

This file starts with some exercises to practice finding complex power and its various derivative quantities (PF, θ , $|S|$, etc). Later longer questions are about maximum power and power superposition.

Recommended: In view of limited time, and some quite difficult work to get familiar with, your focus on the following pages should be Exercises 1, 2 and 4. Exercise 1 could seem rather long, but it's probably useful to work through several of its parts, to get familiar with the calculations. Then you have the exam suggestions below, and the homework.

The following are good ac-power practice questions from old exams.

(There are plenty more power-related questions in the old exams, but many of them also require concepts from later Topics, about transformers or three-phase circuits. Only the basic power and maximum-power questions are included below.)

2016-10'IT'omtenta Q6 nice simple power-calculation

2015-05'IT'omtenta Q6 power-factor compensation

2015-03'EM'tenta Q6 part-b, and part-c (power superposition is useful)

2014-08'IT'omtenta Q6 sources with different frequencies

2015-06'EM'omtenta Q6 not classic ac power, but a conceptually interesting part-b

2016-03'IT'omtenta Q6 max-power

2015-10'IT'omtenta Q6 max-power

2014-03'EM'tenta Q7 max-power

2014-05'EM'omtenta Q7 max-power

2014-08'IT'omtenta Q5 max-power

2014-10'IT'omtenta Q5 max-power

2015-03'IT'omtenta Q6 max-power

2015-02'E'ks2 Q2 Thevenin, for max-power

2015-03'E'tenta2 Q8 max-power using numbers, needing 'search'

Exercise 1 Practice with classic impedance/admittance

Why is this exercise valuable? Well, a very common *task* is to calculate something like the active power P into an impedance $Z = R + jX$, when the impedance is connected to a given voltage u .

There are various typical *mistakes!*

One is to write something like $P = u^2/R$, which is **wrong** for the case where $Z = R + jX \neq R$. Another is to start with $S = |u|^2/(R - jX)$ and take the real part (right idea), but assuming that this is just $|u|^2/R$ (which is the **wrong** arithmetic).

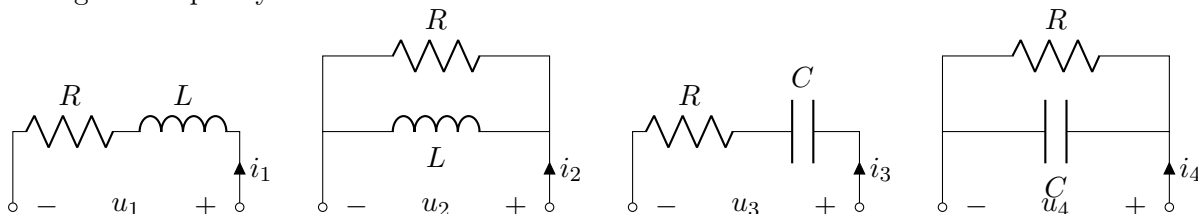
The following should be good training! It also gives practice at handling complex power S , its rectangular components (P and Q), apparent power, and power factor.

Notice an important difference between admittance and impedance.

When we write an impedance such as $Z = R + jX$, this can be represented as a *series* connection of a resistor R and inductor $L = X/\omega$ (or a capacitor if it is $R - jX$). Then if we know the current i , we can immediately find the active power ($|i|^2 R$) or reactive power ($|i|^2 X$) in one of the series components. But if we know only the voltage, then the powers in the two components depend on *both* component values; that makes the equations more complicated.

An admittance $Y = G + jB$ can be represented as a *parallel* connection of a resistor $R = 1/G$, and a capacitor $C = B/\omega$ (or an inductor if it's $G - jB$). In that case the powers are easily found if voltage is known, but not so easily if just the current is known. This is why it can sometimes seem so much easier and sometimes so much harder to do the power calculations.

The diagrams below show four common types of impedance found in ac circuits. In each, a voltage and current are marked. Assume these are rms values of phasors (ac steady-state), all at angular frequency ω .



The **known** quantities are R , L , C , u_1 , u_4 , i_2 and i_3 .

