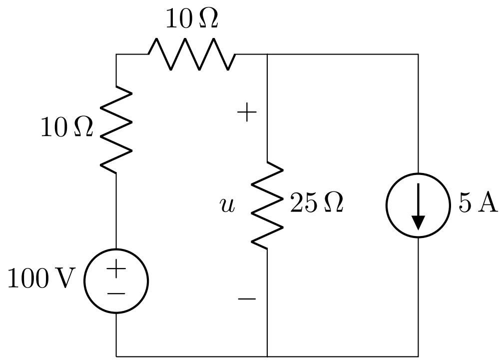
The bottom node can be re-drawn in an equivalent but simpler way.

This shows also that the bottom resistor was short-circuited, hence $i = 0$, by KVL (zero voltage) and Ohm's law (zero voltage means zero current in a resistor).



Exercise 9

Find voltage u.

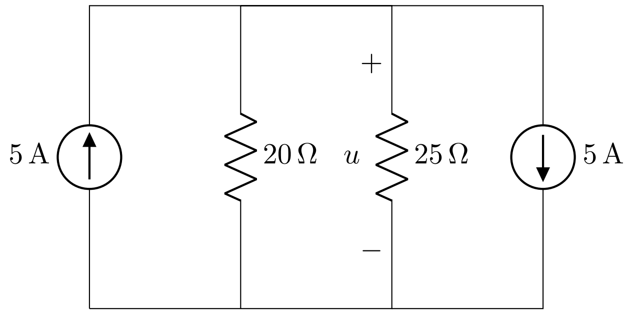


Answer 9

$u = 0\,\mathrm{V}$

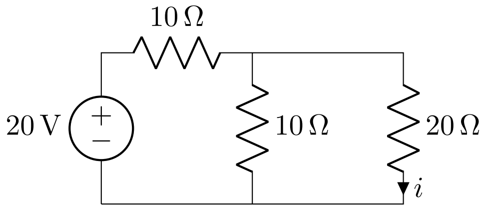
All current from one source passes back through the other: their equivalent is zero.

(This circuit just happens to have component values such that the sources 'balance'.)



Exercise 10

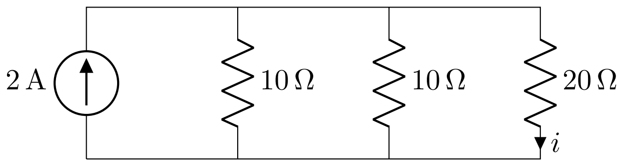
Find current i.



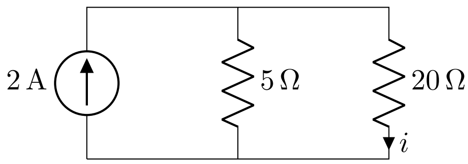
Answer 10

$i = 0.4\,\mathrm{A}$

By source-transformation, the circuit can be drawn in the following way, without changing the current i.



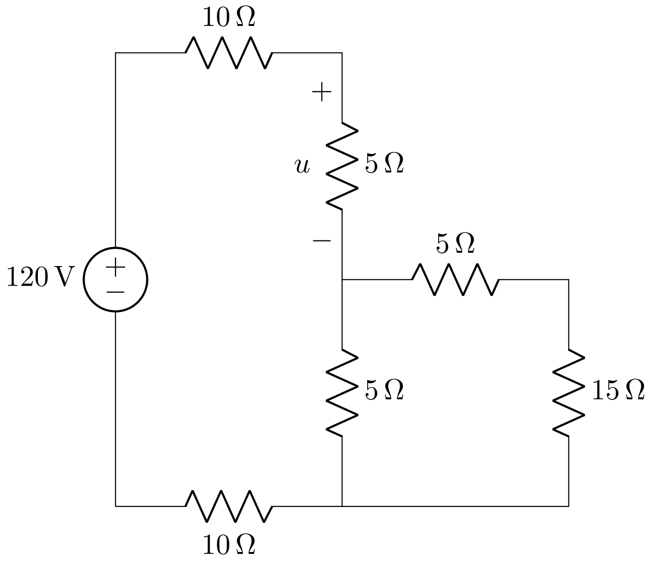
After simplification (combining two parallel $10\,\Omega$ resistors), current-division can be used.



