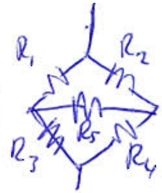
Systematic methods: NODAL ANALYSIS (Pt. 1/2)

We've **seen** "tricks" like: equivalent resistor, remove components, voltage division, current division, source transformation, redraw, - - -

and we've put them together to solve some circuits.

But not all circuits can be attacked using just these:



(remember?)

On the other hand, we claimed that KVL, KCL, and the component definitions, were sufficient for all our circuits! (The trouble is that they quickly give us lots of equations.)

> **We want** a systematic way to get minimal number of equations to solve any circuit containing our known components.

What is the meaning and advantage of being SYSTEMATIC ?

We mean : following a particular set of rules (always)
(by implication : a simple set of rules.)

advantage.

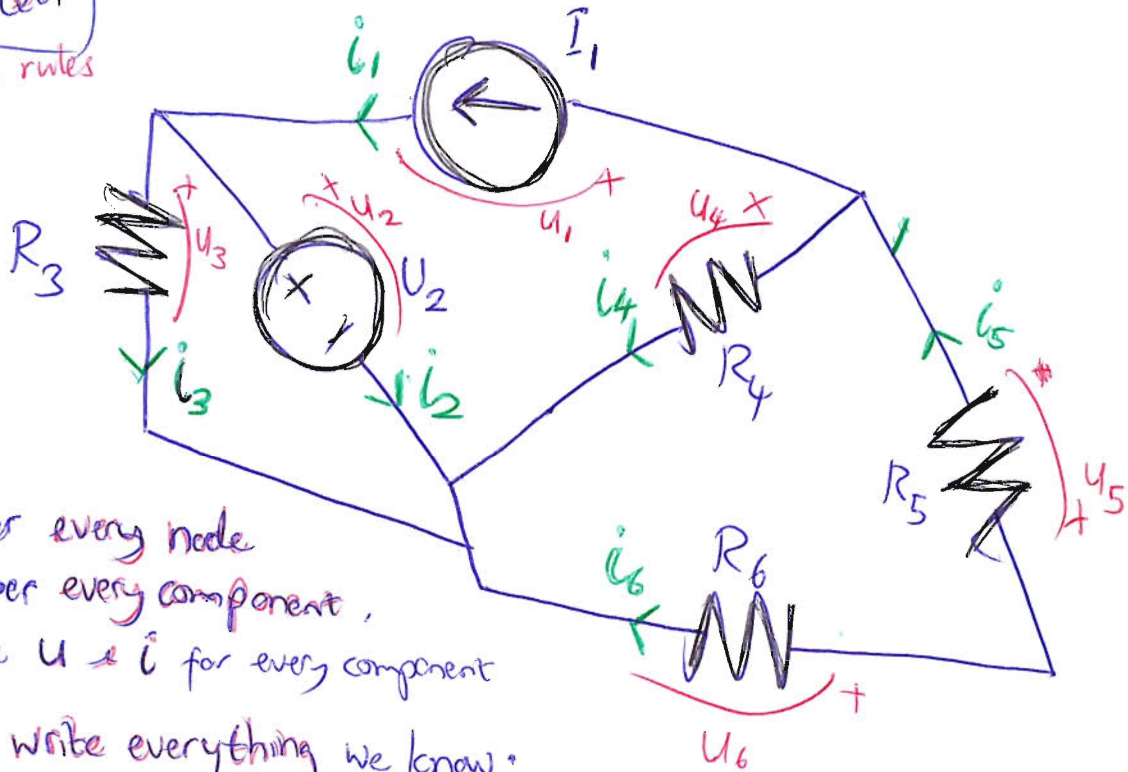
- solution is not so dependent on our luck and skill at seeing which clever trick to use
- we can program a computer (easily) to do circuit analysis.

(disadvantage)

- the equations we generate may be hard to solve "by hand"
- other methods may make us "understand" the circuit better by having to think what is important to our solution

(Bad idea)

using ALL rules



Number every node
Number every component.
Define u & i for every component

Then write everything we know:

$u_2 = U_2$
$i_1 = I_1$
$\frac{u_3}{i_3} = R_3$
etc.
component definitions

$u_3 - u_2 = 0$
$u_4 + u_5 - u_6 = 0$
etc.
KVL in every loop

$i_1 + i_4 - i_5 = 0$
$-i_1 + i_2 + i_3 = 0$
etc.
KCL at every node

That "bad idea" way should work

But it's lots of equations and new variables.