For each of the above circuits (1,2,3,4) calculate the following:

- i) the phasor of the unknown voltage or current (i.e. i_1 , u_2 , u_3 , or i_4).
- ii) the complex power S into the total impedance.
- iii) the active ('real') and reactive ('imaginary') powers P and Q .
- iv) the apparent power $|S|$.
- v) the power factor $\text{PF} = \cos \theta$; include whether lagging or leading.

Try to make the results neat: e.g., separate the real and imaginary parts.

If you want extra work, try swapping which currents and voltages are the known and the unknown. Then you'll have 4 more cases to solve ... but not all with model solutions available.

Answer 1**Case 1:** Series RL, u_1 known.

i) $i_1 = u_1/Z_1 = u_1/(R + j\omega L)$.

ii) $S = u_1 i_1^* = u_1 u_1^*/Z^* = |u_1|^2/(R - j\omega L) = \frac{|u_1|^2}{R^2 + \omega^2 L^2} (R + j\omega L)$.

iii) Solution 'b)' is in a convenient form: $P = \Re\{S\} = \frac{|u_1|^2}{R^2 + \omega^2 L^2} R$, $Q = \Im\{S\} = \frac{|u_1|^2}{R^2 + \omega^2 L^2} \omega L$.

iv) $S_{\text{app}} = \sqrt{P^2 + Q^2} = |S| = \frac{|u_1|^2}{\sqrt{R^2 + \omega^2 L^2}}$.

v) PF = $\frac{P}{|S|} = \frac{R}{\sqrt{R^2 + \omega^2 L^2}} = \frac{R}{|Z|}$. Lagging, because the circuit is inductive ($Q > 0$).

Case 2: Parallel RL, i_2 known.

i) $u_2 = i_2 Z_2 = i_2 \frac{1}{\frac{1}{R} + \frac{1}{j\omega L}} = i_2 \frac{j\omega LR}{R + j\omega L}$.

ii) $S = u_2 i_2^* = i_2 i_2^* \frac{j\omega LR}{R + j\omega L} = \frac{j\omega LR |i_2|^2}{R + j\omega L} = \frac{j\omega LR |i_2|^2}{R^2 + \omega^2 L^2} (R - j\omega L) = \frac{\omega LR |i_2|^2}{R^2 + \omega^2 L^2} (\omega L + jR)$.

iii) $P = \frac{\omega^2 L^2 R |i_2|^2}{R^2 + \omega^2 L^2}$, $Q = \frac{\omega LR^2 |i_2|^2}{R^2 + \omega^2 L^2}$.

iv) $S_{\text{app}} = \frac{\omega LR |i_2|^2}{\sqrt{R^2 + \omega^2 L^2}}$.

v) PF = $\frac{\omega L}{\sqrt{R^2 + \omega^2 L^2}} = \frac{\omega L}{|Z|} \left[= \frac{1/R}{1/|Z|} = \frac{G}{|Y|} \right]$. Lagging, because the circuit is inductive.

Case 3: Series RC, i_3 known.

i) $u_3 = i_3 Z_3 = i_3 (R - j\frac{1}{\omega C})$.

ii) $S = u_3 i_3^* = i_3 Z_3 i_3^* = |i_3|^2 (R - j\frac{1}{\omega C})$.

iii) $P = |i_3|^2 R$, $Q = -\frac{|i_3|^2}{\omega C}$.

iv) $S_{\text{app}} = |i_3|^2 |Z| = |i_3|^2 \sqrt{R^2 + \frac{1}{\omega^2 C^2}}$.

v) PF = $\frac{R}{\sqrt{R^2 + \frac{1}{\omega^2 C^2}}} = \frac{R}{|Z|}$. Leading, because the circuit is capacitive ($Q < 0$).

