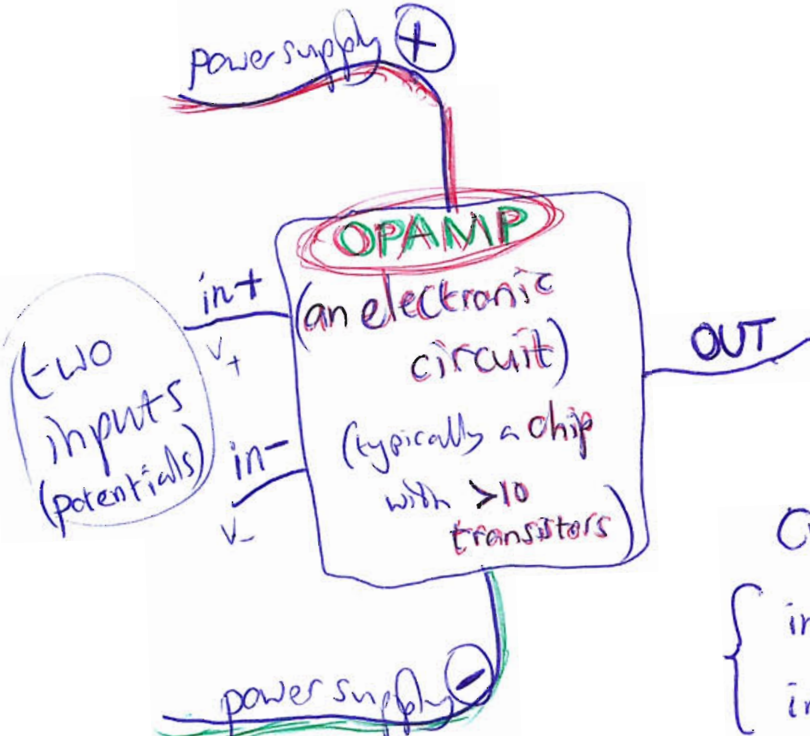
OPAMPs

(operational amplifier
operations förstärkare



<u>Not</u> a "fundamental" component for modelling basic physics like a wire, battery, etc

But very versatile and widely used so we should know about it.

Crude summary of operation:

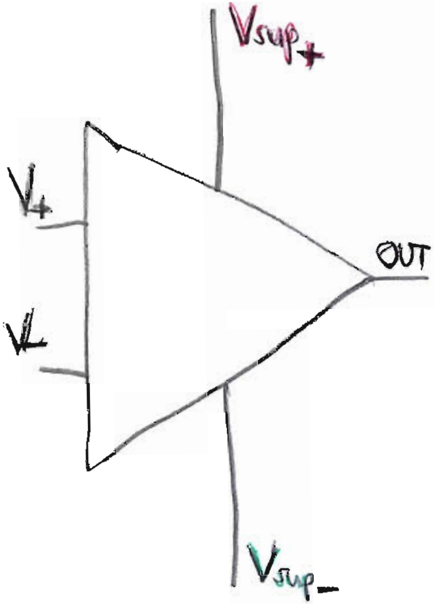
in+ > in− ⇒ connect OUT to power ⊕
in− > in+ ⇒ connect OUT to power ⊖

Alternative (more conventional) summary:
"high gain differential amplifier, with high input resistance"

Different levels of symbol detail

We use this in the course ↓

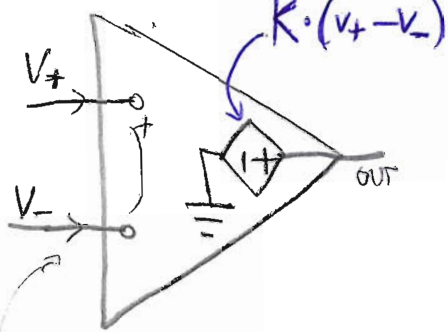
With power supply shown



Here it would make sense to do KCL on the connections ... Output current comes from the supply connections

Hiding the power supply

big number of "gain"



"Ideal" opamp:

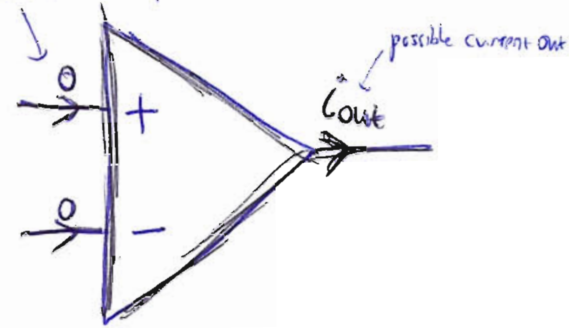
K (open loop gain) $\rightarrow \infty$

no input current (input resistance $\rightarrow \infty$)

Common "simplified" symbol

(Just inputs and output)

(no current in, if ideal)

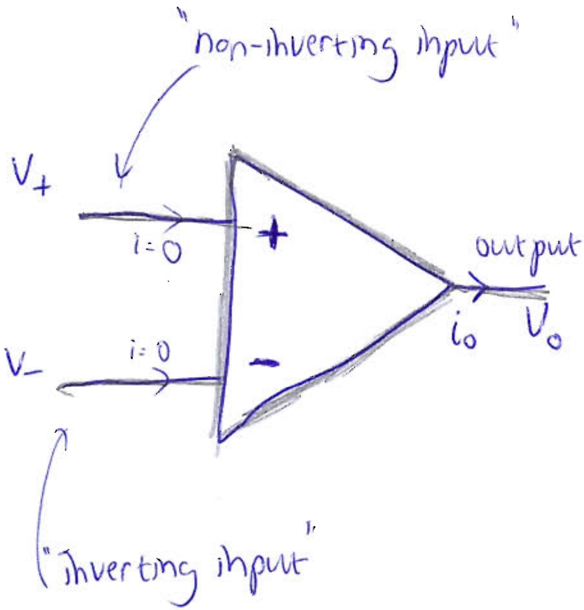


! DON'T FORGET !

i_{out} can be anything ... it's like a "voltage source from earth" but this detail is hidden in the symbol.

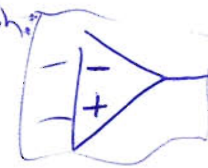
So don't try KCL on the two inputs and output !!

(Get familiar with the common symbol.)



Ideal
- no input current
- infinite gain $\left(\frac{V_0}{V_+ - V_-}\right)$
- any output current → i_0 = ??
  (like voltage source)

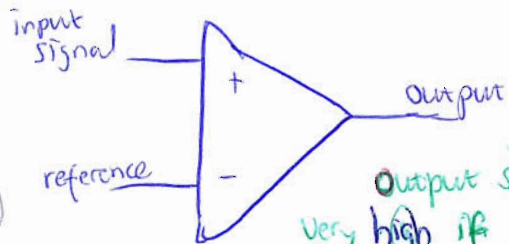
Quite often we draw it the other way up, with:



as that fits more neatly in some circuits.

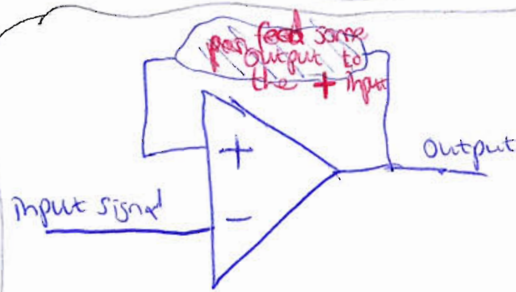
Basic ways of using an opamp

COMPARATOR (no "feedback")



Output shoots up to very high if $V_+ > V_-$ or to very low if $V_- > V_+$

— like a yes/no answer



Like a comparator that doesn't easily change its opinion!