In our simple case: $\underset{\text{components}}{6} + \underset{\text{KVL}}{3} + \underset{\text{KCL}}{4}$ equations.

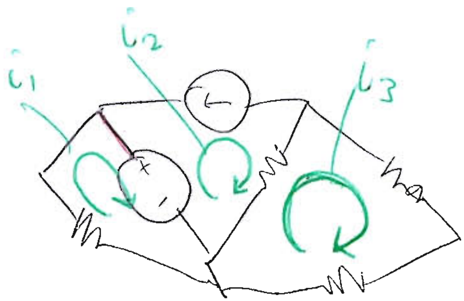
We don't need so many.

One efficient systematic method

"mesh analysis" (maskanalys)

(skip this page unless interested)

NOT in this course



- Identify each inner loop (mesh)
- Define a current in it $\{i_1, i_2, i_3\}$
- Write KVL for each loop.
- Solve for currents i_1, i_2, i_3

$\Rightarrow$ then easy to find other quantities such as current in a component, or voltage.

Summary:

define current

use KVL

Bad points:

- Not so easy to define minimal number of loops in a circuit that can't be drawn flat without crossings. (-|-)
- Mesh currents aren't directly physical currents in some cases.

Our choice of an efficient systematic method: NODAL ANALYSIS

a complement of mesh analysis: define potentials, and use KCL

Basic rules (see additions on later pages!!)

Define one node as zero ('ground' 'earth')

Define potentials at other nodes

Write KCL at these other nodes (not the earth node)

for N nodes,

$\Leftarrow$ N−1 potentials (unknown)

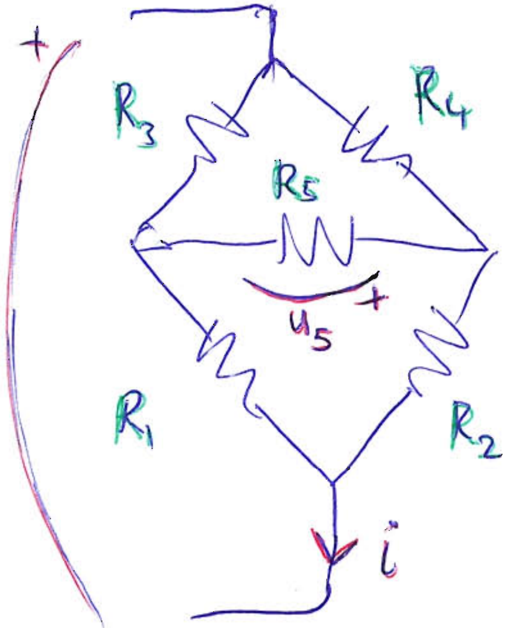
$\Leftarrow$ N−1 independent equations

$\Downarrow$

solvable system

Example: Nodal analysis

Let's take that annoying circuit that we couldn't solve! (We could but not easily by our toolbox of simplifications and tricks.)

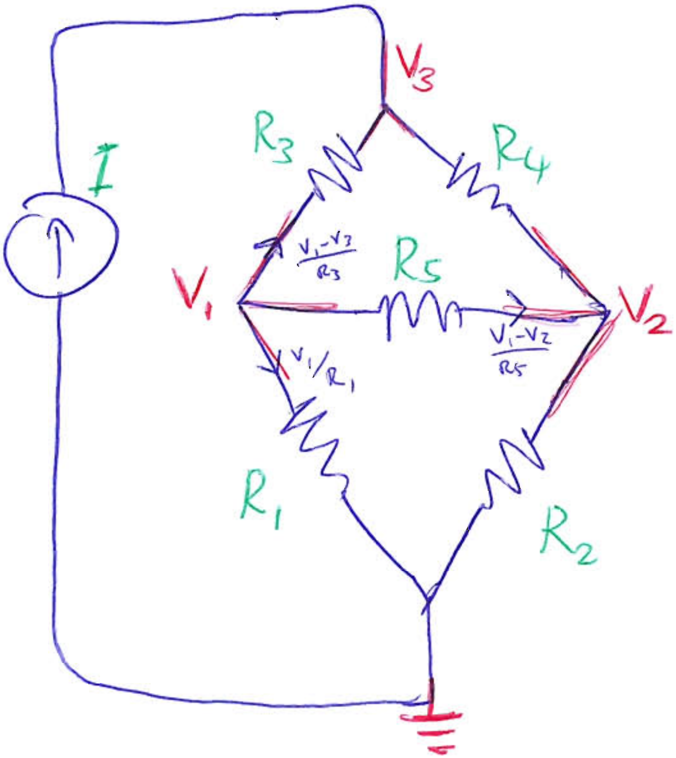


- Suppose we want to find the voltage u_5 across the central resistor. Then we can use the case $R_1 = R_2 = R_3 = R_4 = R_5$ to prove that $u_5 = 0$ for this symmetric case.