Case 4: Parallel RC, u_4 known.

i) $i_4 = u_4/Z_4 = u_4 Y_4 = u_4 (\frac{1}{R} + j\omega C)$.

ii) $S = u_4 i_4^* = u_4 u_4^*/Z_4^* = u_4 u_4^* Y_4^* = |u_4|^2 (\frac{1}{R} - j\omega C)$.

iii) $P = |u_4|^2/R$, $Q = -|u_4|^2 \omega C$.

iv) $S_{\text{app}} = |u_4|^2 \sqrt{\frac{1}{R^2} + \omega^2 C^2}$.

v) PF = $\frac{1/R}{\sqrt{\frac{1}{R^2} + \omega^2 C^2}} = \frac{1/R}{1/|Z|} = \frac{G}{|Y|}$. Leading, because the circuit is capacitive.

You will note that the power factor is not dependent on whether the voltage or current is the known variable, or what value of voltage or current is applied. The power factor is entirely determined by the components and their connection, so the i_x or u_x terms cancel if we calculate PF from P and $|S|$.

For a series connection, PF is easily found from impedances; and for a parallel connection, PF is easily found from admittances.

In an exam the answer should be given in terms of known values, such as R , L , C and ω in the above problems. In the above answers, some other expressions for PF have been given too, in order to show the duality more clearly.

The lagging or leading power factor can be deduced in several ways: double-check the following points in the above examples!

The answer 'i)' shows the phasor relation between the voltage and the current: when the current's reference direction is into the +-side of the voltage's reference direction, then the current lags the voltage if the impedance has an inductive part, or leads it if the impedance has a capacitive part, or is in phase if the impedance is purely resistive.

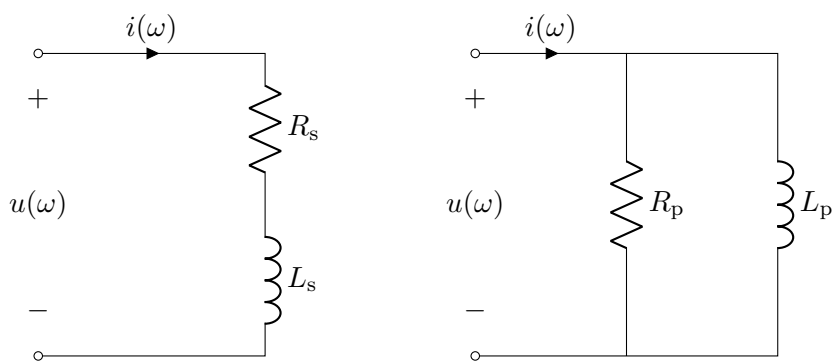
The answer 'iii)' shows the reactive power into the impedance: because of the convention that $S = ui^*$ (instead of $S = u^*i$), we know that an impedance with an inductive part will 'consume reactive power', i.e. the reactive power input is positive.

Just look at whether there's a capacitor or inductor in the load: leading power-factor if it's a capacitor, lagging if it's an inductor!

Exercise 2

Don't waste ages on the algebra if you get stuck. Try attacking it another way, or just look at the answer a few times. This is quite a useful 'result', about series or parallel equivalents of impedances: it's relevant to some of the past exam questions, and in some real situations.

Find the necessary inductance L_p and resistance R_p , in the parallel circuit on the right, to make it behave exactly like the series circuit on the left, at a given ω .



In other words, express L_p and R_p in terms of L_s , R_s , and ω , so that the relation u/i (impedance) is the same in both circuits.

Answer 2

We want to find L_p and R_p in terms of L_s , R_s , and ω , so that the parallel impedance $L_p \parallel R_p$ is equal to the series impedance $L_s + R_s$ at frequency ω .

Let's start by finding the *admittance* of the series circuit. We can expect that to be a good choice, as we know that R_p and L_p in the parallel circuit are the components that respectively determine the real and imaginary part of the admittance.

$$Y_s = \frac{1}{Z_s} = \frac{1}{R_s + j\omega L_s} = \frac{R_s - j\omega L_s}{R_s^2 + \omega^2 L_s^2}.$$

This has to be made to equal the admittance of the parallel circuit, which is

$$Y_p = \frac{1}{R_p} - j\frac{1}{\omega L_p}.$$

To achieve this, equate the real and imaginary parts between Y_s and Y_p ,

$$R_p = \frac{R_s^2 + \omega^2 L_s^2}{R_s}$$

$$L_p = \frac{R_s^2 + \omega^2 L_s^2}{\omega^2 L_s}$$

We can also show this in the other direction: find the components needed in the series circuit on the left, in order to make it behave like some given component values in the parallel circuit on the right.