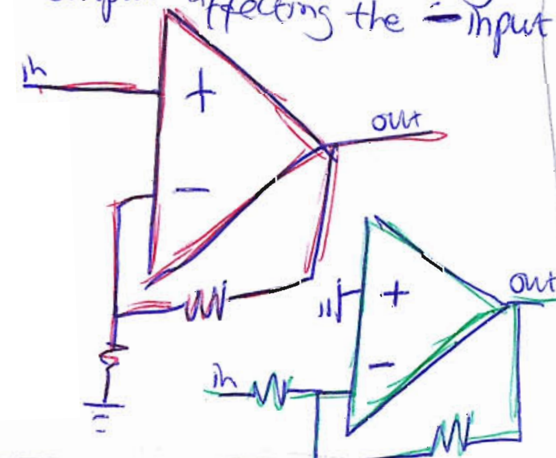
HYSTERESIS (positive feedback)

SEE MORE DETAIL IN "CHAPTER" PDF-FILE (computer written)

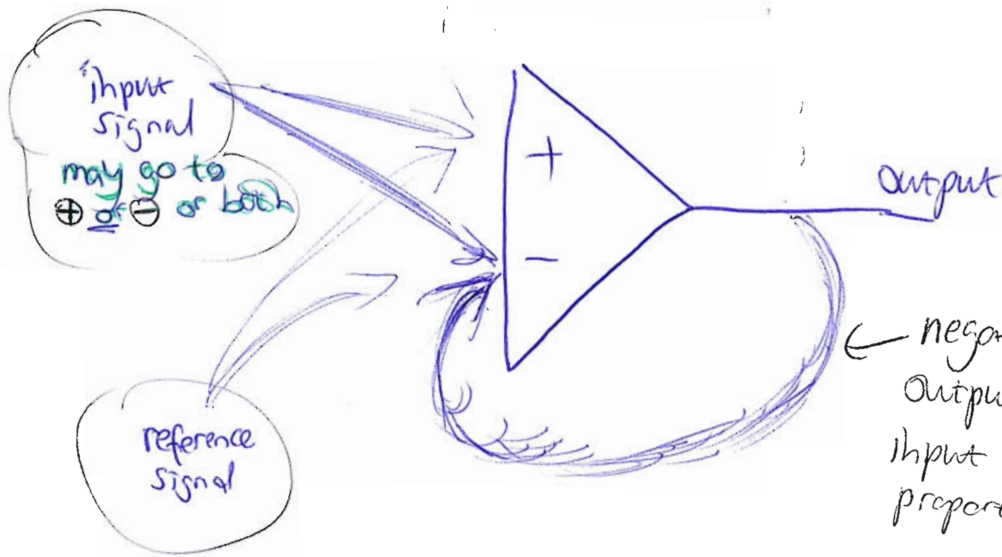
We only really study negative feedback in this course

WELL BEHAVED AMPLIFIER WITH GAIN SET BY FEEDBACK COMPONENTS (negative feedback)

Various ways, all involving output affecting the − input



NEGATIVE FEEDBACK – the way of taming a wild high-gain amplifier into a controlled amplifier of chosen gain.

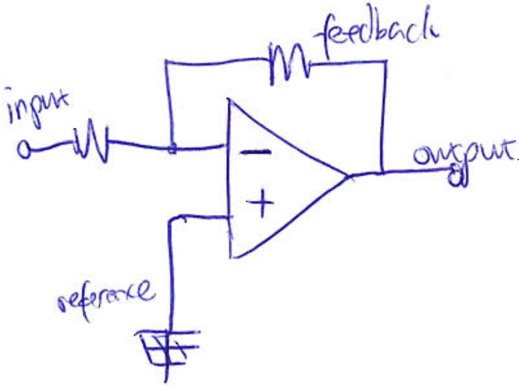


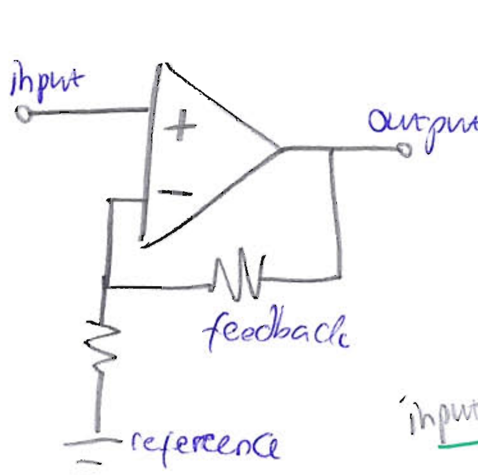
← negative feedback makes the output affect the **inverting** input by some controlled proportion

