Simplifications to set solutions

Use a mixture of methods, sometimes iteratively.

REDRAW make neater, easily understood

REDUCE remove details not needed for what we're studying

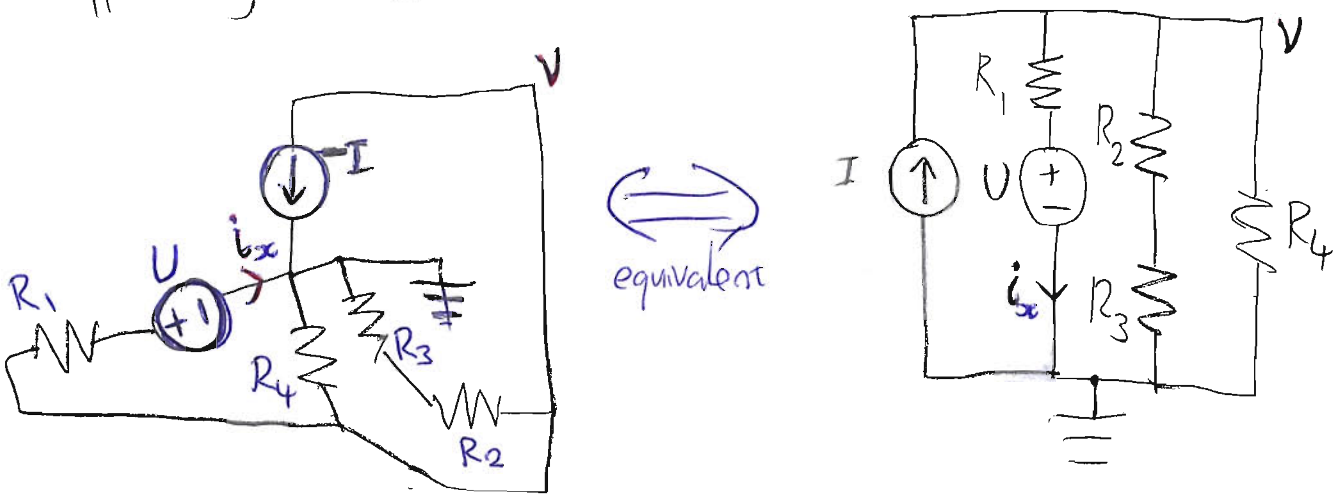
COMBINE join multiple components into an equivalent

DIVIDE {voltage / current} division – useful rules

TRANSFORM choose more convenient equivalents

REDRAWING

the same circuit (components, connections, markings i_x, u_y)
but differently drawn



Redrawing might make a solution much easier to see.
Or it might waste time and allow a transcription error...
THE BEST CHOICE depends on you and the circuit.

tips for re-drawing :

- **be systematic** and CHECK : in a complicated case, number the nodes
  check each component is present
  check each component connects to right nodes
  check directions relative to nodes
  count connections (terminals) on each node.

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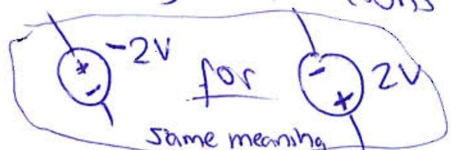
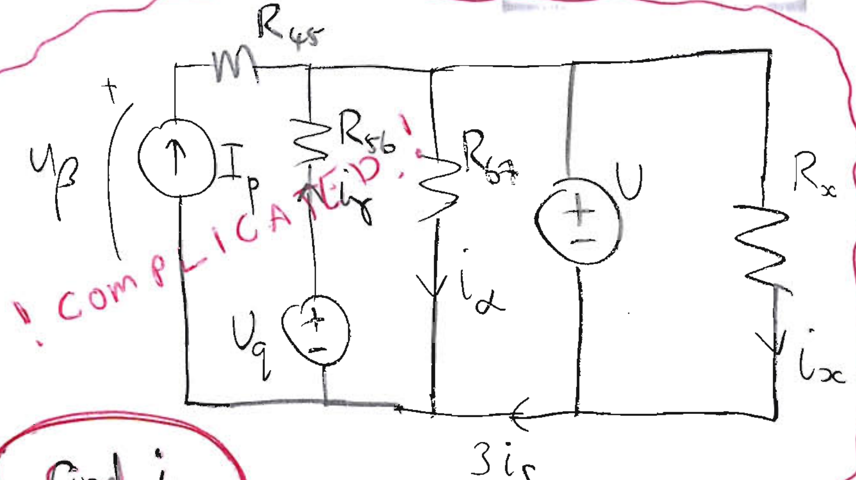
  - identify nodes with many connections ... place bottom or top.
  - consider swapping



