KTH, Electric Circuit Analysis, EI1120

Topic 05 Practice Exercises

As well as the exercises on the following pages, the following old exam-questions are good practice.

The earlier exercises are easier than typical exam questions, and the last few are rather *more* algebraicly demanding than the exams.

Recommendation: do not bother with the final three exercises unless you've somehow done all the others and the homework and some exam questions, and really want more! They were mainly for Elektro students who might go into electronics in their future courses.

Some general questions about opamps in dc circuits:

2015-06˙EM˙omtenta Q3

2014-09˙E˙ks1 Q3

2014-05˙EM˙omtenta Q3

2014-02˙EM˙ks1 Q3

2014-03˙EM˙tenta Q3 (tricky last-part)

Some questions where an opamp is included in a classic question of "use nodal analysis to write equations, but don't solve them":

2014-10˙E˙tenta1 Q2

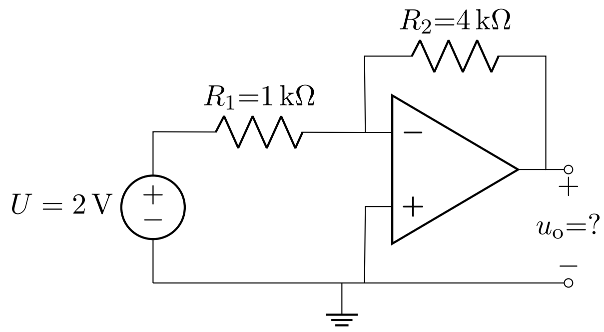
2015-01˙E˙omtenta1 Q2

Exercise 1

Find the output voltage u_o.

Think of the ideal opamp, together with the two resistors, as an amplifier circuit; its input is the voltage source and its output is marked as u_o.

What is the input resistance of this amplifier (i.e. the resistance that the voltage source 'sees' in the thing it's connected to)?



Answer 1

$u_o = -8\,\text{V}$

This is a standard case of an inverting amplifier. Using KCL at the inverting input, or just looking for "current balance" between R_1 and R_2, the output voltage must be 4 times the input, and inverted (negated). See Notes on Topic 05 for a derivation.

Reminder: you are advised to have read the Notes (Chapter) before trying the exercises.

$R_\text{i} = 1\,\text{k}\Omega$ (where R_i denotes input resistance).

The non-inverting input is fixed to 0 V. By negative feedback we expect the inverting input also to be fixed to that potential. The inverting input in this circuit is called "virtual earth": it is not part of the earth node, but it is forced to the same potential as the earth node, by currents flowing from/to the opamp output through R_2.

So resistor R_1 has always got 0 V at the right. Thus, the current into it from the voltage source is $\frac{U}{R_1}$. The input resistance is simply $R_\text{i} = R_1$.

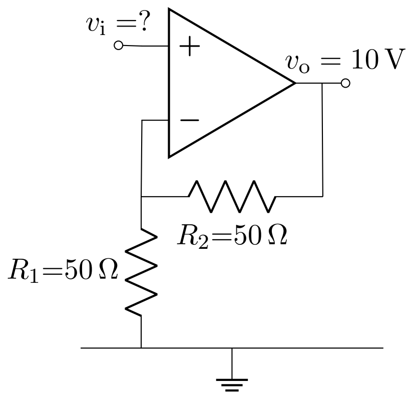
The general meaning of "input resistance" is the differential change $\mathrm{d}u \,/\, \mathrm{d}i$, where the voltage at the input is u, and the current into the input is i. For our case, this is the same as u/i, because the input is a resistor (R_1) connecting to a zero-potential node: the current is thus $i = u/R_1$. As an example of another situation, consider if the opamp's non-inverting input had been connected to a potential v_+. Then R_1 would connect to a node at potential v_+, and the input current would be $i = (u - v_+)/R_1$. However, the value of $\mathrm{d}u \,/\, \mathrm{d}i$ would still be R_1.

(The output resistance of this circuit is zero: the output is fixed to whatever is needed in order to force the inverting input to zero potential; the opamp output will provide whatever current is needed to maintain this output voltage. So the ratio u/i for the *output* voltage and current is zero: it's a flat line in the u-i plane, just like any voltage source.)

Exercise 2

Find the potential v_i.

What is the input resistance looking between the node v_i and earth?



Answer 2

$v_\mathrm{i} = 5\,\mathrm{V} \quad R_\mathrm{i} = \infty$.

This is a classic non-inverting amplifier.