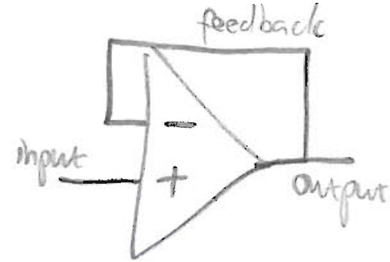
e.g. if $V_+ > V_-$, output goes up so it helps V_- go up, so the difference $V_+ - V_-$ is reduced.

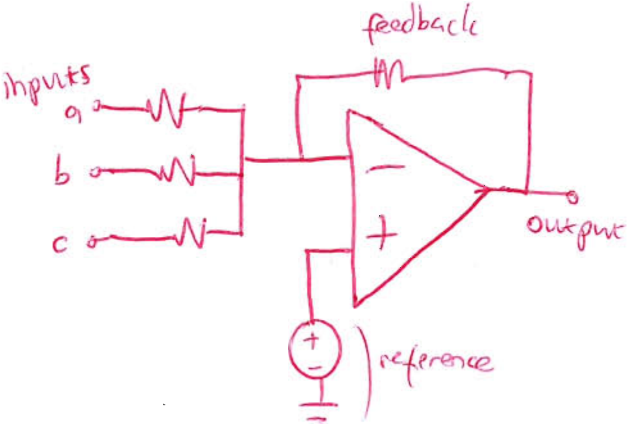
Infinite Gain AND Negative feedback $\Rightarrow$ $V_+ = V_-$

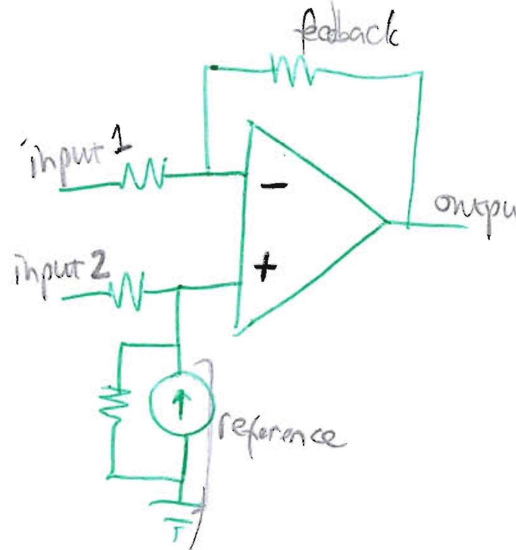
examples of negative feedback











Summary of "rules" for opamp circuit solution:

negative feedback and infinite gain → can assume that $V_+ = V_-$

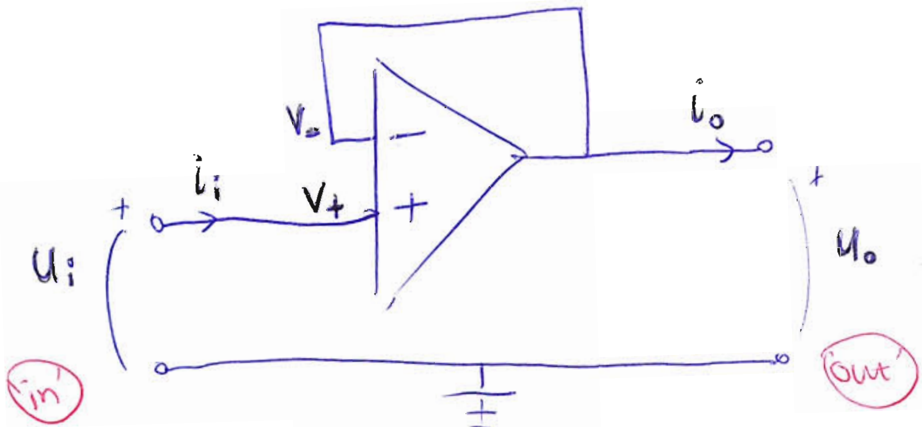
infinite input resistance → no current into inputs

output is like voltage source (getting its current from earth node) → output current is determined by what's connected ... voltage is whatever the feedback needs in order to ensure that $V_- = V_+$

