KTH, Electric Circuit Analysis, EI1120

Topic 07 Equilibrium and Continuity Practice Exercises

Many of the questions here are about equilibria, e.g. $u(0^-)$, $u(\infty)$, $i(t_1^-)$, etc.

Some have parts about continuity also: typically $u(0^+)$, $i(t_0^+)$, etc.

The same is true for the linked exam questions, below.

Using the principles of equilibrium and continuity is not a very big step, even if it looks worrying! The most difficult part is often the analysis of the dc circuits that remain after using the equilibrium or continuity assumptions; the skills are then the ones from Topics 1–4!

The extra parts that are needed here in Topic 7 are about correct application of a few simple rules: you need to build confidence in doing this, and to establish your chosen method for writing diagrams in a way that keeps track of which components have been replaced by others in parts of the calculation (e.g. an inductor by a current-source).

Mistakes can easily happen through doing too much "in the head", or by being careless when copying the circuit with changes.

For further practice after these exercises, try some past exams. As you will see in the links below, this type of question has come up regularly in this course for several years. For example, some of the following!

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2014-10˙E˙tenta1 Q4

2014-02˙EM˙ks2 Q2

2015-02˙EM˙ks2 Q1

2015-03˙EM˙tenta Q4

2015-06˙EM˙omtenta Q4

2014-05˙EM˙omtenta Q5

2014-03˙EM˙tenta Q5

2013-03˙EM˙tenta Q4

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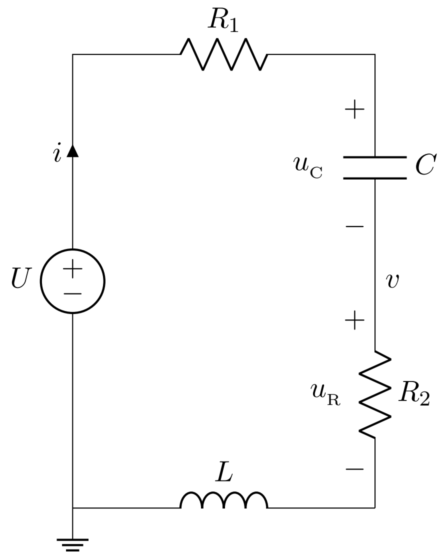
Exercise 1

What are the equilibrium values of i, v, u_{R} and u_{C} in the following circuit:



Answer 1

$i = 0$ the loop is 'blocked' by the capacitor (open-circuit under equilibrium assumptions).

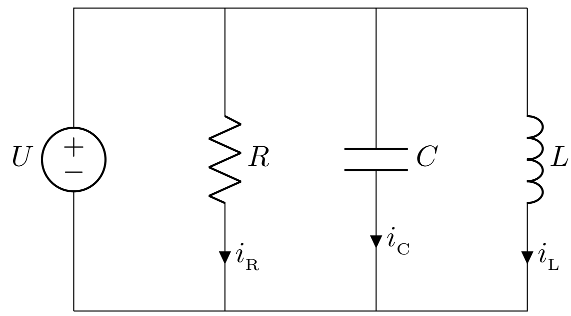
$v = 0$ u_{R} is zero (see below) and the inductor is assumed to be a short-circuit.

$u_{\mathrm{R}} = 0$ current of $i = 0$ (see above), together with Ohm's law.

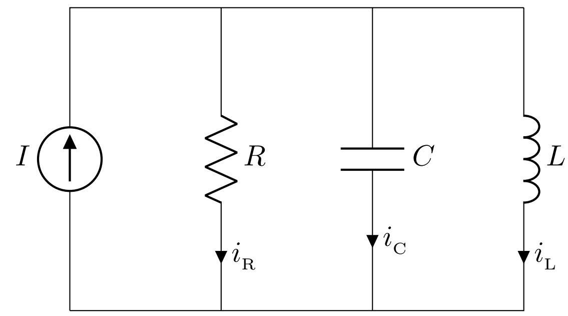
$u_{\mathrm{C}} = U$ KVL around the loop, bearing in mind $v = 0$ and $i = 0$.

Exercise 2

Which of the marked currents and voltages in each circuit *has* an equilibrium value? What is that value?



1)



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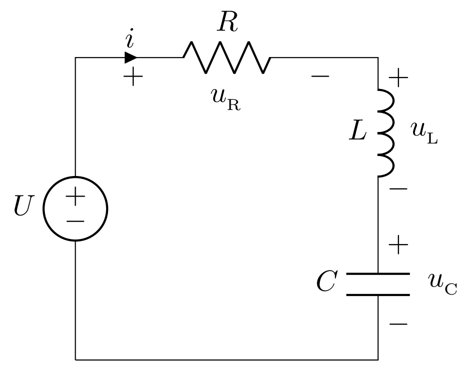

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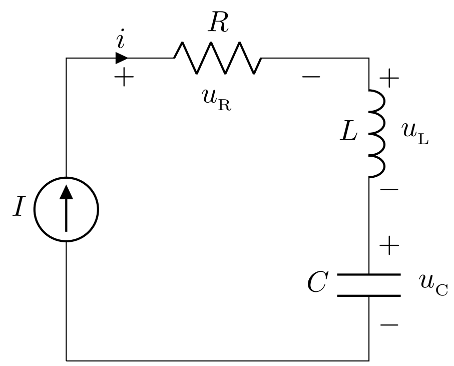


4)

Answer 2

1) Voltage source, parallel.