For that case, it's easier to equate the impedances, since the sought variables are series components and are therefore easily written in impedance form. A similar approach to the calculation gives the following:

$$Z_s = R_s + j\omega L_s$$
$$Z_p = \frac{1}{Y_p} = \frac{1}{\frac{1}{R_p} + \frac{1}{j\omega L_p}} = \frac{j\omega L_p R_p}{R_p + j\omega L_p} = \frac{(R_p - j\omega L_p)j\omega L_p R_p}{R_p^2 + \omega^2 L_p^2} = \frac{\omega^2 L_p^2 R_p + j\omega L_p R_p^2}{R_p^2 + \omega^2 L_p^2}.$$

By having got the final answer for Z_p in pure rectangular form, its real and imaginary parts can be equated with those from Z_s , to give

$$R_s = \frac{\omega^2 L_p^2 R_p}{R_p^2 + \omega^2 L_p^2},$$

$$L_s = \frac{L_p R_p^2}{R_p^2 + \omega^2 L_p^2}.$$

Some general ramblings about series/parallel conversion

As was seen in Exercise question 1, it is easier to calculate currents and powers with a known voltage for parallel-connected RL or RC components: the R component then entirely determines the active power, and the L or C entirely determines the reactive power. We don't have to handle a complex number in the denominator as we would have to for series-connected components. The opposite is true if the current is the known quantity: the series connection becomes simpler. Another way of thinking of this is that series components are conveniently seen

as impedances (they sum to the total impedance), and parallel components are conveniently seen as admittances (they sum to the total admittance).

We know that any group of R , L and C components with two terminals can be reduced to a single impedance at a particular frequency ω . This follows from how multiple resistors (dc) can be made into an equivalent: this is simply extended to the case of complex numbers (impedances). The same result can be found from thinking of a Thevenin equivalent: the equivalent source's voltage is clearly zero if there is no independent source, so the Thevenin equivalent is just an impedance. The impedance will be "in the right half-plane", i.e. its real part will be non-negative; we would need to have dependent sources in order to get a negative resistance.

Any impedance with non-negative real part can be represented with (at most) one resistor and one inductor *or* capacitor, in series or in parallel. (The values of the two components will need to be different depending on whether the series or parallel connection is chosen.)

So, it should be possible to *convert* a series pair of components to a parallel pair (or vice versa), that have the same impedance. Note that the conversion is valid at just one frequency; it's based on the assumptions of sinusoidal steady state that are used for the phasors method.

Exercise 3

Assume the components in Exercise 1 have the following values:

$R = 100$, $L = 133\text{e-}3$, $C = 13.3\text{e-}6$ and let the known inputs to the circuit be these phasors at angular frequency ω ,

$$u_1 = 1\text{e}3, u_4 = \exp(4.605+2j), w = 2\pi*60 .$$

For circuits 1 and 4 shown in Exercise 1, find the numerical values of parts $|S|$ and PF.

You are *encouraged* to use a computer. and to ignore the symbolic solutions you got in the previous question. Instead of using those solutions, you can work in steps, defining intermediate variables as necessary.

For example, for circuit 2 (parallel R,L with known current i_2), we could define¹ the known current as $i_2 = 4+3j$, then calculate in the following way.

```
% total impedance: easily found by summing admittances
Y2 = 1/R + 1/(1j*w*L)
Z2 = 1/Y2
% voltage across this impedance
u2 = Z2 * i2

% calculate complex power into circuit
S2 = u2 * conj(i2)

% now find the derived quantities of apparent power and pf
P = real(S2)
Q = imag(S2)
Sapp2 = abs(S2) % ONE OF THE REQUESTED SOLUTIONS
pf2 = P/Sapp2 % THE OTHER REQUESTED SOLUTION
% and let's be terribly clever and automatic! (neglecting if Q==0)
if imag(S2)<0, disp('leading PF'); else disp('lagging PF'); end

% and it's sensible to compare this to the result from
% the expression that we derived,
Sapp2ex = w*L*R*abs(i2)^2 / sqrt(R^2+w^2*L^2)
pf2ex = w*L/sqrt(R^2+w^2*L^2)
```

Results: both ways of calculating $|S|$ and PF give the same solution, of $|S| = 1120.5\text{ VA}$ and $\text{PF} = 0.448$ (lagging). The unit ‘VA’ (volt ampere) is commonly used for apparent power, in contrast to writing watts for active power; this is to indicate that it doesn’t represent an overall transfer of energy per cycle. We’re assuming here that all the numeric values of inputs and components were in unprefixed SI units.