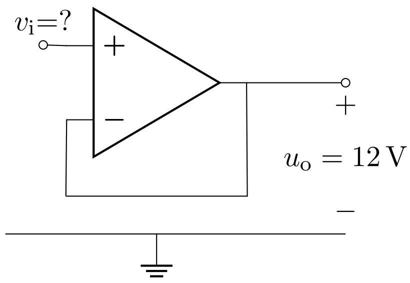
The divider makes v_+ be half of the output (for this choice of resistances).

As the feedback requires $v_- = v_+$, this means the circuit's output v_o must be twice the input v_i.

The input goes directly into only the non-inverting input, so no current is taken into the input. (In a real opamp, the input resistance is not, of course, infinite.)

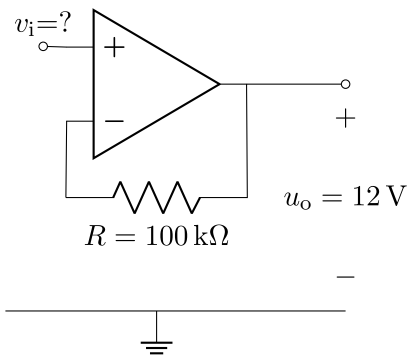
Exercise 3

a) Find the potential v_i in the following circuit:



(A thought: Does it make sense to talk of a potential [which means relative to the shown earth] or to mark a voltage u_o, in spite of the upper part of the circuit showing no component or node connecting to the earthed node at the bottom? These could seem like two separate 'circuits'. But remember: an opamp symbol hides some details!)

b) What is v_i if the circuit is modified to the following:



Answer 3

a) This is a buffer. Its output follows the input. This can be inferred from the assumption that, given negative feedback and an ideal opamp, the two input potentials are equal, $v_+ = v_-$.

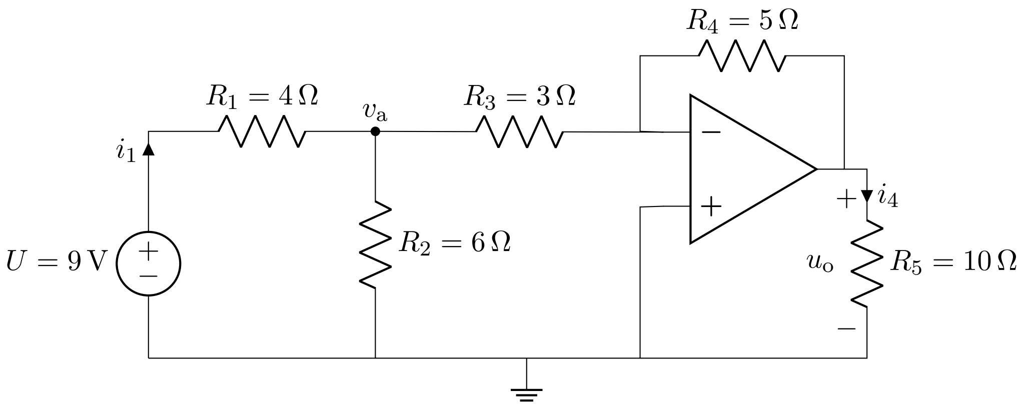
$v_\mathrm{i} = 12\,\mathrm{V}$

(About the 'thought': As long we remember the "hidden source" from earth to the opamp output, then there are connections between all the nodes in this circuit.)

b) With the resistor added, there is no change: still $v_\mathrm{i} = 12\,\mathrm{V}$.
There is no current in the path between the opamp output and inverting input: that's because the inverting input has an infinite input resistance (in the 'ideal opamp'). With no current, there can be no voltage across a resistor (Ohm's law). So regardless of the value of R, we see $v_- = u_\mathrm{o}$, and hence $u_\mathrm{o} = v_\mathrm{i}$.

Exercise 4

Find potential v_a, voltage u_o, currents i_1 and i_4, and the input resistance "seen by the voltage source".



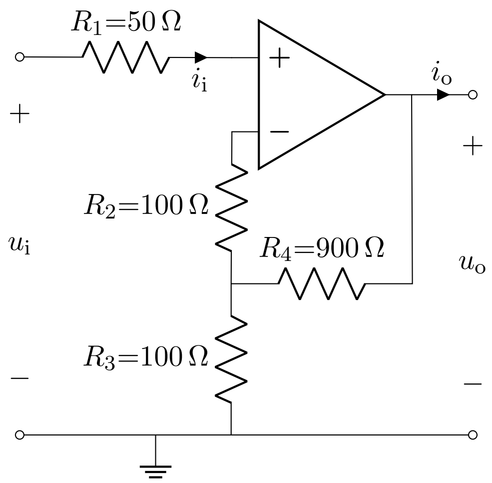
Answer 4

$v_a = 3\,\mathrm{V}, \quad u_\mathrm{o} = -5\,\mathrm{V}, \quad i_1 = 1.5\,\mathrm{A}, \quad i_4 = -0.5\,\mathrm{A}, \quad R_i = 6\,\Omega$