- Perhaps we'd also like to find the relation u/i which tells us the equivalent resistance of this circuit.

so apply the nodal analysis rules.

(Let's start with the case where we define current i by a source I)



- Set a zero node ⏚
- mark potentials at the other nodes
- write KCL at each of these:

KCL on outward currents

node 1

$$\frac{V_1-0}{R_1}+\frac{V_1-V_2}{R_5}+\frac{V_1-V_3}{R_3}=0$$

node 2

$$\frac{V_2-0}{R_2}+\frac{V_2-V_1}{R_5}+\frac{V_2-V_3}{R_4}=0$$

node 3

$$\frac{V_3-V_1}{R_3}+\frac{V_3-V_2}{R_4}-I=0$$

$\boxed{\text{That gave us 3 (independent!) equations and 3 unknowns. It's solvable.}}$

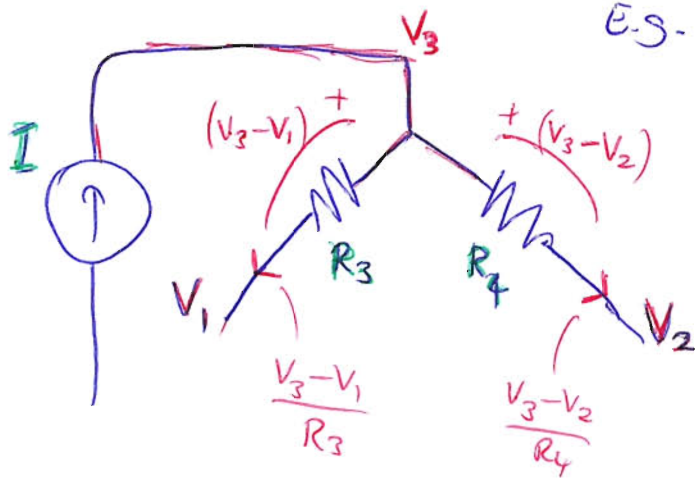
Quite hard work to solve symbolically for all of V_1, V_2, V_3.
(But a computer is happy to do it.)

- We could solve for just V_3 to help find the circuit's equivalent resistance $\left(\frac{V_3}{I}\right)$
- Or for V_1 & V_2 to find u_5
- It's easier if R_1, R_2 etc have numeric values: then we get simple coefficients,

e.g. $\frac{1}{R_1} + \frac{1}{R_2} \underset{\text{becomes}}{\Longrightarrow} 2{,}56$

Zoom in ... how did we write KCL:

e.g. node 3 in the earlier circuit.



$$\Rightarrow \frac{V_3 - V_1}{R_3} + \frac{V_3 - V_2}{R_4} - I = 0$$

Quick way of handling resistor!

$\Rightarrow$ Current out from here to there is:

$$\frac{V_{here} - V_{there}}{R}$$

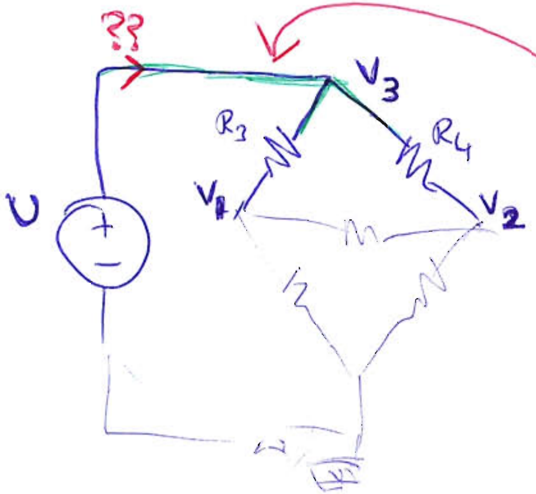
Ok We can handle current sources and resistors in KCL

current by definition

potential difference and Ohm's law tells us the current

But what if there had been a VOLTAGE source ??