→ THIS, WITH {KCL, KVL, Ohm} is all you need!

opamp circuits can be solved using these rules

Let's use these rules on a simple example
(a "buffer" or "follower" circuit, in fact)



What is the "gain" (förstärkning) of this circuit from input to output?

I.e. what is $\frac{U_o}{U_i}$?

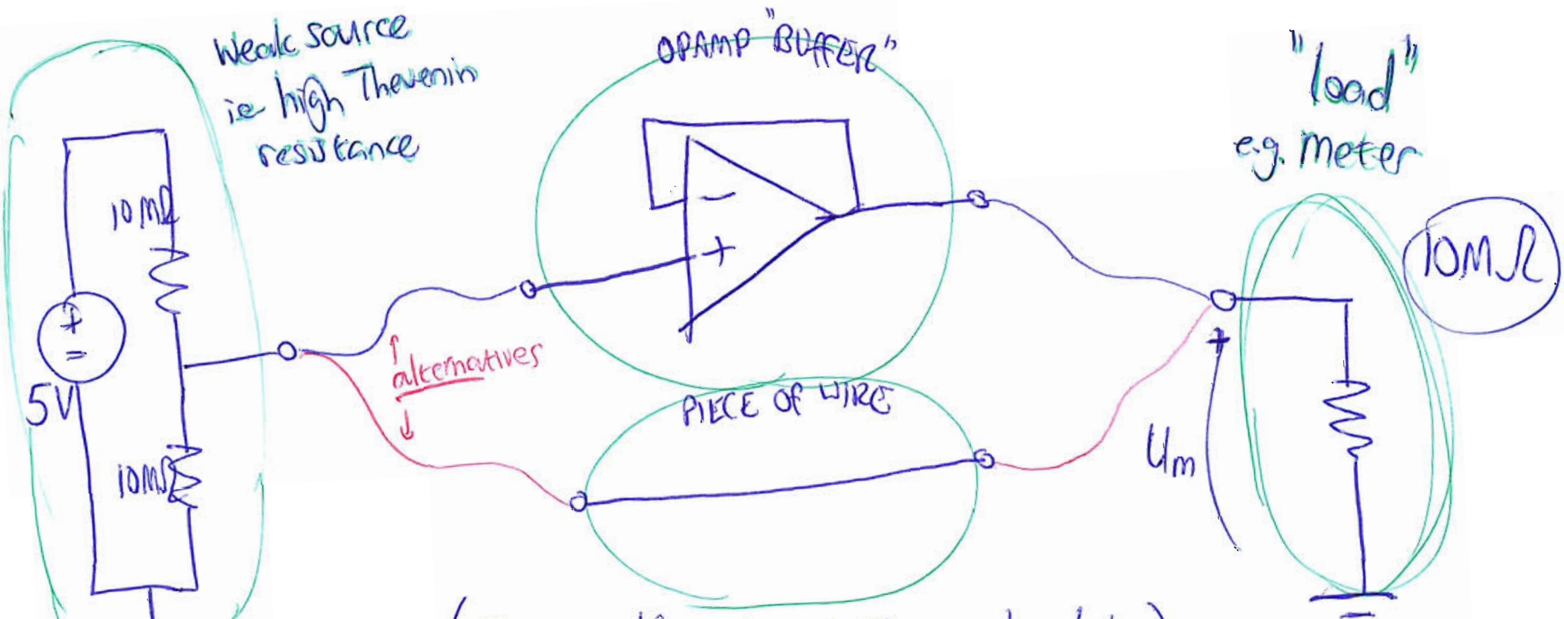
What do we know?

$V_+ = U_i$
$V_- = U_o$
} shown by the connections (a node has one potential!)