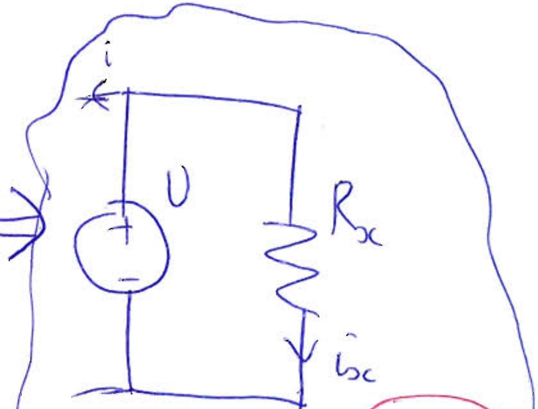
  - keep mainly vertical and horizontal.
  - etc

REDUCE

When redrawing, we might not need all the details



find i_x



KVL + Ohm $\Rightarrow i_x = \frac{U}{R_x}$

If the task is to find just i_x then in this circuit we don't need to know what happens the other side of the voltage source! KVL in one loop fixes the value of i_x.

This simplified circuit is ok for finding i_x. (But not for "what power does the source U deliver!")

.... but sometimes every component and malloing *is* relevant to the solution, so we can't reduce except by COMBINING components into *equivalents*.

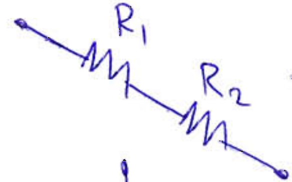
(i_x in the circuit that we redrew is an example where all components are relevant.)

So how do we combine components?

COMBINE

Probably familiar:

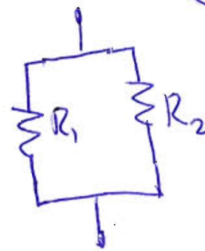
SERIES resistors



$\Longrightarrow$

$R_1 + R_2 = R_{equivalent}$

PARALLEL resistors



$\Longrightarrow$

$$\frac{1}{\frac{1}{R_1} + \frac{1}{R_2}} = \frac{R_1 R_2}{R_1 + R_2} = R_{equivalent}$$

combining into an equivalent means replacing with a component of the same u, i behaviour.

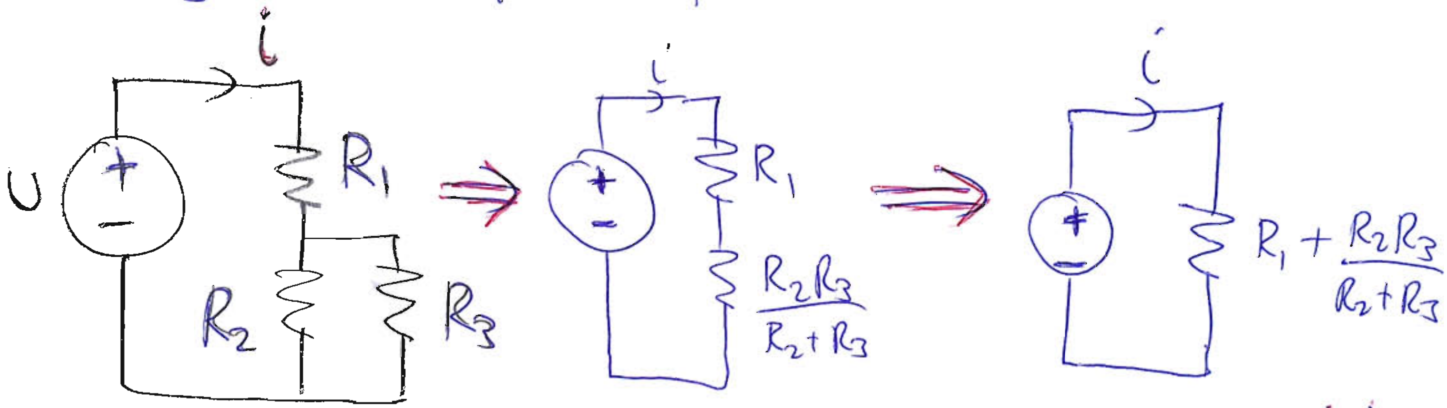
actual circuit, with two terminals

an equivalent that "looks the same" at the two terminals

There are also theorems for components connected with three or more terminals. We stay with just two for now!