$v_- = v_+, \quad i_3 + i_2 - i_1 = 0, \quad R_i = \frac{9\,\mathrm{V}}{i_1}$

Exercise 5

Find $\frac{u_o}{u_i}$ in the following circuit:



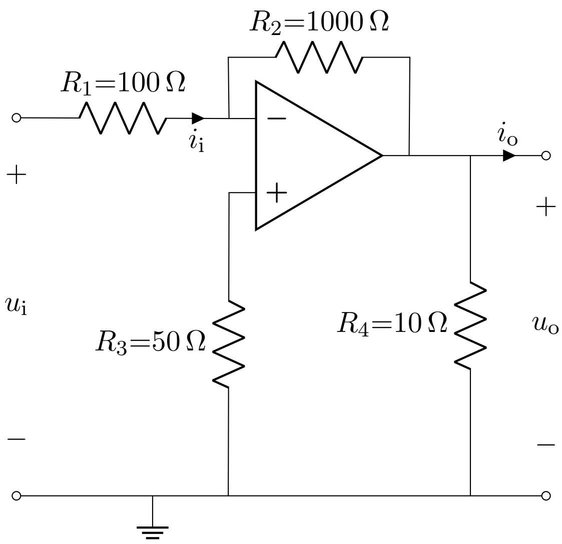
Answer 5

$$\frac{u_\mathrm{o}}{u_\mathrm{i}} = \frac{R_3 + R_4}{R_3} = 10$$

Note the irrelevance of R_1 and R_2. Both are the only path to an opamp input, so no current flows in them. By Ohms's law, the voltage across each of them is thus zero, so both sides have the same potential.

Exercise 6

Find $\frac{u_o}{u_i}$ in the following circuit:



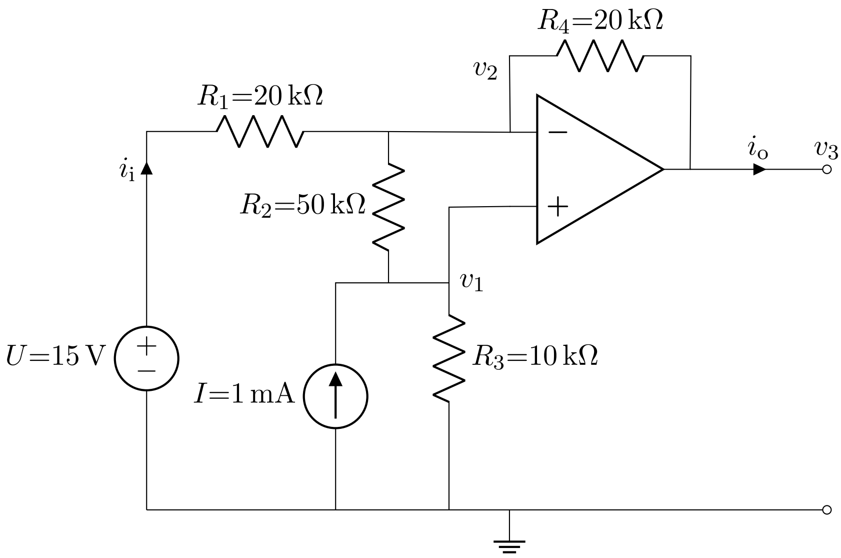
Answer 6

$\frac{u_\mathrm{o}}{u_\mathrm{i}} = -10$

Note the irrelevance of R_3 (explained in second part of Q1) and of R_4 (which is on the opamp output but doesn't affect the feedback).

Exercise 7

Find v_1, v_2 and v_3 in the following circuit:



Answer 7

$v_1 = v_2 = 10\,\mathrm{V}$

$v_3 = 5\,\mathrm{V}$

Nodal analysis is one good option.