We've always said "it tells us nothing about the current"



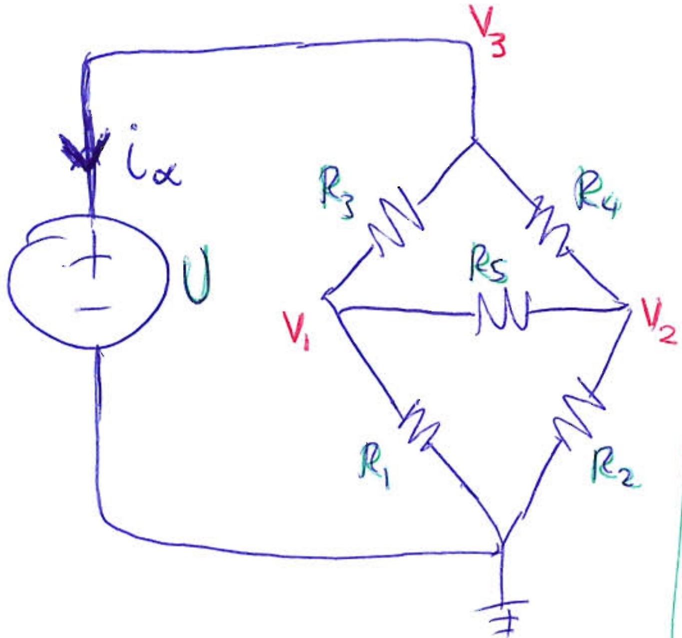
how to get KCL here ?

currents in resistors are ok (as before)

What about voltage source's current ??

SOLUTION: define an unknown current

... but then we have more unknowns than equations ...



KCL, using defined unknown i_α :

(1) $$\frac{V_1}{R_1} + \frac{V_1 - V_2}{R_5} + \frac{V_1 - V_3}{R_3} = 0$$

(2) $$\frac{V_2}{R_2} + \frac{V_2 - V_1}{R_5} + \frac{V_2 - V_3}{R_4} = 0$$

(3) $$\frac{V_3 - V_1}{R_3} + \frac{V_3 - V_2}{R_4} + i_\alpha = 0$$

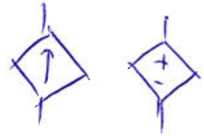
3 equations, 4 unknowns of V_1 V_2 V_3 i_α.

KEY: The voltage source gave an extra unknown i_α (BAD)
BUT it provides an extra equation too : $\boxed{V_3 - 0 = U}$ (GOOD)

Now we have the rules for writing nodal analysis equations in circuits with VOLTAGE SOURCES, CURRENT SOURCES and RESISTORS.

(We could handle a circuit of 1000 components, if we have a computer to solve the equations for us...)

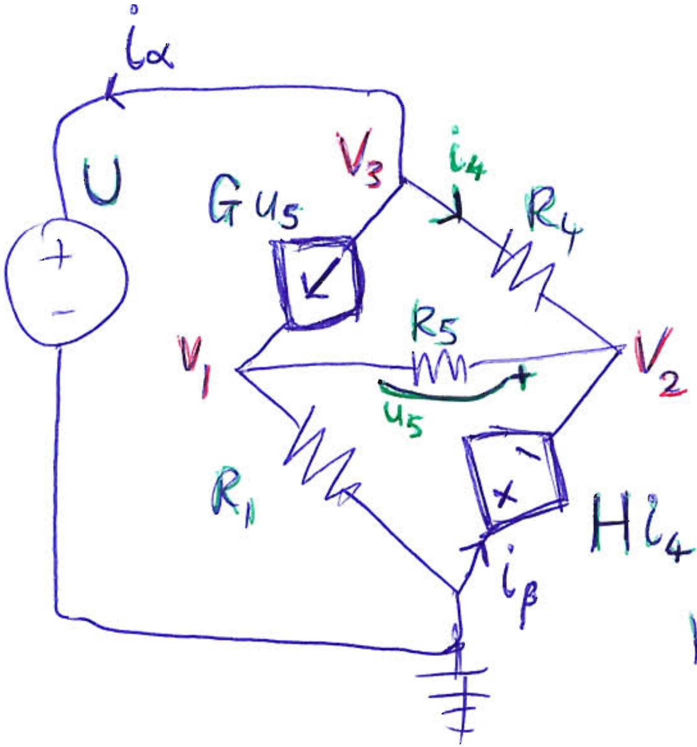
What's missing ....?