$i_{\mathrm{R}} = U/R$

$i_{\mathrm{C}} = 0$

i_{L} : no equilibrium: continuous increase at $\frac{\mathrm{d}i_{\mathrm{L}}}{\mathrm{d}t} = U/L$.

2) Current source, parallel.

$i_{\mathrm{R}} = 0$ because the inductor in equilibrium is like a short-circuit

$i_{\mathrm{C}} = 0$ equilibrium behaviour of a capacitor (and note the inductor being like a short-circuit in parallel is another reason).

$i_{\mathrm{L}} = I$ like a short-circuit in equilibrium, so all the current goes through this.

3) Voltage source, parallel.

$i = 0$ blocked by capacitor's open-circuit.

$u_{\mathrm{R}} = 0$ Ohm's law, given that $i = 0$.

$u_{\mathrm{C}} = U$ KVL around the circuit, given that $u_{\mathrm{L}} = 0$ and $i = 0$.

$u_{\mathrm{L}} = 0$ assumption of short-circuit in equilibrium; also we know $i = 0$ and $\frac{\mathrm{d}i}{\mathrm{d}t} = 0$ (as we know $i = 0$ is a long-term final-value).

4) Current source, series.

$i = I$ "obvious", or KCL at current-source output: note how the current-source "wins" over the rule that a capacitor's current is zero in equilibrium; that's because we know it can be that the equilibrium just doesn't happen, in which case the assumptions based on e.g. $\frac{\mathrm{d}u}{\mathrm{d}t} = 0$ are invalid.

$u_{\mathrm{R}} = IR$ Ohm's law, given that $i = 0$.

u_{C} : no equilibrium: continuous increase at $\frac{\mathrm{d}u_{\mathrm{C}}}{\mathrm{d}t} = I/C$; so $u_{\mathrm{C}}(t) = \frac{I}{C}t$ (if $u_{\mathrm{C}}(0) = 0$!) as capacitor voltage is integral of the (constant) current.

$u_{\mathrm{L}} = 0$ current is known to be constant, i.e. $\frac{\mathrm{d}i}{\mathrm{d}t} = 0$; this implies inductor voltage of zero.

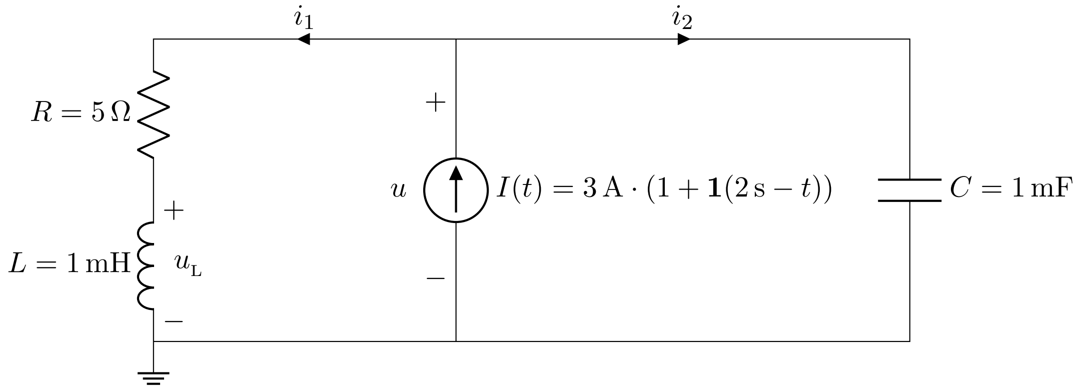
Exercise 3

Find u, u_{L}, i_1, and i_2, at the following times:

a) $t = 2^{-}\,\mathrm{s}$

b) $t = 2^{+}\,\mathrm{s}$

c) $t \to \infty\,\mathrm{s}$



Answer 3

a) $t = 2^-$ s, equilibrium just before the step, $I(2^-\,\mathrm{s}) = 6\,\mathrm{A}$.
(Replace capacitor with open-circuit, inductor with short-circuit, and solve.)

$u = 30\,\mathrm{V}$ all current through left branch, no voltage across inductor

$u_{\mathrm{L}} = 0$ inductor in equilibrium implies zero voltage (zero $\frac{\mathrm{d}i}{\mathrm{d}t}$)

$i_1 = 6\,\mathrm{A}$ no current in open-circuit (capacitor); do KCL on top node

$i_2 = 0$ no current in the open-circuit (the capacitor in equilibrium)

b) $t = 2^+$, just after the step, $I(2^+\,\mathrm{s}) = 3\,\mathrm{A}$. Continuity can be used.

$u = 30\,\mathrm{V}$ the current source is in parallel with the capacitor, and voltage on a capacitor is a continuous variable. Note that we could not analyse the left part of the circuit as easily as was done in part 'a)', since the inductor can no longer be *assumed* to have zero voltage when not in equilibrium (but see the next line).