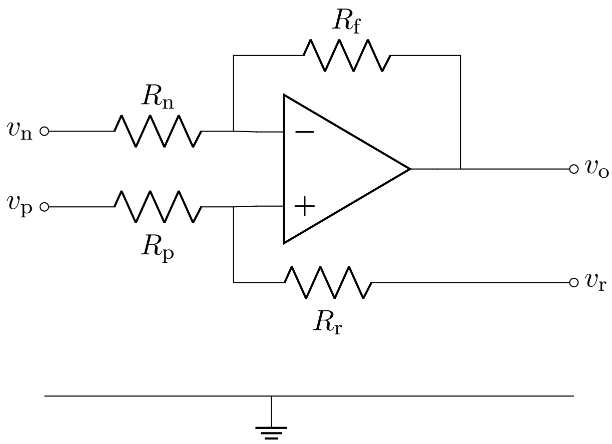
We'll instead briefly show a quick way based on seeing a sequence of steps:

Assume $v_1 = v_2$.
Therefore, R_2 has no current: it can be ignored.
All current I must flow down in R_3: hence $v_1 = IR_3$.
We now know $v_1 = v_2 = 10\,\mathrm{V}$.
The current $i_\mathrm{i} = \frac{U - v_2}{R_1} = 0.25\,\mathrm{mA}$.
This must pass also in R_4: so $\frac{v_3 - v_2}{R_4} = -0.25\,\mathrm{mA}$.
Hence, $v_3 = 5\,\mathrm{V}$.

Exercise 8

a) Find v_o, if v_n, v_p and v_r are known.

b) Now assume $R_\mathrm{n} = R_\mathrm{p} = R_1$ and $R_\mathrm{f} = R_\mathrm{r} = R_2$, and $v_r = 0$. Simplify the above expression.



Remember: in this sort of circuit, where we show 'terminals' at the ends of the nodes, you can *assume something is connected* to them! Otherwise there wouldn't be much happening. So in the diagram above, there *can* be currents in the resistors. This might seem strange, but it is because we are wanting to show the amplifier circuit as a 'building block' that can be connected inside a bigger circuit.

Answer 8

a) Initially we can use KCL to write

$$\frac{v - v_\mathrm{n}}{R_\mathrm{n}} + \frac{v - v_\mathrm{o}}{R_\mathrm{f}} = 0,$$

and

$$\frac{v - v_\mathrm{p}}{R_\mathrm{p}} + \frac{v - v_\mathrm{r}}{R_\mathrm{r}} = 0.$$

Rearrange the second in terms of v, then substitute this into the first equation and simplify:

$$v_\mathrm{o} \;=\; \frac{R_\mathrm{p}}{R_\mathrm{p}+R_\mathrm{r}}\left(1+\frac{R_\mathrm{f}}{R_\mathrm{n}}\right)v_\mathrm{r} \;+\; \frac{R_\mathrm{r}}{R_\mathrm{p}+R_\mathrm{r}}\left(1+\frac{R_\mathrm{f}}{R_\mathrm{n}}\right)v_\mathrm{p} \;-\; \frac{R_\mathrm{f}}{R_\mathrm{n}}v_\mathrm{n}.$$

In approximate words, the output is a weighted sum of the three input potentials, with the v_n part negated. By suitable choice of component values, this can be made to amplify the difference between the v_p and v_n inputs (i.e. like a plain opamp but with a lower and more controlled gain).

Very different forms of the expression might be written. As a simple numerical verification of whether they agree, you could try

```
Rr = 20e3
Rp = 40e3
Rn = 50e3
Rf = 75e3
vn = 0.51
vp = 0.53
vr = -0.1
```

Using these values in the above expression,

```
vo = vr*(1+Rf/Rn)*Rp/(Rp+Rr) + vp*(1+Rf/Rn)*Rr/(Rp+Rr) - vn*Rf/Rn
```

evaluates to `vo = -0.4900`. This can be compared to the result from other, supposedly similar, expressions.

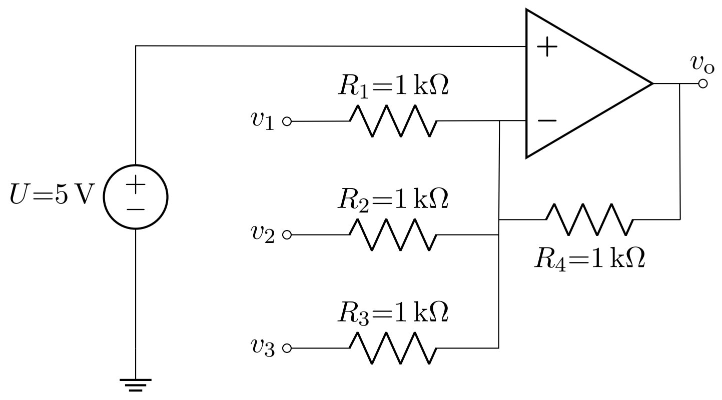
b) Make the substitutions and simplify. The result is

$$v_\mathrm{o} = \frac{R_2}{R_1}\left(v_\mathrm{p} - v_\mathrm{n}\right).$$

This is a simple form of differential amplifier, with output proportional to the difference between two inputs.

