NODAL ANALYSIS (Part 2):

- writing **simpler** equations
- "**supernode**" approach

last time

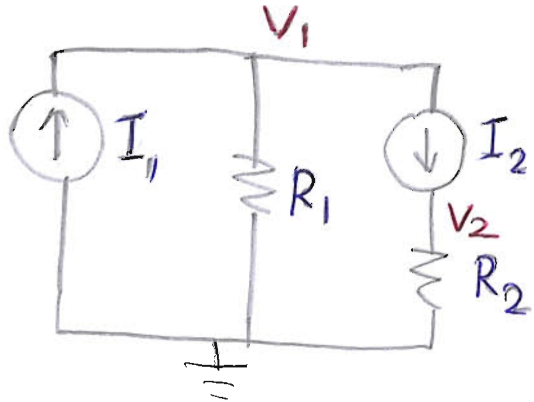
- "extended" **nodal analysis**
  - simple rules (GOOD)
  - generates **lots** of equations and unknowns (BAD for humans)
  - solves all potentials and voltage-source currents

- then we looked at combining nodal analysis with simplifications, when not needing all the variables to be solved.
  - shorter equations

Now we look at some more ways to use nodal analysis in a way that's helpful for hand-calculations

VOLTAGE SOURCES in nodal analysis

It's 'traditional' to start as we did (this year) by considering the case of **resistors and current sources** for nodal analysis:

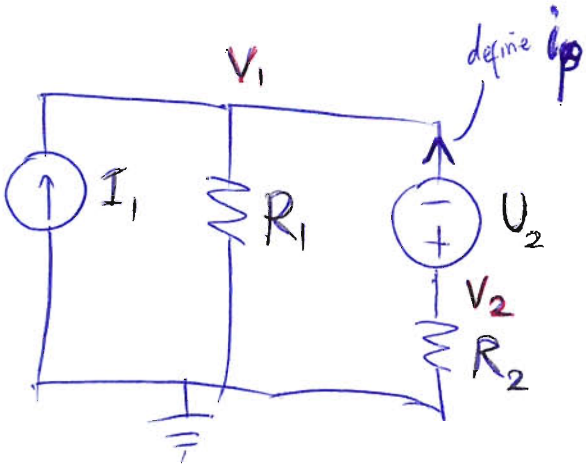


$$\begin{array}{ccccc} \text{known} & & \text{unknown} & & \text{known} \\ \begin{bmatrix} \frac{1}{R_1} & 0 \\ 0 & \frac{1}{R_2} \end{bmatrix} & & \begin{pmatrix} V_1 \\ V_2 \end{pmatrix} & = & \begin{bmatrix} I_1 - I_2 \\ I_2 \end{bmatrix} \end{array}$$

$$\underbrace{[\underline{A}] \cdot [\underline{x}] = [\underline{b}]}_{\text{Classic matrix equation}}$$

This fits nicely with linear algebra courses.
The translation from circuit to equation is easily described (few rules).
(But it gives N-1 equations for an N-node circuit).

When we introduced voltage sources in nodal analysis, we got EVEN MORE equations and variables!



$$
\begin{array}{ccc}
\text{known} & \text{unknown} & \text{mixed known/unknown} \\
\begin{bmatrix} \frac{1}{R_1} & 0 \\ 0 & \frac{1}{R_2} \\ -1 & 1 \end{bmatrix} & \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = & \begin{bmatrix} I_1 + i_\beta \\ -i_\beta \\ U_2 \end{bmatrix}
\end{array}
$$

Or in non matrix form:

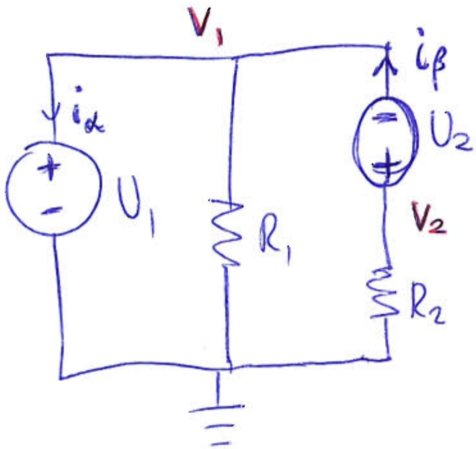
$$\frac{V_1}{R_1} - I_1 - i_\beta = 0 \qquad \text{KCL}(1)$$

$$\frac{V_2}{R_2} + i_\beta = 0 \qquad \text{KCL}(2)$$

$$V_2 - V_1 = U_2 \qquad \text{(voltage source)}$$

This makes it look harder.
But that's mainly because we've chosen to solve all the potentials and the current i_α, and to follow a single set of rules always...

going further ... only voltage sources and resistors ...