$V_+ = V_-$ } ideal opamp with negative feedback

$\Rightarrow$ $U_i = U_o$
i.e. gain $= 1$!

So is there any point to this circuit? Better than a piece of wire?

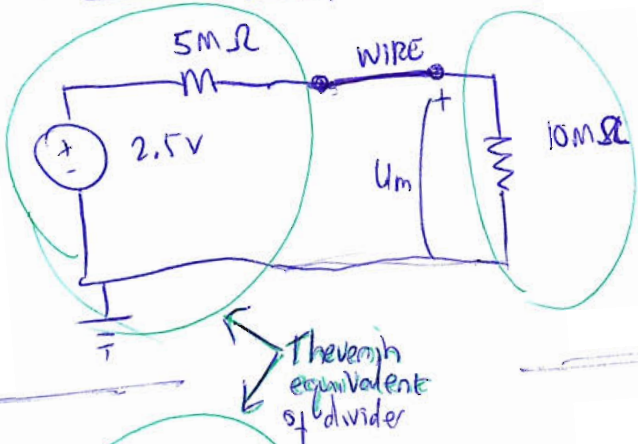


this earth and all others in the circuit are the same node (secret connection to make diagram neater)

(This is like the divider in the lab.)

- What's the difference in measured voltage U_m when using opamp buffer compared to "piece of wire"?
- Note both have "gain of 1" ... can they be different?

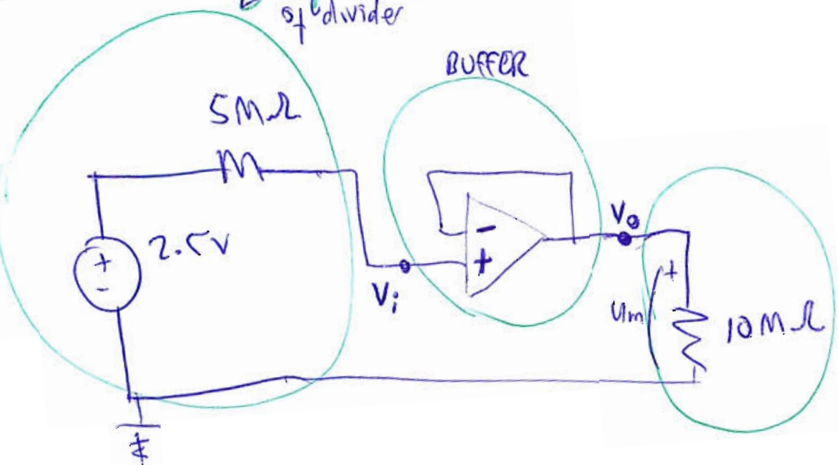
There is a difference!



Voltage division: $2.5V \cdot \frac{10M\Omega}{10M\Omega + 5M\Omega}$

$U_m = 1.67V$

(That's what we saw in the lab)