Exercise 9

Find v_o as a function of v_1, v_2, and v_3.



Answer 9

The output potential includes a sum of the input potentials and a constant. This is multiplied by a (negative) gain. So the circuit is an inverting amplifier with multiple inputs. Its output depends on the "weighted sum" of the inputs (with the equal resistors shown in the example, the weights are equal).

Nodal analysis (i.e. KCL in one node) is a convenient approach. KCL in the inverting-input node can be written with the assumption that the inverting- and noninverting-input have the same potential:

$$\frac{v_o - U}{R_4} + \sum_{n=1}^{3} \frac{v_n - U}{R_n} = 0$$

from which

$$v_o = U - R_4 \left[\frac{v_1}{R_1} + \frac{v_2}{R_2} + \frac{v_3}{R_3} - U \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \right) \right],$$

which simplifies for this case where $R_1 = R_2 = R_3 = R_4$, to

$$v_o = 20\,\mathrm{V} - (v_1 + v_2 + v_3)\,.$$

A practical point: it's not many real opamps that can tolerate an output voltage of 20 V. Perhaps one would have to ensure that the sum of the inputs is always significantly positive, to avoid the opamp hitting its limits.

An abstract point: the inputs (v_1 etc.) should not be seen as being open-circuited (disconnected). If they all were, then the inverting input of the opamp would be held to the output voltage of the opamp: the circuit would be just a buffer amplifier, with the output equal to U. These input nodes, to be *able* to have different potentials, are considered to have something (not shown) connected to them, to supply whatever current is needed to force them to some known potentials v_1 etc.

Exercise 10

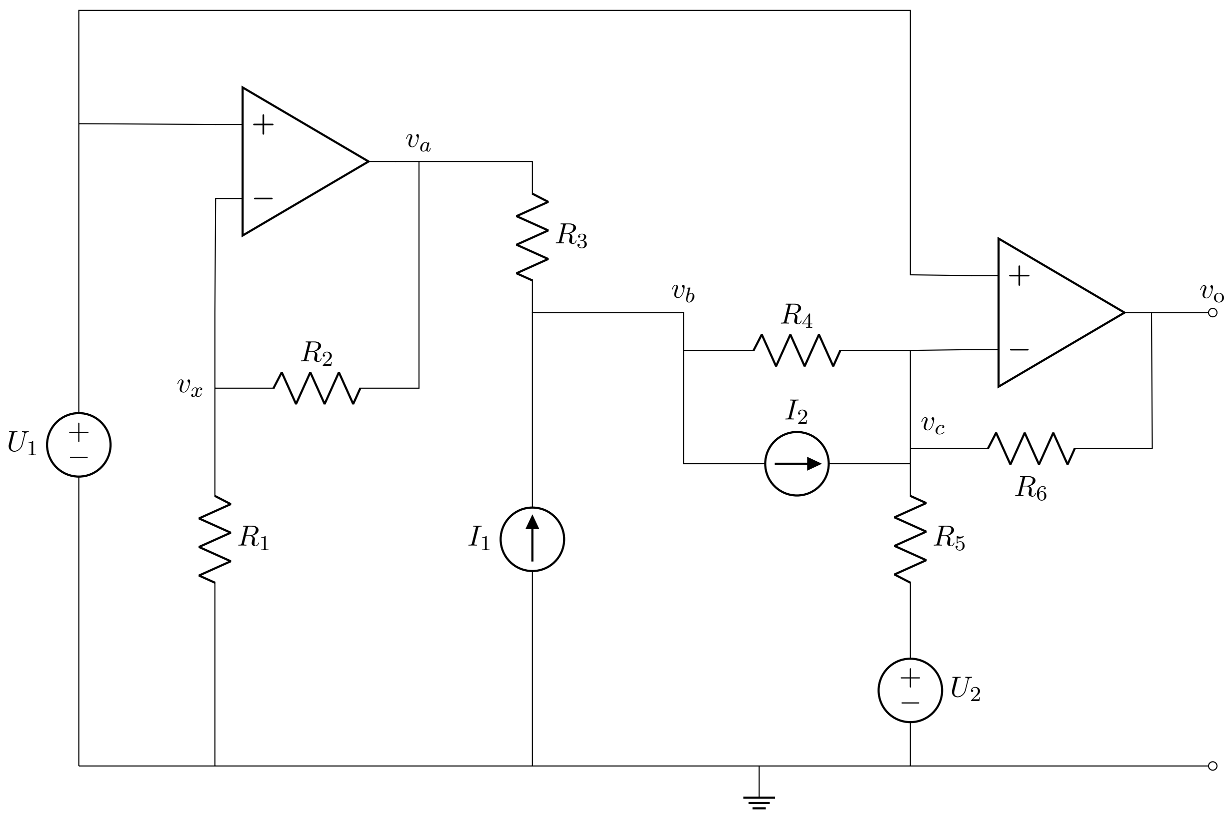
Write a set of equations that could be solved to find (at least) v_a, v_b, v_c, and v_o.