The idea is to show that it’s quick and easy to do ac power calculations in the everyday, practical way that people tend to, without symbolic calculations on paper. The symbolic way is sometimes useful for showing a general principle, or finding a value that isn’t easily obtained by step-by-step solutions, for getting more understanding of how the circuit’s components affect the solution. That’s why we study it in the basic course. But it’s good to realise that ac analysis in practice is not something to be scared of as a time-consuming and complicated process!

¹None of the results should depend on the phase of the current; the power factor won’t even depend on the magnitude of the current. The reason we choose a not-very-simple value is in order to demonstrate that when we’re using the computer we are not inconvenienced by situations where the angles aren’t specially chosen to be simple!

Answer 3

The following shows just the equations taken from the symbolic solutions. It's encouraged that you also check this by a calculation in the style of the example that was after the question, for circuit 2. This sort of check helps to catch errors in our derivation of the symbolic solution.

```
% set the parameters
u1 = 1e3, u4 = exp(4.605+2j), w = 376, R = 100, L = 133e-3, C = 13.3e-6

% circuit 1: apparent power and power-factor
mS1 = abs(u1)^2 / sqrt( R^2 + w^2*L^2 )
      8944.0
PF1 = R / sqrt( R^2 + w^2*L^2 )
      0.89440

% circuit 4: the same
mS4 = abs(u4)^2 * sqrt( 1/R^2 + w^2*C^2 )
      111.77
PF4 = (1/R) / sqrt( 1/R^2 + w^2*C^2 )
      0.89440
```

Exercise 4

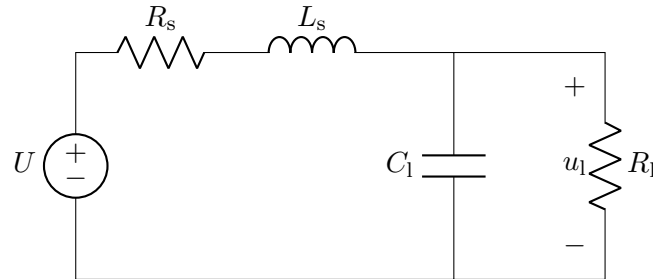
Maximum [active-]power.

The voltage source in this diagram is ac with angular frequency ω and rms magnitude U .

Let's define it as the phase reference, i.e. its phase can be called zero.

A source-impedance $Z_s = R_s + j\omega L_s$ is in series with the voltage source.

A load consisting of two parallel components R_l and C_l is connected to the output, as shown.



a) Find R_l and C_l to give maximum power to the *load* (i.e. maximise the active power in Z_l), given that U , ω , R_s and L_s are fixed.

Hint: The question is a clear “maximum power” one. The aim, then, is to make a “load” and a “source” have impedances that are complex conjugates. But be careful about the load impedance: remember the components are in parallel, not series.

b) Take $U = 230$, $\omega = 2\pi \cdot 50$, $R_s = 5$, $L_s = 10\text{e-}3$.

(You can assume SI units, if you want to write units.)

What are the numeric values needed for R_l and C_l , from part ‘a’?

What is the active power to the load at maximum power?

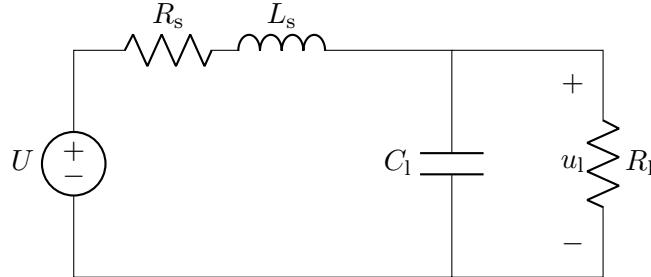
Answer 4

Maximum [active-]power.