Several other approaches can also be used, such as combining the parallel $20\,\Omega$ and $10\,\Omega$ resistors in the original circuit, then using current division.

Exercise 11

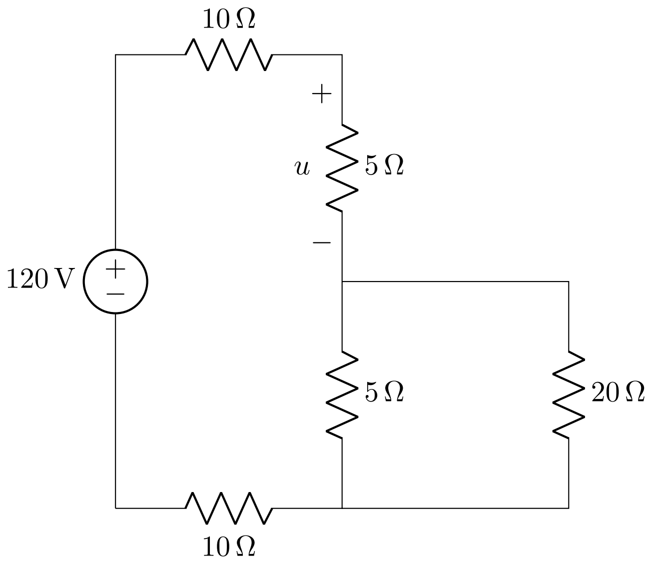
Find voltage u.

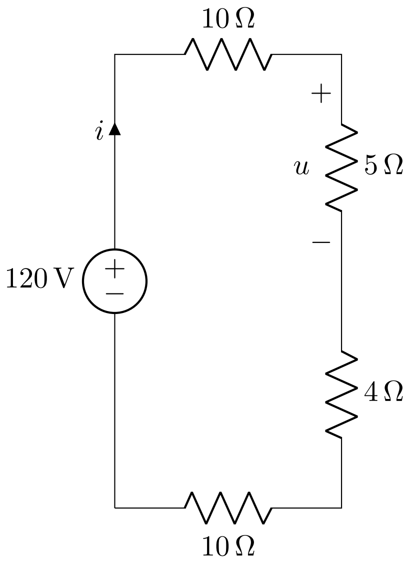


Answer 11

$u = 20.69\,\mathrm{V}$

Probably the best way to start is to turn the circuit into a single loop, by combining the parallel branches: first the $5\,\Omega$ and $15\,\Omega$ in series, then the two branches in parallel.



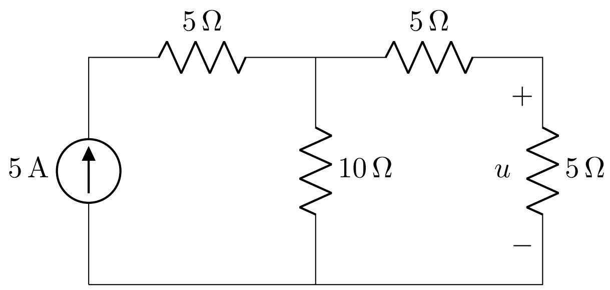


After this, one approach is to find the current i, then from this find the voltage u by Ohm's law. (We can find i by adding all the resistors in series, $i = \frac{120\,\mathrm{V}}{29\,\Omega} = 4.14\,\mathrm{A}$.)

Another way would be to notice that we can use voltage division between the $5\,\Omega$ resistor and the total resistance of all the others in series. This could be proved in the same way as we proved voltage division for the basic case of two resistors.