Suggestions:

i) Start by just looking … can you work out any of the sought potentials?
This is *not* recommended as a reliable way to handle tasks of "write the equations" with opamps: there can be circuits where it's hard to work in steps. However, this particular circuit *can* be solved fairly easily in this way.

ii) Write equations using the extended method, without simplifications.
This is often (always?) the best choice for handling a task of "write the equations".

iii) Use the supernode method to write equations.
This may involve more thinking than 'ii)' (above), but the equations will be fewer.



Answer 10

i)

Both opamps have negative feedback, so we assume the condition $v_- = v_+$ is fulfilled.

$$v_c = U_1$$

and

$$v_a = \frac{R_1 + R_2}{R_1} U_1$$

The potential v_b can then be found by a single KCL equation, as it is at a node that only connects to these potentials v_a and v_c, and to earth. (Although v_b affects v_o, v_o does not affect v_b.)

$$\text{KCL}(b)_{\text{out}} \qquad \frac{v_b - v_a}{R_3} - I_1 + I_2 + \frac{v_b - v_c}{R_4} = 0,$$

into which we substitute the known values of v_a and v_c,

$$\frac{1}{R_3}\left(v_b - \frac{R_1 + R_2}{R_1} U_1\right) - I_1 + I_2 + \frac{v_b - U_1}{R_4} = 0,$$

then rearrange for the one unknown, v_b,

$$v_b = \frac{R_3 R_4 (I_1 - I_2) + \left(R_3 + R_4\left(1 + \frac{R_2}{R_1}\right)\right) U_1}{R_3 + R_4}.$$

The output voltage v_o can be determined from KCL at the node marked with potential v_c:

$$\frac{v_o - v_c}{R_6} = \frac{v_c - U_2}{R_5} + \frac{v_c - v_b}{R_4} - I_2.$$

The earlier expressions for v_b and v_c can then be substituted into this, to remove all unknowns except v_o,

$$v_o = R_6 \left[\left(\frac{1}{R_6} + \frac{1}{R_5} + \frac{1}{R_4} - \frac{\frac{R_3}{R_4} + \left(1 + \frac{R_2}{R_1}\right)}{R_3 + R_4}\right) U_1 - \frac{1}{R_5} U_2 - \frac{R_3}{R_3 + R_4} I_1 - \frac{R_4}{R_3 + R_4} I_2\right].$$

As we only were required to "write the equations", these final steps of rearrangement were not really needed.

continued

ii)

By the complete "extended nodal analysis" approach, we start with 7 KCL equations, for every node other than the earth node.

It would be sensible to say we just won't bother defining a new potential above source U_1, as we already know this potential. But let's stick to the methodical process, like a computer! We'll name as v_1 and v_2 the potentials above sources U_1 and U_2 respectively. The potential above R_1 is already marked as v_x.

The currents in the voltage sources (into the +) and in the opamp outputs (outward) can be named i_1, i_2, i_a and i_o respectively, for the sources U_1 and U_2 and the left and right opamps. We will assume that the output terminal, marked v_o, is

$$\begin{aligned}
i_1 &= 0 \quad \text{above } U_1 \\
\frac{v_x}{R_1} + \frac{v_x - v_a}{R_2} &= 0 \quad \text{above } R_1 \\
\frac{v_a - v_x}{R_2} + \frac{v_a - v_b}{R_3} - i_a &= 0 \quad \text{left-opamp output node} \\
\frac{v_b - v_a}{R_3} - I_1 + I_2 + \frac{v_b - v_c}{R_4} &= 0 \quad \text{node } v_b \\
\frac{v_c - v_b}{R_4} - I_2 + \frac{v_c - v_2}{R_5} + \frac{v_c - v_o}{R_6} &= 0 \quad \text{node } v_c \\
\frac{v_2 - v_c}{R_5} + i_2 &= 0 \quad \text{above } U_2 \\
\frac{v_o - v_c}{R_6} - i_o &= 0 \quad \text{right-opamp output node}
\end{aligned}$$

To this, we must add information given by the voltage sources, *and* by the assumption of ideal opamps and negative feedback.