The ac voltage source has angular frequency ω and rms magnitude U .

Define the source-impedance $Z_s = R_s + j\omega L_s$.

The load is two parallel components, R_1 and C_1 , which are the unknowns.



a) Task: Determine both load components for maximum active power transfer to the load.

The maximum [active] power criterion is that the load impedance must be the complex conjugate of the source impedance: Z_s^* . The load impedance here is the *parallel* combination of the two load components, so we *cannot* just say that e.g. R_s needs to be equal to R_1 .

We can, however, say that if $Z_1 = Z_s^*$, then $1/Z_1 = 1/Z_s^*$. This means we can work with the admittances instead of the impedances: the maximum power criterion is $Y_1 = Y_s^*$. The advantage of using admittance is that the real and imaginary parts of load admittance correspond respectively to the resistor and capacitor (in their parallel connection),

$$Y_1 = \frac{1}{R_1} + j\omega C_1,$$

so we can directly equate these with the real and imaginary parts of Y_s^* , to get the solution.

First, find Y_s ,

$$Y_s = \frac{1}{Z_s} = \frac{1}{R_s + j\omega L_s} = \frac{R_s - j\omega L_s}{R_s^2 + \omega^2 L_s^2},$$

then conjugate it and equate it to the sought Y_1 ,

$$\frac{1}{R_1} + j\omega C_1 = \frac{R_s + j\omega L_s}{R_s^2 + \omega^2 L_s^2}.$$

By separating the real and imaginary parts in the above equation,

$$R_1 = \frac{R_s^2 + \omega^2 L_s^2}{R_s} \quad C_1 = \frac{L_s}{R_s^2 + \omega^2 L_s^2}$$

b) Take $U = 230$, $\omega = 2\pi \cdot 50$, $R_s = 5$, $L_s = 10e-3$.

Find the numeric values of R_1 and C_1 from part 'a', and the load's active power at this maximum-power point.

```

U = 230, w = 2*pi*50, Rs = 5, Ls = 10e-3,
Rl = (Rs^2 + w^2*Ls^2) / Rs
Cl = Ls / (Rs^2 + w^2*Ls^2)
Zs = Rs + 1j*w*Ls           % source impedance
Zl = 1 / ( 1/Rl + 1j*w*Cl ) % load impedance
Ul = U * Zl / ( Zl + Zs )   % load voltage (by voltage division)
Pl  = abs(Ul)^2 / Rl        % load active power (one method)
Pl_ = real( abs(Ul)^2 / Zl ) % load active power (another method)

% the following are the results:
Rl  = 6.9739
Cl  = 2.8678e-04
Zs  = 5.0000 + 3.1416i
Zl  = 5.0000 - 3.1416i
Ul  = 115.000 - 72.257i
Pl  = 2645.0
Pl_ = 2645.0

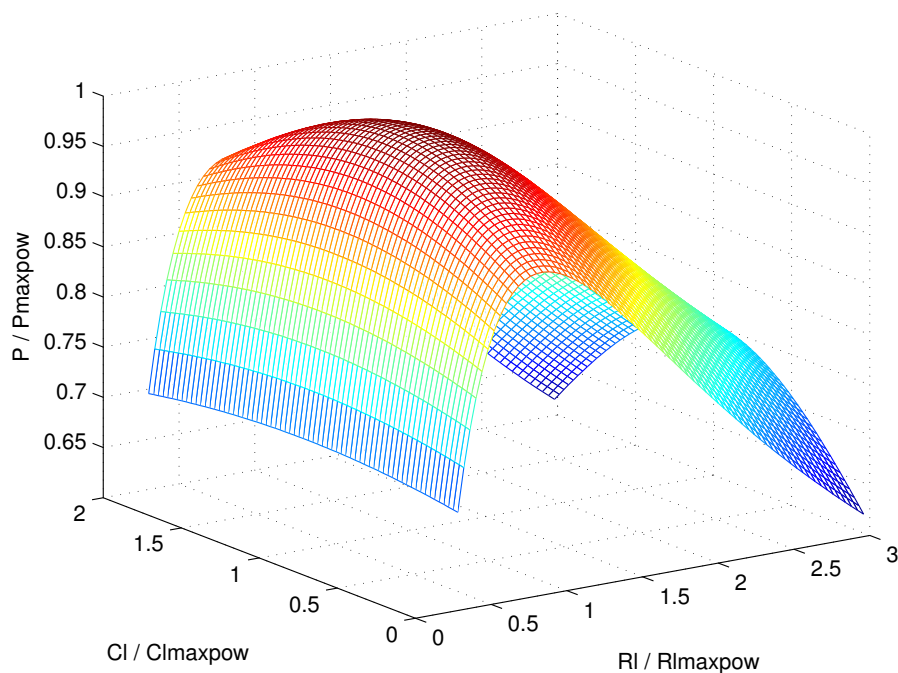
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Note the advantage of defining all the input numbers as variables, then working in small steps. This makes it possible to redefine a variable without rewriting all the equations, and to check intermediate values, and to keep each single expression short and readable. We see for example that Z_s and Z_l are indeed complex conjugates, as expected.

Numerical/graphical check of the maximum-power point.

We could check further, numerically, to ensure we did find a maximum-power point instead of having just messed up the equations.

Here, we take higher and lower values of the load resistance and capacitance, and do a surface plot, to check that the maximum is indeed at the calculated values. We find that it is indeed at the calculated values (relative resistance or capacitance equal to 1) that the maximum power occurs.



The code used is the following.