We could write even more complicated equations, for four unknowns:

$$\underbrace{\begin{pmatrix} \frac{1}{R_1} & 0 \\ 0 & \frac{1}{R_2} \\ 1 & 0 \\ -1 & 1 \end{pmatrix}}_{\text{known}} \underbrace{\begin{pmatrix} V_1 \\ V_2 \end{pmatrix}}_{\text{unknown}} = \begin{pmatrix} i_\beta - i_\alpha \\ -i_\beta \\ U_1 \\ U_2 \end{pmatrix} \begin{matrix} \left.\begin{matrix} \\ \\ \end{matrix}\right\} \text{unknown} \\ \left.\begin{matrix} \\ \\ \end{matrix}\right\} \text{known} \end{matrix}$$

and solve for V_1, V_2 and i_α, i_β if we care!

Or we can simply notice that we can IMMEDIATELY WRITE DOWN the solved values of the potentials!

Look at the circuit — "potential vandra" a bit:

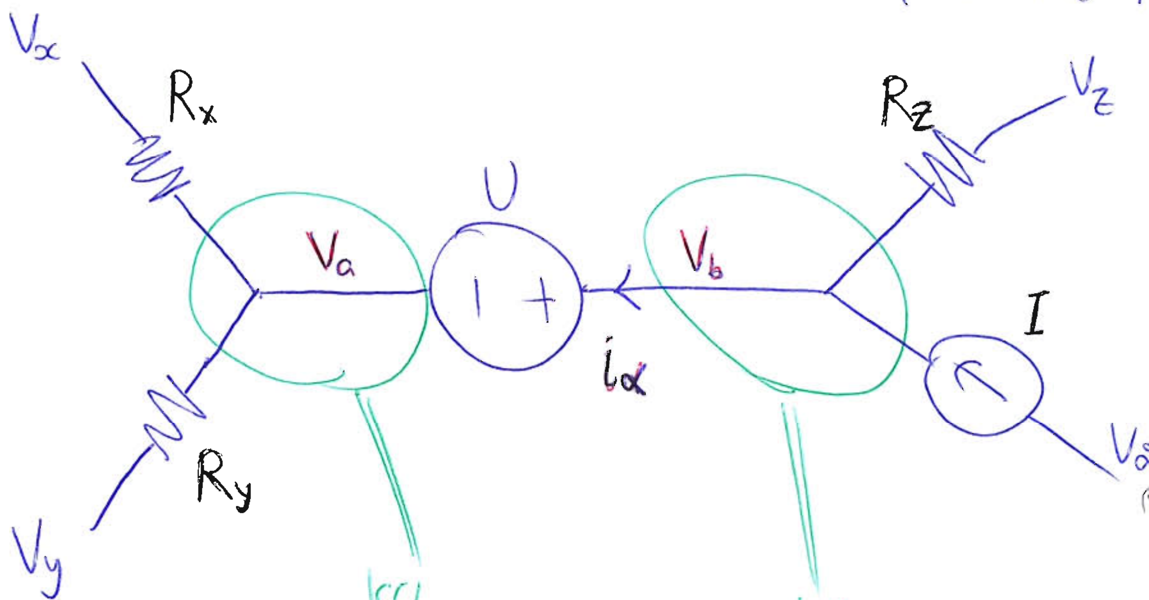
$$\boxed{\begin{aligned} V_1 &= U_1 \\ V_2 &= U_1 + U_2 \end{aligned}}$$

(This immediate solution could be seen from the matrix equations too, but we'd like to avoid writing equations at all if we can instead write the solution directly!)

The circuit on the previous slide was a special case where all nodes were connected by voltage sources, so all potentials could at once be determined by simple use of KVL.

But we can in any circuit use voltage sources as a way (ta fördel av befintliga) to simplify (instead of complicate) the nodal equations --- this is useful for solving by hand.

Consider part of a circuit (two nodes, connecting to four other nodes)



potential doesn't affect nodes V_a V_b

KCL

$$-i_\alpha + \frac{V_a - V_x}{R_x} + \frac{V_a - V_y}{R_y} = 0$$

KCL

$$i_\alpha - I + \frac{V_b - V_z}{R_z} = 0$$

This is what we did in extended nodal analysis ... add: $U = V_b - V_a$

to compensate for the extra unknown i_α

$\Rightarrow$ 3 equations

But suppose we don't care about i_α.
(We just want the potentials ...)