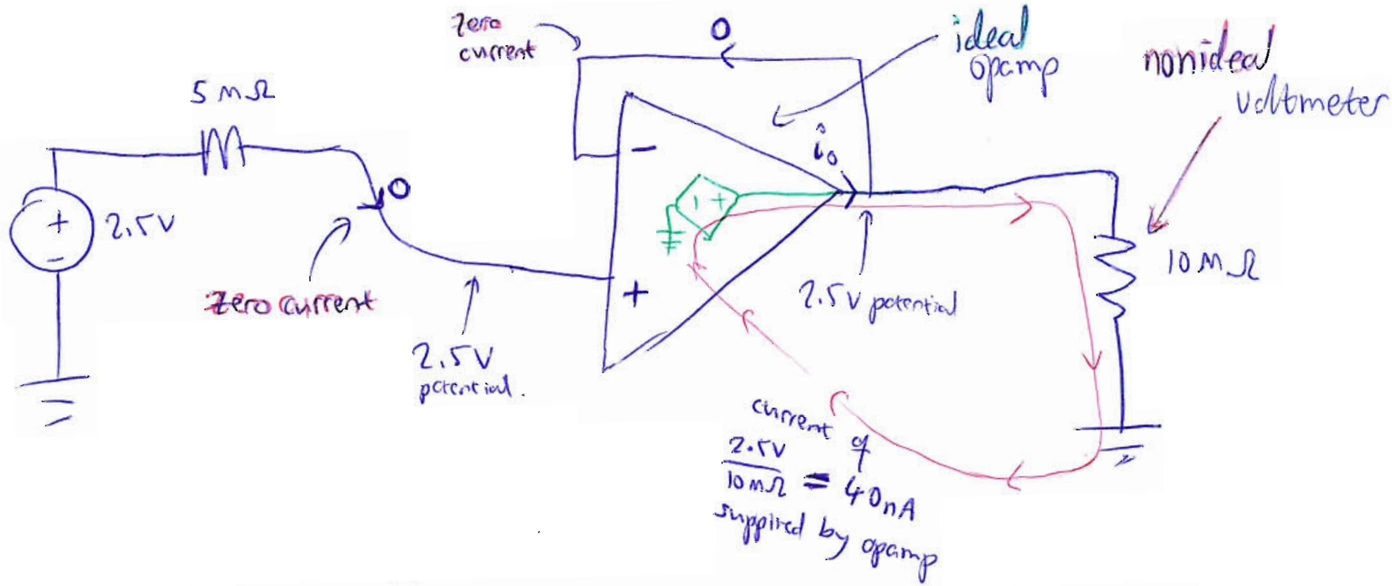


No current into opamp input. Thus, no current taken from the source, so we don't affect the voltage at the source.

Opamp output provides current to ensure that $V_o = V_i$ is seen by the meter.

$V_i = 2.5V, \quad V_o = 2.5V, \Rightarrow \quad U_m = 2.5V$

What about KCL in that example (buffer) ?!



Note the hidden connections between earth symbols, and the hidden earth "in" the opamp.

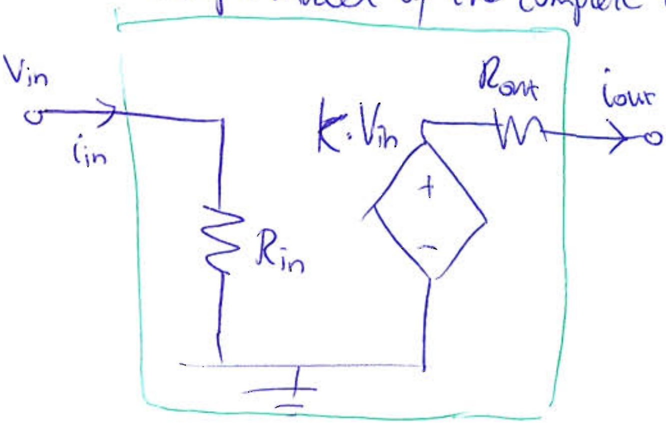
See how the opamp itself feeds the current to the voltmeter open circuit (no output current)

$$K = \frac{V_{out}}{V_{in}} \quad \left(\text{"gain"}\right)$$

$$R_{in} = \frac{V_{in}}{i_{in}} \quad \left(\begin{array}{l}\text{input}\\ \text{resistance}\end{array}\right)$$

$$R_{out} = \frac{\Delta V_{out}}{\Delta i_{out}} \quad \left(\begin{array}{l}\text{output}\\ \text{resistance}\end{array}\right)$$

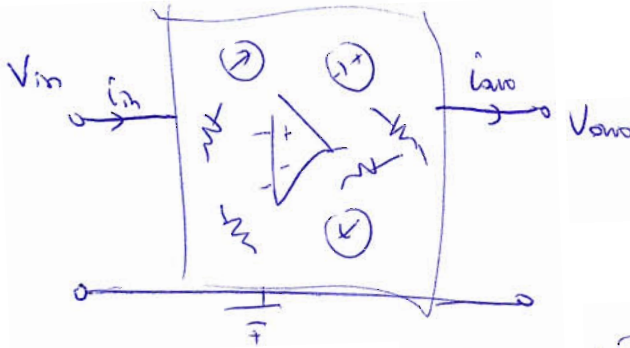
simple model of the complete amplifier.