Exercise 12

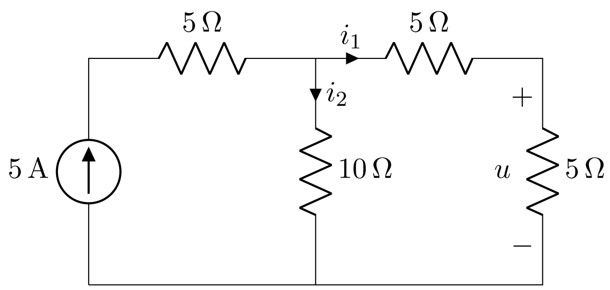
Find voltage u.



Answer 12

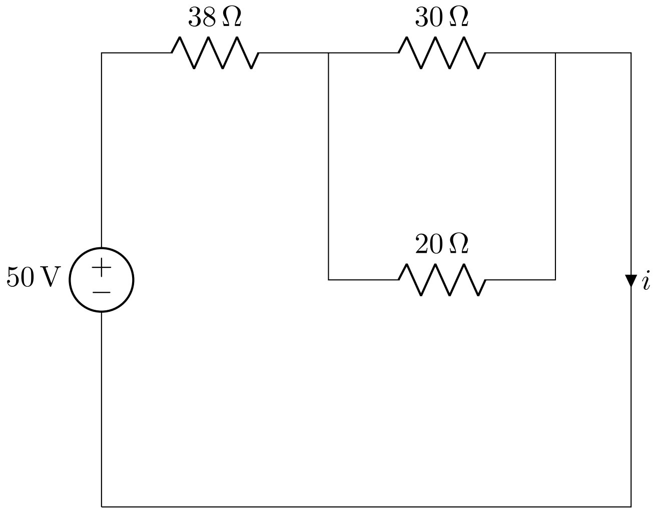
$u = 12.5\,\mathrm{V}$

The resistor in series with the current source is irrelevant to u.
The source current must split in the branches so that $5\,\mathrm{A} = i_1 + i_2$
The series resistors have a $10\,\Omega$ equivalent: current division gives $i_1 = 5\,\mathrm{A}/2$.
By Ohm's law, $u = i_1 \cdot 5\,\Omega = 12.5\,\mathrm{V}$.



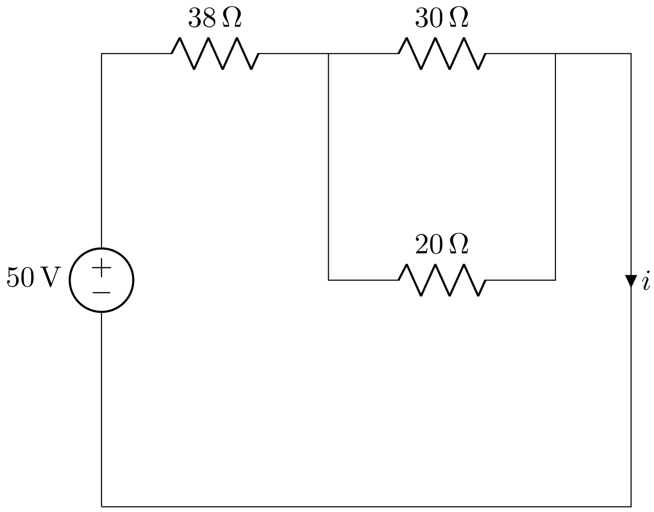
Exercise 13

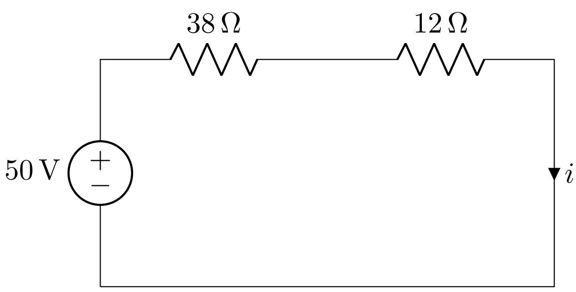
Find current i.