$$\begin{aligned}
v_1 &= 0 + U_1 \quad \text{left source} \\
v_2 &= 0 + U_2 \quad \text{right source} \\
v_x &= v_1 \quad \text{inputs of left opamp} \\
v_c &= v_1 \quad \text{inputs of right opamp}
\end{aligned}$$

In these 11 equations, there are 11 unknowns: 7 node potentials (with 7 KCL equations) and 4 voltage-source currents (with 4 equations that relate node potentials). Some simplifications could easily have been made when writing these equations, to avoid having so many equations, when only a few of the unknowns are wanted in our solution. These simplifications could alternatively have been made by substituting equations such as $v_1 = U_1$ into the other equations.

continued

iii)

The main feature of a supernode-based method is that we will not use KCL on any voltage-source: all voltage sources are contained *within* supernodes.

There are then different styles to choose between, about how much we should simplify the equations at the start. For example, should we from the beginning write just one variable for the potential of an opamp's '+' and '−' inputs? In the following, we will try to simplify as much as possible when writing the equations.

The opamp outputs are treated as voltage sources, i.e. as dependent sources with the other side connected to the earth node. Remember also that we don't need KCL on any part of an earth node or earth supernode. As the opamp outputs are part of the earth supernode, we don't use KCL on them.

When ignoring the earth supernode, which contains the two voltage-sources and the opamp outputs, there are just three nodes where KCL is needed:
the one labelled as potential v_x,
the one labelled as potential v_b,
the one labelled as potential v_c.

We will have to use the unknown potentials of the opamp outputs in some of these equations. This is different from the usual situation with dependent voltage-sources, where the source voltage can be expressed in terms of other variables. To compensate, we know that the two inputs of an opamp have the *same* potential: in this case, we can even see immediately that that potential must be U_1 for both opamps. We therefore substitute this for the opamp input potentials: $v_x = v_c = U_1$.

$$\begin{aligned}
\mathrm{KCL}_{x(\mathrm{out})} \qquad 0 &= \frac{U_1}{R_1} + \frac{U_1 - v_a}{R_2} \\
\mathrm{KCL}_{b(\mathrm{out})} \qquad 0 &= \frac{v_b - v_a}{R_3} + \frac{v_b - U_1}{R_4} + I_2 - I_1 \\
\mathrm{KCL}_{c(\mathrm{out})} \qquad 0 &= \frac{U_1 - v_b}{R_4} - I_2 + \frac{U_1 - U_2}{R_5} + \frac{U_1 - v_\mathrm{o}}{R_6}
\end{aligned}$$

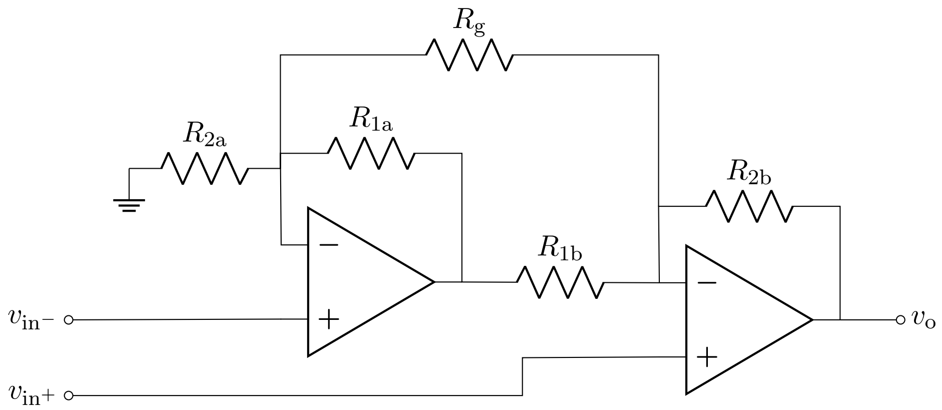
Those are three equations in three of the requested unknowns: v_a, v_b and v_o. We have mentioned earlier that $v_c = U_1$, but we can mention it again to make clear that it is one of the necessary equations for this task:

$$v_c = U_1$$

The above four equations are a sufficient solution to the task.

Exercise 11

Find v_o as a function of v_{in^+} and v_{in^-}.



Assume $R_{1a} = R_{1b} = R_1$, and $R_{2a} = R_{2b} = R_2$.

In the final equation we can use just R_1 and R_2 (and R_g) as the component values.