example

Combining resistors often helps get solutions.



combine parallel

combine series

⇓ calculate

$$i = \frac{U}{R_1 + \dfrac{R_2 R_3}{R_2 + R_3}}$$

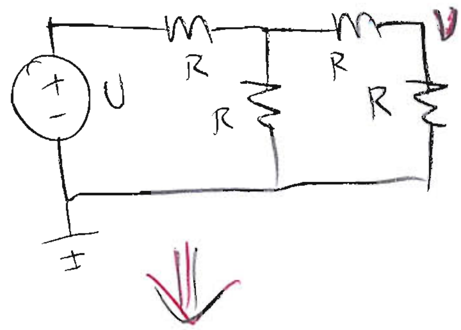
⇓ simplify

$$i = \frac{U(R_2 + R_3)}{R_1 R_2 + R_1 R_3 + R_2 R_3}$$

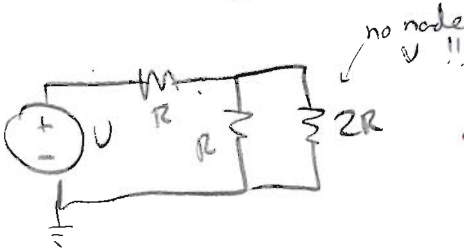
Could have solved by writing several KVL or KCL equations

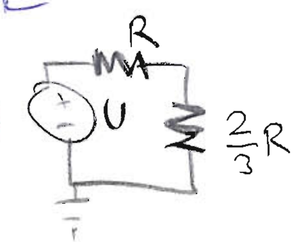
But combining can be neat.

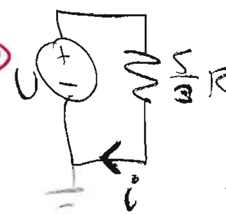
Another example: find V given that $U = 10V$ and $R = 2\Omega$



It's not trivial. Some simultaneous KVL or KCL could do it. Or we can combine resistors. If we combine, we lose the node where V is defined! But we can do this to solve something else, then work backward for V.

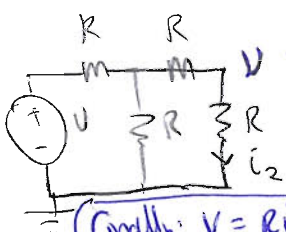






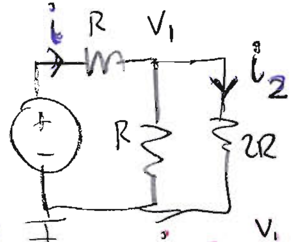
now define current i,

$$i = \frac{U}{R} \cdot \frac{3}{5}$$

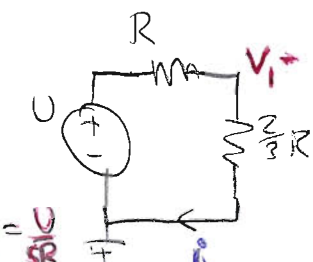


$$V = \frac{U}{5} = 2V$$

$$\boxed{\text{finally: } V = R i_2 = \frac{RU}{5R} = \frac{U}{5}}$$



$$i_2 = \frac{V_1}{2R} = \frac{2U}{5 \cdot 2R} = \frac{U}{5R}$$



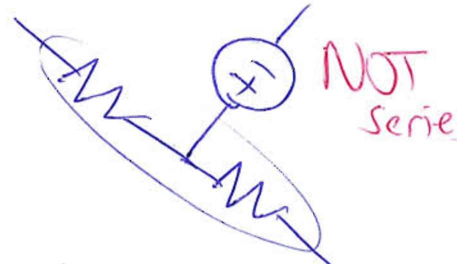
start removing combinations

$$V_1 = \frac{2}{3}R \cdot \frac{U}{R}\frac{3}{5} = \frac{2}{5}U$$

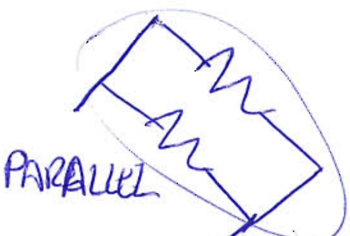
A warning.