$u_{\mathrm{L}} = 0$ this is *not* directly due to continuity: an inductor's voltage is not a continuous variable. However, we see that $u(2^+\,\mathrm{s}) = u(2^-\,\mathrm{s})$ (continuity of the capacitor voltage), and $i_1(2^+\,\mathrm{s}) = i_1(2^-\,\mathrm{s})$ (continuity of inductor *current*), so the voltage Ri_1 across the resistor is also unchanged: taking KVL around the left loop, u_{L} must also be unchanged!

$i_1 = 6\,\mathrm{A}$ this current goes through the inductor; so continuity applies!

$i_2 = -3\,\mathrm{A}$ KCL in the top node, using continuity of inductor current i_2.

c) $t = 2^-$ s, equilibrium a long time after the step, $I(2^-\,\mathrm{s}) = 3\,\mathrm{A}$.
Notice that we are finding an equilibrium in the same circuit as in part 'a)', and the only difference is the value of the only source. It is now half as much. So all quantities are halved. See the reasoning in part 'a)'.

$u = 15\,\mathrm{V}$

$u_{\mathrm{L}} = 0$

$i_1 = 3\,\mathrm{A}$

$i_2 = 0$

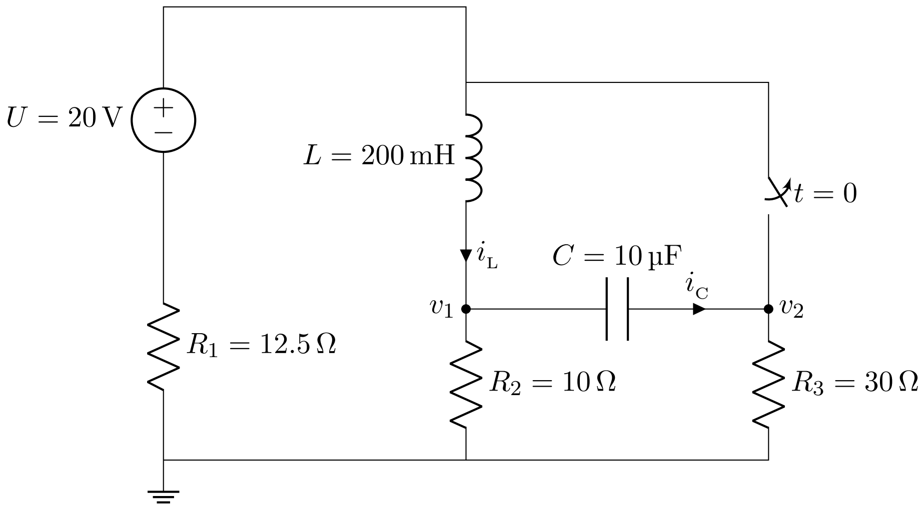
Exercise 4

What are i_{L}, i_{C}, v_1 and v_2, at the following times?

a) $t = 0^-$

b) $t = 0^+$

c) $t \to \infty$



(Did the values of C and L matter?)

Answer 4

a)

$i_{\mathrm{L}}(0^-) = 0.75\,\mathrm{A}$ current from source divides between R_2 and R_3

$i_{\mathrm{C}}(0^-) = 0$ equilibrium behaviour of capacitor is open-circuit

$v_1(0^-) = 7.5\,\mathrm{V}$ KCL with i_{L} and i_{C}, then Ohm's law

$v_2(0^-) = 7.5\,\mathrm{V}$ connected directly to v_1 via the switch and short-circuited inductor

b)

$i_{\mathrm{L}}(0^+) = i_{\mathrm{L}}(0^-) = 0.75\,\mathrm{A}$ continuity

$i_{\mathrm{C}}(0^+) = \frac{R_2}{R_3+R_2} i_{\mathrm{L}}(0^+) = 0.19\,\mathrm{A}$ current division: the capacitor has zero voltage (continuity) so it is like a voltage source of zero, i.e. a short circuit, meaning that current division can be used.

$v_1(0^+) = 0.75\,\mathrm{A} \cdot \frac{10\,\Omega \cdot 30\,\Omega}{10\,\Omega + 30\,\Omega} = 5.63\,\mathrm{V}$ this is based on how the capacitor voltage is zero, by continuity (see how it was shorted by the switch and the equilibrium inductor at times $t < 0$); so $v_1 = v_2$ and this determines the current through the resistors R_2 and R_3; but the total current in is through L, which by continuity is still passing only its old equilibrium value of current; the potentials v_1 and v_2 therefore fall (discontinuously) at $t = 0$, and a voltage appears across L.

$v_2(0^+) = v_1(0^+)$ continuity of the capacitor voltage, $v_1 - v_2$, which was zero at $t = 0^-$.

c)

$i_{\mathrm{L}}(\infty) = 0.89\,\mathrm{A}$

$i_{\mathrm{C}}(\infty) = 0$

$v_1(\infty) = 8.9\,\mathrm{V}$ Ohm's law (after KCL with i_{C} and i_{L})

$v_2(\infty) = 0$ Ohm's law, with no path for current in R_3