```

clear all

U = 230, w = 2*pi*50, Rs = 5, Ls = 10e-3,
Zs = Rs + 1j*w*Ls; % source impedance

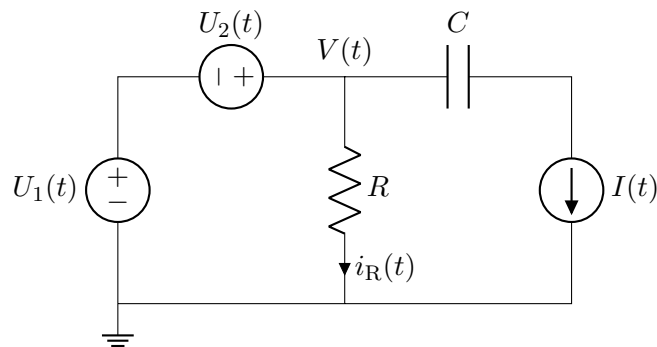
% values needed for maximum power, according to our calculations
Rl_ = (Rs^2 + w^2*Ls^2) / Rs; % the believed Rl for max power
Cl_ = Ls / (Rs^2 + w^2*Ls^2); % the believed Cl for max power
% calculate the maximum power
Zl = 1 / ( 1/Rl_ + 1j*w*Cl_ );
Ul = U * Zl / ( Zl + Zs );
Pl = abs(Ul)^2 / Rl_