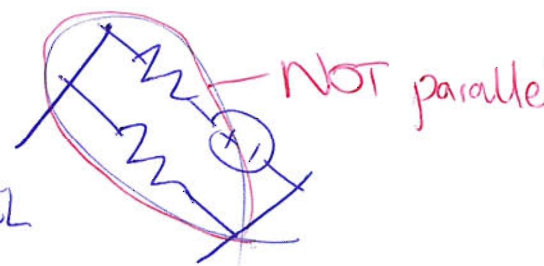
Resistors in series or parallel can be made into an equivalent.

<u>SERIES</u> means all current in one passes in the other too ... they should have one terminal of each connected, but not to a third component



<u>PARALLEL</u> means both connect directly between the same two nodes.





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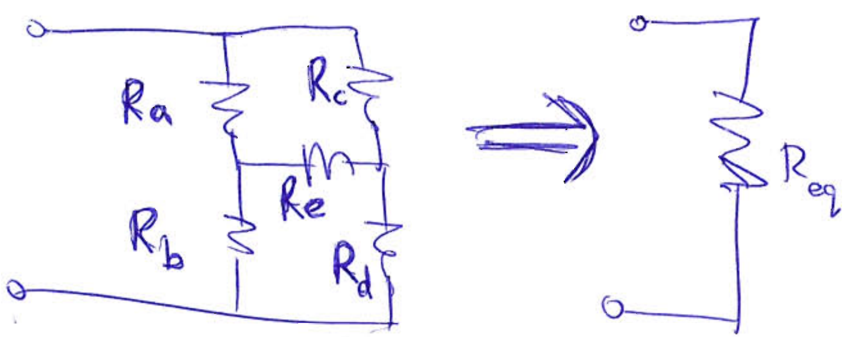
combining resistors to an equivalent two-terminal model is typically useful when solving something elsewhere in the circuit.

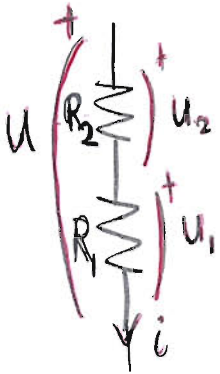
Or sometimes we solve elsewhere, then solve something inside the set of resistors that we had equivalenced (like the previous example).

Making these equivalents is not always possible by just the series/parallel rules:



⇐ an R_{eq} exists but you won't find it by series/parallel reasoning as nothing is series or parallel here!

Resistors in series and parallel : how to show the relation.



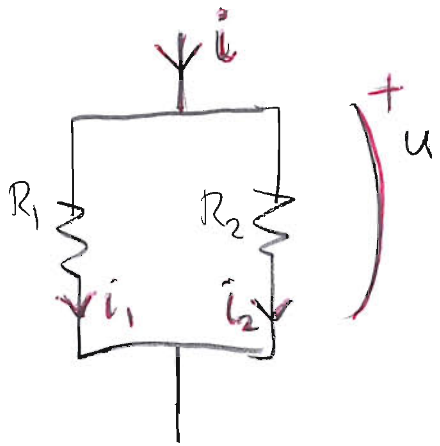
$$\left.\begin{array}{l} u = u_1 + u_2 \end{array}\right\} \text{KVL (potential vandring)}$$

$$\left.\begin{array}{l} u_1 = iR_1 \\ u_2 = iR_2 \end{array}\right\} \text{Ohm}$$

implicitly, by KCL, we see i is same in both resistors.