... we haven't considered DEPENDENT voltage or current sources

Good news they're nothing special — treat the same as an independent source, and then define the controlling variable

Example with dependent sources:



NOTE: i_4 and u_5 are the controlling variables of the dependent sources

We define the current in each voltage source: here we have i_α and i_β

KCL:

$$① \quad \frac{V_1}{R_1} + \frac{V_1 - V_2}{R_5} + (-G u_5) = 0$$

$$② \quad \frac{V_2 - V_3}{R_4} + \frac{V_2 - V_1}{R_5} + (-i_\beta) = 0$$

$$③ \quad G u_5 + \frac{V_3 - V_2}{R_4} + i_\alpha = 0$$

Include voltage-source equations:

$$V_3 = U, \quad V_2 = -H i_4.$$

And define controlling variables in terms of existing variables:

$$i_4 = \frac{V_3 - V_2}{R_4}, \quad u_5 = V_2 - V_1$$

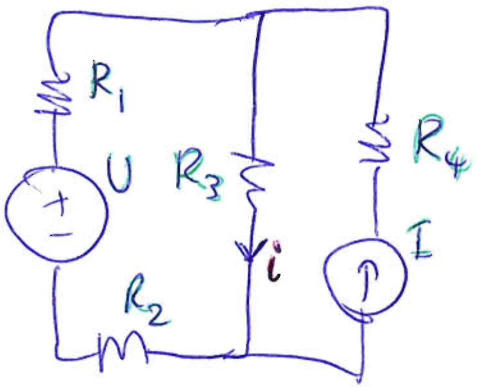
The previous page shows a very general and reliable method, based on very few rules, for handling conversion of circuit diagrams to equations.

I used to call it "the simple method". It's commonly called "extended nodal analysis", as it includes and solves for the unknown currents in voltage sources.

There are several ways that we can simplify the equations before or during writing them! This is useful for "solving by hand". (It's particularly easy when we don't care about some of the variables' solutions.)

The rest of this topic is about these various ways of writing more human-friendly nodal analysis equations!

Example of nodal analysis with simplification before writing the equations ...



"find i" (by nodal analysis, symbolically without computer)

By the machine-like use of rules from the earlier cases, we would write 4 KCL equations, with 4 potentials, and an unknown current in source U ..

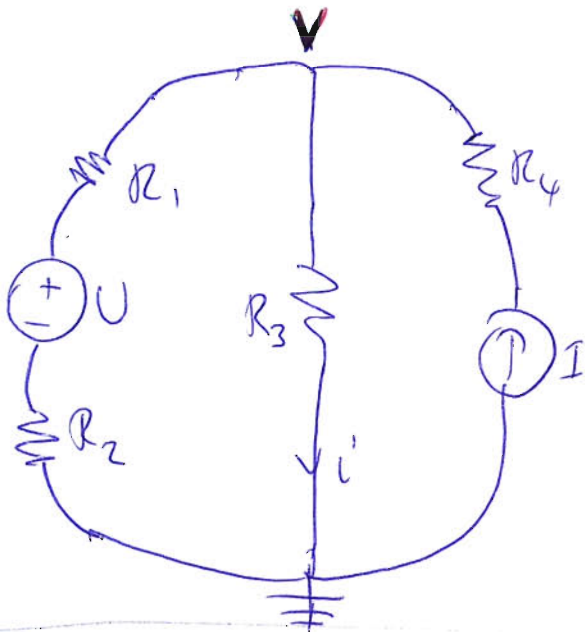
$\Rightarrow$ 5 equations, 5 unknowns

Then solve for $i = \dfrac{V_{top} - V_{bottom}}{R_3}$ $\Leftarrow$ names of the nodes above & below R_3

Example cont.d.

Instead: recognise that we only want the potential at one side of R_3 (call the other side zero).

... then we need only one KCL -- can we adequately express the currents into the node?



KCL (node V)
outward current

$$\frac{V}{R_3} + \frac{V-U}{R_1+R_2} - I = 0$$

middle branch by ohm's law

left branch by KVL and ohm's law

right branch by KCL & current source definition

The previous example gave us a neat, quick solution.

We could have written a more simple circuit first, but one gets familiar with treating whole branches

Often, nodal analysis is a good choice: "Write KCL"!

Often, it needs to be simplified by avoiding variables we don't want, or simplifying branches, if it is to give nice equations.

Next session further simplifications: "supernode".