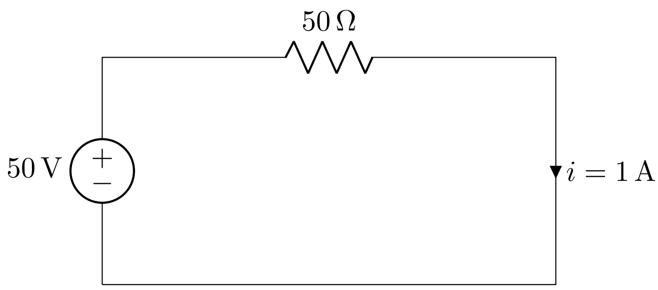


Answer 13

$i = 1\,\mathrm{A}$

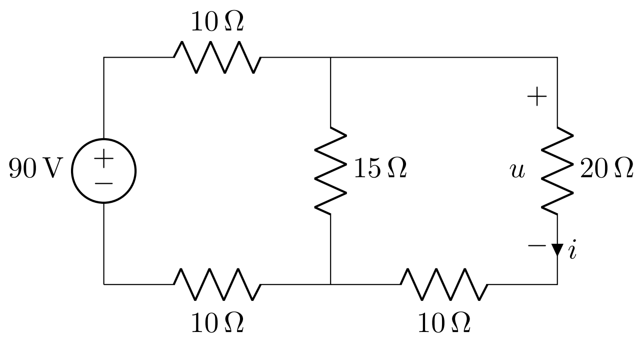






Exercise 14

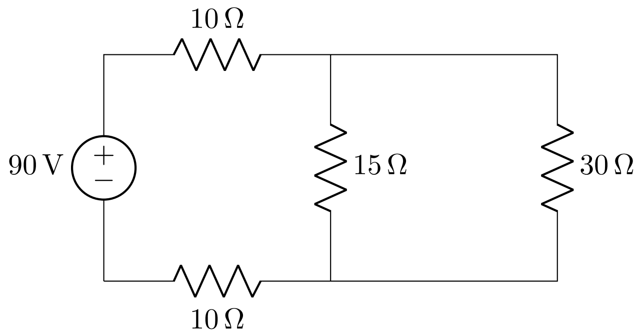
Find voltage u and current i.

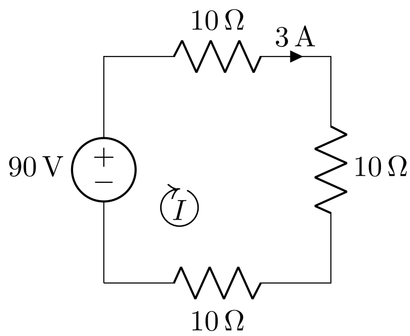


Answer 14

$i = 1\,\mathrm{A} \quad u = 20\,\mathrm{V}$

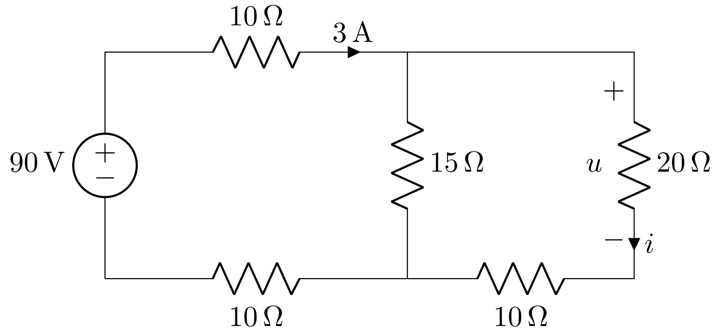
One method starts by simplifying the three resistors on the right into a single one, to get a simple series circuit where the current from the source can be found.





$(I = 3\,\mathrm{A})$

The above steps might look a bad idea, as the marked current we want to find (i) has then disappeared. But because we now have found the current from the source, we can put back the actual resistors and calculate the current i. Current-division is a good method in this case. The $3\,\mathrm{A}$ current has a $15\,\Omega$ and a $30\,\Omega$ path in parallel.



(Without even using the current-division formula, we can think that twice as much current goes through the twice as easy path, i.e. the $15\,\Omega$ one. Thus, the current in the other path must be $\frac{1}{3}$ of the total.)