$$\Longrightarrow u = i(R_1 + R_2) = i\,R_{eq}$$

a resistor of R_{eq} will behave with the same u, i relationship



$$\left.\begin{array}{l} i = i_1 + i_2 \end{array}\right\} \text{KCL}$$

$$\left.\begin{array}{l} i_1 = \frac{u}{R_1} \\ i_2 = \frac{u}{R_2} \end{array}\right\} \text{Ohm}$$

implicitly, by KVL, we see u is same for both

$$\Longrightarrow i = u\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

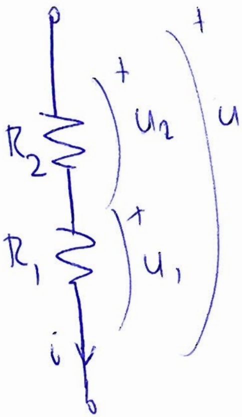
so $$u = i \cdot \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}} = i\,R_{eq}$$

$$= \frac{i\,R_1 R_2}{R_1 + R_2}$$

DIVIDERS

$\left\{\begin{array}{l}\text{Voltage}\\ \text{Current}\end{array}\right\}$ division are quick formulas useful with series or parallel resistors

Voltage division



Voltage u is divided between R_1 e R_2.

How can we find u_1 if we know u ?

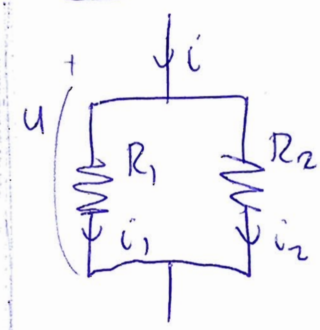
$$u_1 = i R_1$$

$$i = \frac{u}{R_1 + R_2}$$

equivalent resistance

$$\boxed{\therefore\ u_1 = u \frac{R_1}{R_1 + R_2}}$$

Current division



Current i is divided between R_1 and R_2.

How can we find i_1 if we know i ?

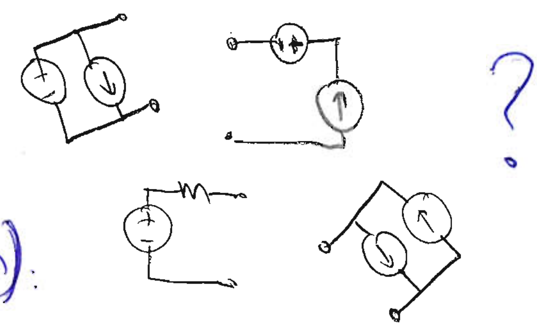
$$i_1 = \frac{u}{R_1} = \frac{i\left(\frac{R_1 R_2}{R_1 + R_2}\right)}{R_1}$$

equivalent resistance

$$\boxed{\therefore\ i_1 = i \frac{R_2}{R_1 + R_2}}$$

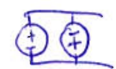
Other combinations of two components, connected in parallel or series to two terminals, are also of interest.

Can we simplify/combine these pairs?

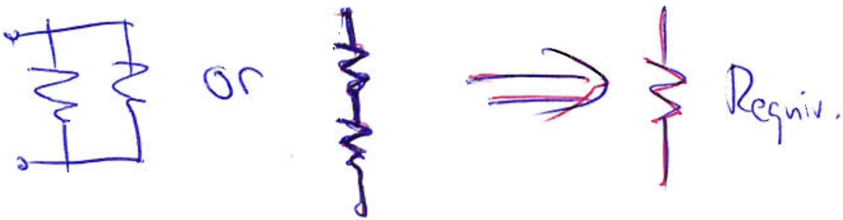


There are 12 pairs, if using U, I, R, series or parallel (without considering direction).

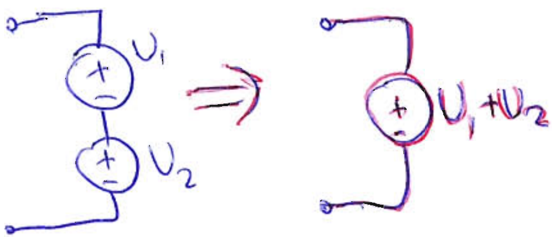
They each give one of 4 types of behaviour:

0. contradiction/badly defined: 
1. behave like 1 component with a value depending on both: R_1 R_2 → R_1+R_2
2. behave like just 1 of the components — the other is hidden (irrelevant)
3. cannot be simplified to one component, but has well defined behaviour.