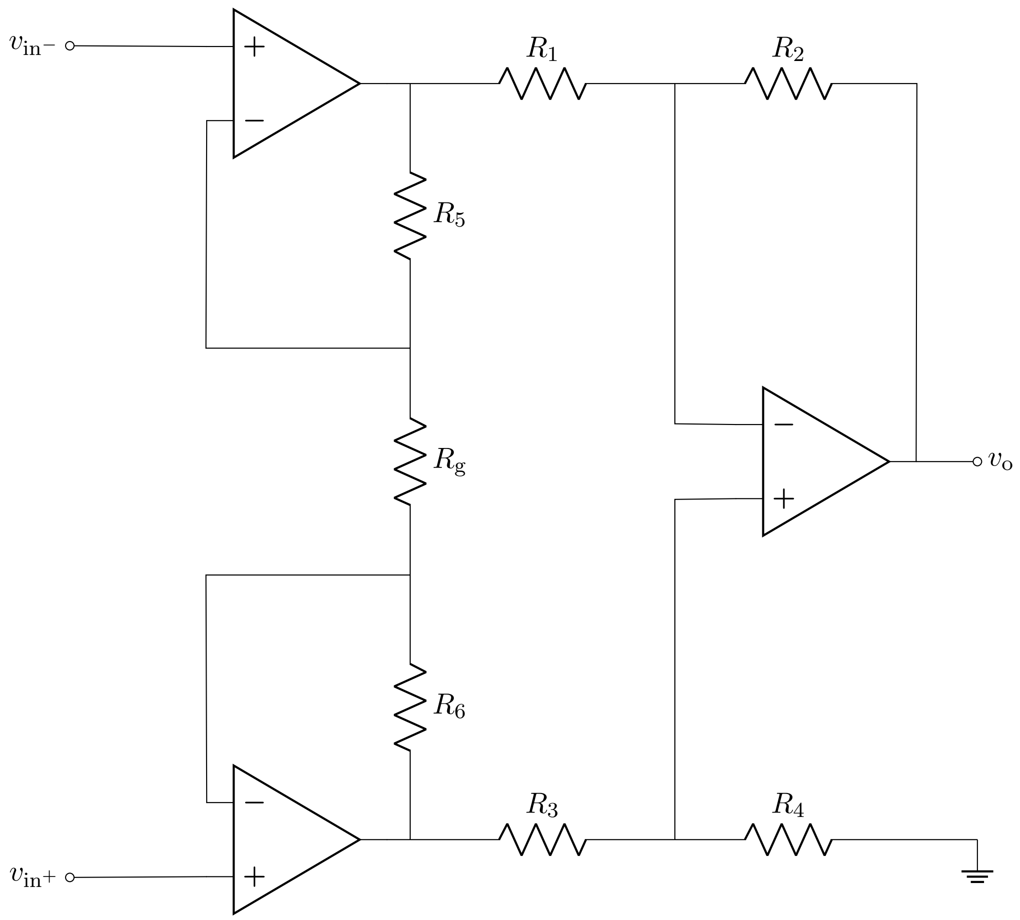
The unique names might be useful during your solution, to keep track of which resistor is being considered.

Answer 11

$$v_{\rm o} = \left(1 + \frac{R_2}{R_1} + \frac{R_2}{R_{\rm g}}\right) v_{\rm in^+} - \left(\frac{R_2(R_1 + R_2)}{R_1^2} + \frac{R_2}{R_{\rm g}}\right) v_{\rm in^-}.$$

Exercise 12

a) Find v_{o} as a function of v_{in^+} and v_{in^-} and the component values.



b) Given the following relations (sometimes used in this type of circuit),

$$\begin{aligned} R_5 = R_6 &= R_a \\ R_1 = R_3 &= R_b \\ R_2 = R_4 &= R_c \end{aligned}$$

express and simplify the earlier solution in terms of R_a, R_b, R_c and R_{g}.

Answer 12

a)

$$v_\text{o} = v_{\text{in}^+}\left[\frac{R_4(R_1+R_2)(R_6+R_g)}{R_1R_g(R_3+R_4)} + \frac{R_2R_5}{R_1R_g}\right] - v_{\text{in}^-}\left[\frac{R_4R_6(R_1+R_2)}{R_1R_g(R_3+R_4)} + \frac{R_2R_5}{R_1R_g} + \frac{R_2}{R_1}\right]$$

b)

After the substitutions, this becomes much simpler, reducing to

$$v_\text{o} = \left(1 + \frac{2R_a}{R_\text{g}}\right)\frac{R_c}{R_b}\left(v_{\text{in}^+} - v_{\text{in}^-}\right).$$

Here are some references that might be of interest about this circuit, which is a form of "instrumentation amplifier" made from opamps.

Wikipedia

ElectronicDesign

Ecircuitcenter