Then we can do some immediate reduction to the three equations of this voltage source (previous page):

Add the KCL to each other:
(to eliminate the unknown current)

$$\boxed{-I + \frac{V_a - V_x}{R_x} + \frac{V_a - V_y}{R_y} + \frac{V_b - V_z}{R_z} = 0}$$

(New KCL for both nodes around the source)

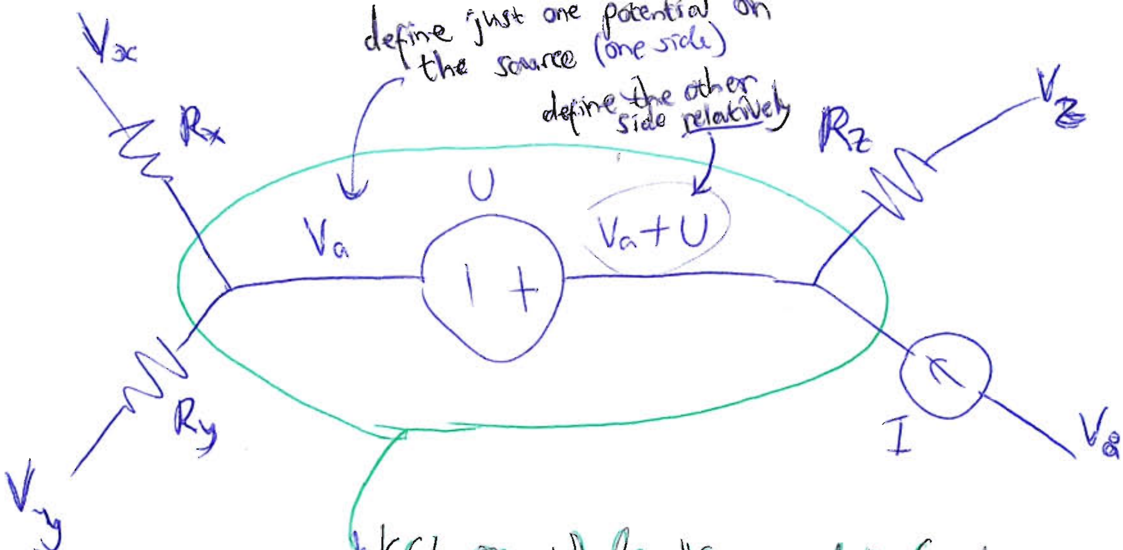
Substitute to source's equation $V_b - V_a = U$ into this to eliminate one of the potentials (e.g. V_b)

$$\boxed{-I + \frac{V_a - V_x}{R_x} + \frac{V_a - V_y}{R_y} + \frac{\overbrace{V_a + U}^{V_b} - V_z}{R_z} = 0}$$

(only one potential at the voltage source is solved for)

This removal of i_α is sometimes called the 'supernode' method.

Supernode method seen in the circuit (instead of the algebra):



do KCL on whole "supernode" (nodes joined by voltage source)

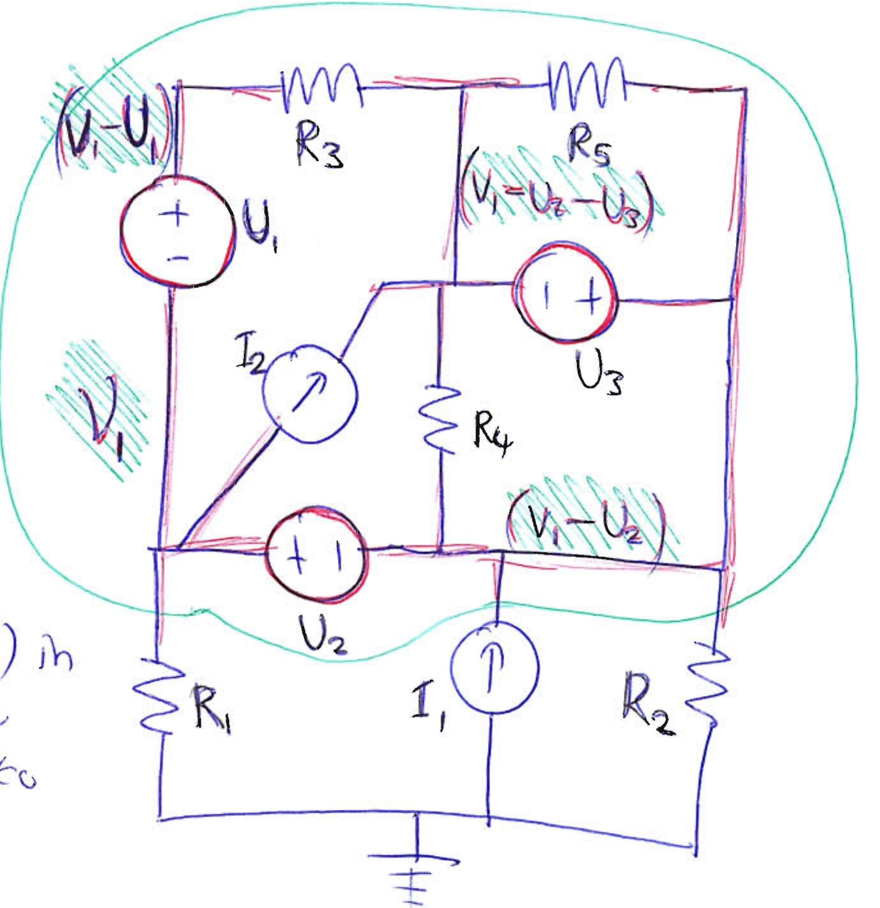
$$-I + \frac{V_a - V_x}{R_x} + \frac{V_a - V_y}{R_y} + \frac{V_a + U - V_z}{R_z} = 0$$

This provides a neat, "circuit thinking" way to write one instead of three equations for the voltage source and its surrounding pair of nodes.