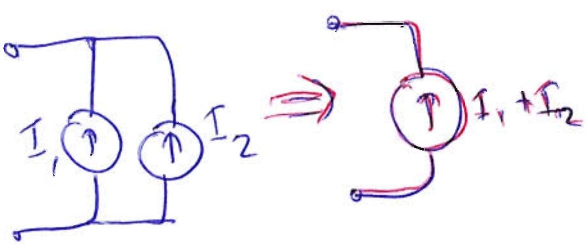
Two similar components:



Seen already

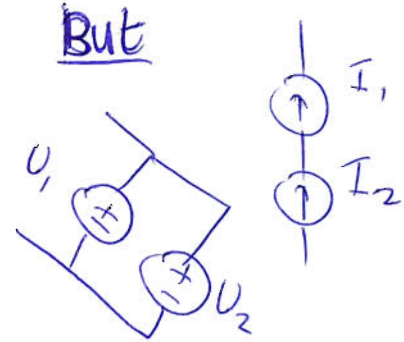


by KVL



by KCL

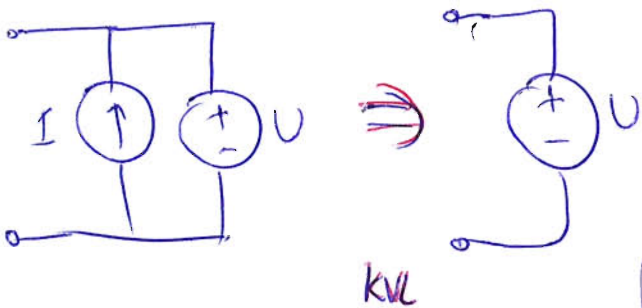
But

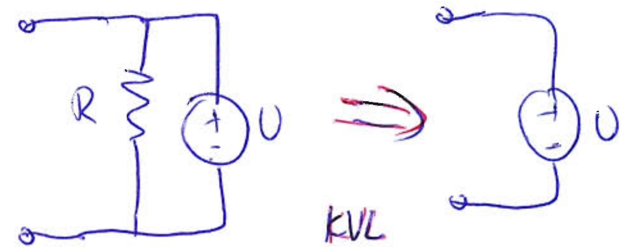


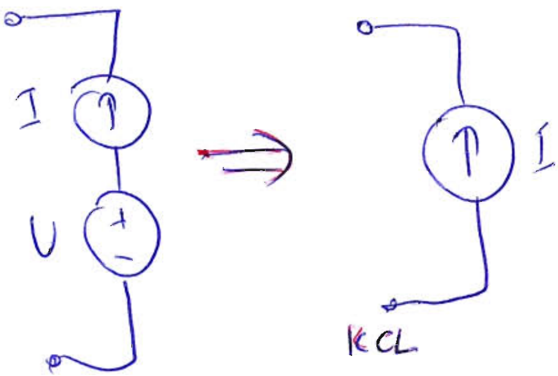
are strange "forbidden" cases ... What does Kirchhoff say if $U_1 \neq U_2$ or $I_1 \neq I_2$?

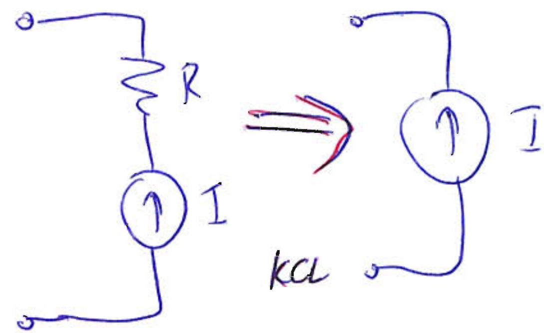
BADLY DEFINED circuit ... contradiction.