% make vectors of other Rl and Cl values above and below
% the values that we calculated as giving "maxpower"
CL=Cl_*(0.02:0.03:4);
RL=Rl_*(0.02:0.04:4);
S = NaN*ones(numel(RL),numel(CL)); % initialise matrix for powers
% the following could be done in matrix form in one line ... this is simpler
for i=1:length(RL), for j=1:length(CL);
    Cl=CL(j); Rl=RL(i);
    Zl=1/(1/Rl+1j*w*Cl);
    Ul = U * Zl / ( Zl + Zs );
    S(j,i)=abs(Ul)^2 / Zl;
end ; end
% 3d surface plot, load active power against relative Rl and Cl
mesh( RL/Rl_, CL/Cl_, real(S)/Pl )
xlabel('Rl / Rlmaxpow');
ylabel('Cl / Clmaxpow');
zlabel('P / Pmaxpow');

```

Exercise 5 Power superposition.

Power superposition is not going to a major part of exam tasks. But it's a simple and fundamental principle, which can be convenient to understand conceptually as well as to use as a shortcut in calculations. This question gives some more practice with ac analysis and ac power, as well as with power superposition.



The time-functions of the three independent sources are:

$$\begin{aligned}U_1(t) &= \hat{U}_1 \cos(3\omega t + \phi) \\U_2(t) &= \hat{U}_2 \sin(2.2\omega t - \phi) \\I(t) &= -\sqrt{2} I_{\text{rms}} \cos(5.56\omega t + 27.4^\circ).\end{aligned}$$

a) What is the mean power dissipated in resistor R ? (Assume this is the “mean” over a sufficiently long time that many cycles of the ac waveforms have happened.) Power superposition gives a quick solution. Irrelevance can also be useful.

b) What is the mean power out of source $U_1(t)$?

c) If you feel a little crazy for work, you could try repeating part ‘a’, but not using the principle of power superposition or irrelevance (i.e. find a time-solution for $i_R(t)$, then integrate it over a suitable time-period, and check against ‘a’. Quite difficult and time-consuming. . . .

Answer 5

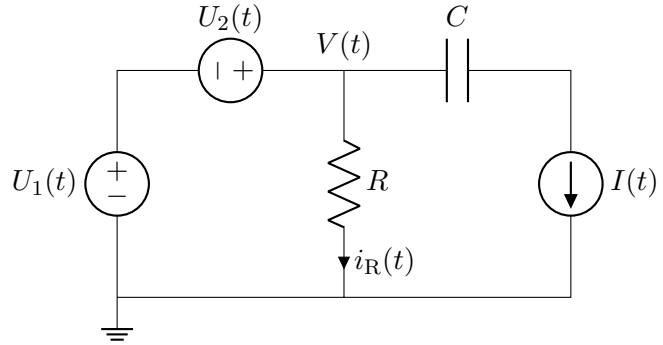
The studied circuit has three independent sources, whose time-functions are the following:

$$U_1(t) = \hat{U}_1 \cos(3\omega t + \phi),$$

$$U_2(t) = \hat{U}_2 \sin(2.2\omega t - \phi),$$

$$I(t) = -\sqrt{2} I_{\text{rms}} \cos(5.56\omega t + 27.4^\circ).$$

Many of these values will not be important for the solution.



a) Find the [long-term] mean power dissipated in resistor R .

Irrelevance tells us we can throw away C and $I(t)$ from the start: the voltage across R is fixed by the series pair of voltage sources, $V(t) = U_1(t) + U_2(t)$. The current in R is therefore

$$i_R(t) = \frac{\hat{U}_1 \cos(3\omega t + \phi) + \hat{U}_2 \sin(2.2\omega t - \phi)}{R}$$

We could square this and multiply by R to get instantaneous power, then integrate (in the time-domain) to get a mean value of power; some of the resulting $\cos() \sin()$, $\cos^2()$, etc. terms could be eliminated as having zero mean.

A quicker way to see the concept is power superposition. The frequencies of the two relevant sources are different, so power superposition can be used.

The mean power due to $U_1(t)$ alone is $\frac{\hat{U}_1^2}{2R}$, and the mean power due to $U_2(t)$ alone is $\frac{\hat{U}_2^2}{2R}$.

Therefore, the total mean power in R , by power superposition, is

$$P = \frac{\hat{U}_1^2 + \hat{U}_2^2}{2R}$$

The factor 2 is there because \hat{U}_1 and \hat{U}_2 are peak values.

b) What is the mean power out of source U_1 ?

This is simply the power out of source U_1 with the other sources set to zero. In that case the source U_1 is connected only to R , as U_2 is a short-circuit and I is an open-circuit. The power out of the source is thus $P = \hat{U}_1^2/(2R)$. With the other sources active instead, and U_1 set to zero, there can be no power in or out of source U_1 , as it has zero voltage. A way of seeing this same idea without power superposition is to realise that the current in source U_1 has three different frequencies, but only the current with the same frequency as the source will give a mean power over a long time.

Solution: $P = \frac{\hat{U}_1^2}{2R}$, out of source U_1 .