Any set of nodes linked by voltage sources can be treated as a supernode

This example has a supernode consisting of four nodes linked by three voltage sources.

The potential of one node (V_1) in the supernode is marked. The others are expressed relative to this: $(V_1 - U_2)$ etc.



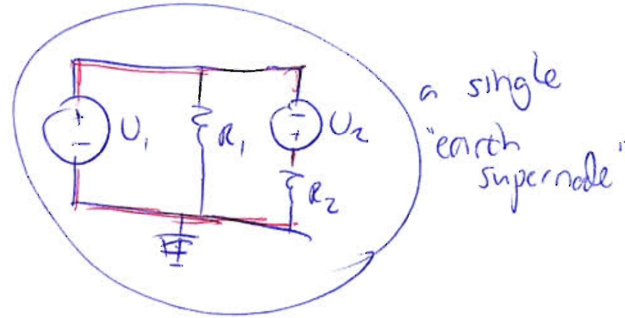
Solve this circuit: KCL at supernode:

$$\boxed{\frac{V_1}{R_1} - I_1 + \frac{V_1 - U_2}{R_2} = 0}$$

Easy! One equation and unknown. Then $V_1 + U_1$, $V_1 - U_2$ etc can be expressed.

If a supernode contains the reference (earth) node, we don't need KCL on it at all.!

The first example showed this case:

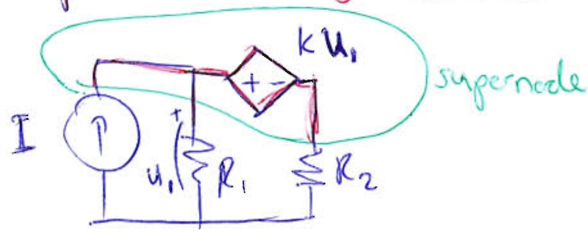


We can understand this, as there's no need to define any unknown potential (like V_1) for the supernode if one of its nodes is defined as zero potential — all the nodes linked by voltage sources have their potentials defined already.

DEPENDENT SOURCES

Already seen (last time) (part 1): dependent sources are treated in nodal analysis similarly to the independent sources of the same type ... (but we need to express their controlling variable in terms of existing variables)

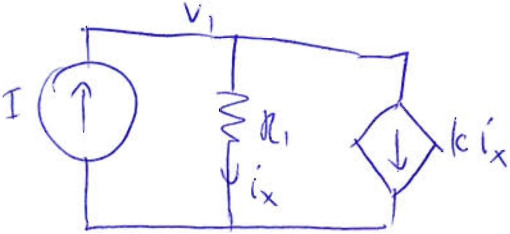
Thus: dependent voltage sources also join nodes into supernodes:



By "supernodes" we have a formalised way to write the equations in a simpler way from the start. (A similar set of equations could be made by algebraic steps from the "extended method" equations)

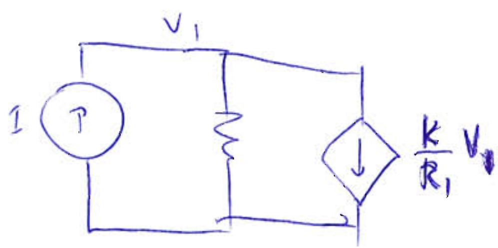
"Simplifying when writing" can also be used with controlling variables.

Instead of:



$$\Rightarrow \begin{cases} \dfrac{V_1}{R_1} + k i_x - I = 0 & \text{(KCL)} \\ i_x = \dfrac{V_1}{R_1} & \text{(define } i_x \text{ from circuit)} \end{cases}$$

Write:



$$\Rightarrow \left(\frac{V_1}{R_1} (1+k) - I = 0 \quad \text{(KCL)} \right.$$

final words

NODAL ANALYSIS provides a good way to make circuits into equations.

The "extended method" is simple in principle, but usually heavy to solve

Various "tricks" including simplifying branches first (last session), using supernodes, and substituting controlling-variable definitions, can help us write more solution-friendly equations from the beginning.

In many circuits (in our courses) we find that writing "nodal equations" is a useful step if we can't easily see a quick way of simplifying the circuit further.