Sources U, I together | or with resistor

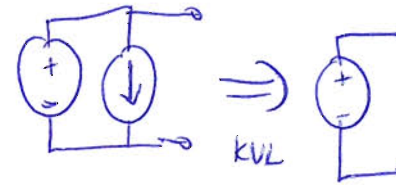






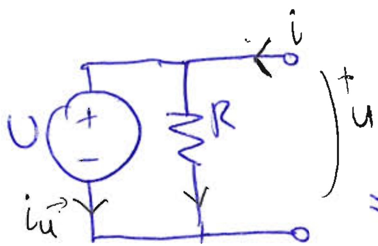


So SERIES CURRENT source or PARALLEL VOLTAGE source **WINS**

If you're suspicious about the earlier page, eg. 

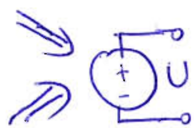
then try this reasoning ...

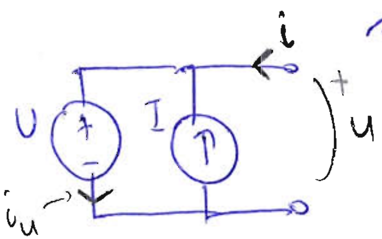
We have:



KVL: $u = U$

KCL: $i = \frac{U}{R} + i_u$





KVL: $u = U$

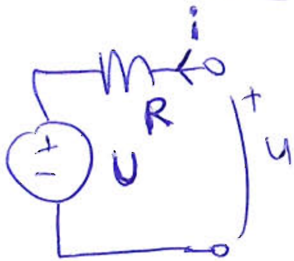
KCL: $i = -I + i_u$

the voltage sources by definition fix U and have i_u undetermed

(we can solve i_u by solving the whole circuit including whatever connects to the u, i terminals)

so the total circuit also has a fixed $u = U$ (KVL) and an undefined i ... just like a voltage source with value U

Final cases — not like any single component



"Thevenin source"

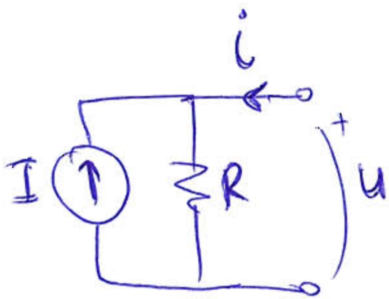
$$u = U + iR$$

depends on both R & U

notice: these are the same if we choose that $U = IR$

$$u = (I + i)R = IR + iR$$

depends on both R & I.

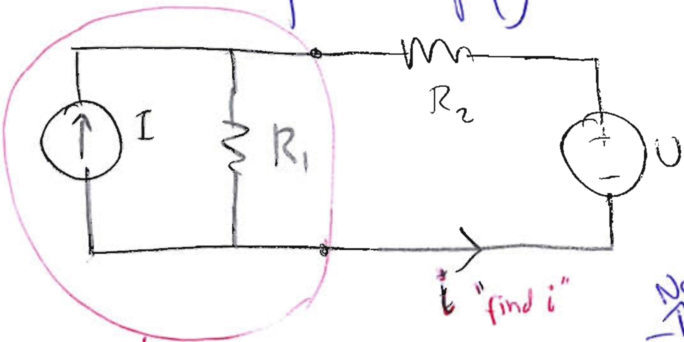


"Norton source"

Task: prove the above relations using KVL/KCL/Ohm's law.

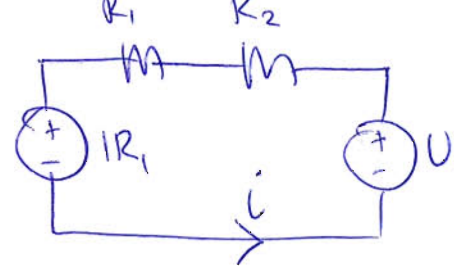
SOURCE TRANSFORMATION: swap between equivalent Thevenin and Norton sources.

This can help to simplify ...



Norton source.

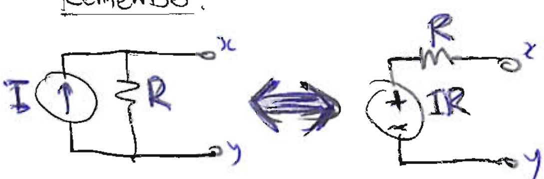
Norton - Thevenin transformation



Now it is easier. KVL gives:

$$i = \frac{U - IR_1}{R_1 + R_2}$$

Remember:



Norton Thevenin

Equivalent