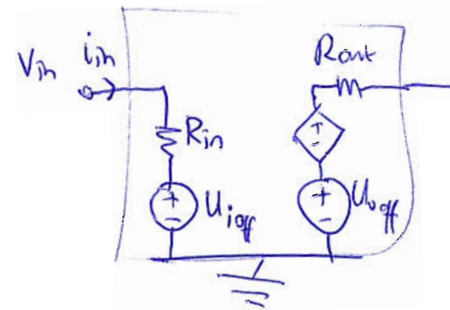
Resistors in the circuit, or nideal opamp can cause V_{out} not to be ideal V-source

$$V_{out} = K \cdot V_{in} - i_{out} R_{out}$$

More generally: amplifiers may contain sources, may have output with zero input, etc.



$\Longrightarrow$



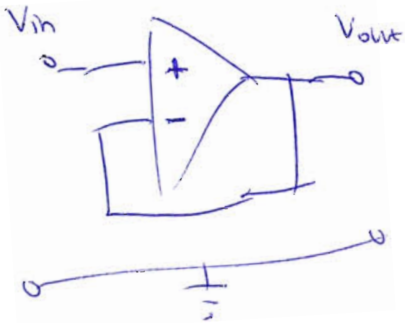
Input and output can both look like Thevenin sources.

Need to define:

$$\begin{cases} K = \dfrac{\Delta V_{out}}{\Delta V_{in}} & \text{open circuit} \\ R_{in} = \dfrac{\Delta V_{in}}{\Delta i_{in}} \\ R_{out} = \dfrac{\Delta V_{out}}{\Delta i_{out}} \end{cases}$$

general concept of gain, input resistance and output resistance.

So our buffer:

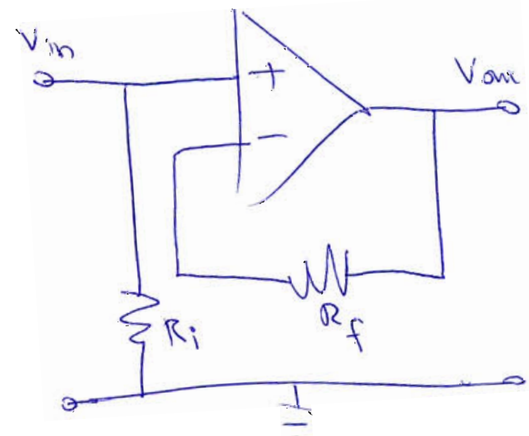


Gain $K = 1$

$R_{in} = \infty$

$R_{out} = 0$

What about this?



$R_{in} = R_i$

$R_{out} = 0$

$K = 1$

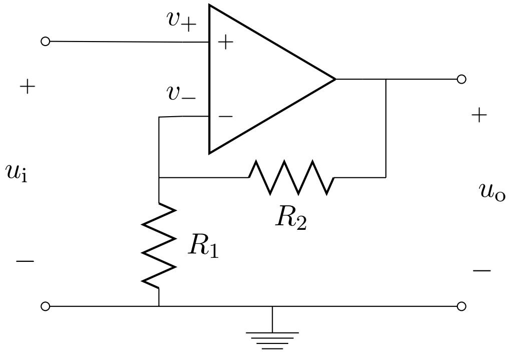
why?

Non-inverting amplifier

The *non-inverting amplifier* puts the amplifier's input into the opamp's non-inverting input v_+.

That's similar to the buffer, which gave 100% feedback so that the output would need to have the same potential as the input.

But here, instead of feeding back 100% of the output to the input, the output is *divided*, by resistors R_1 and R_2.



That means the output has to 'work harder' to force $v_- = v_+$. If we divide by 10, the output must become 10 times as much as the input!

Analysis:

R_1 and R_2 are series-connected (because no current goes into the ideal opamp input that's connected between them).

Voltage division gives

$$v_- = \frac{R_1}{R_1 + R_2} u_o.$$

Given that

$$v_- = v_+ \quad \text{and} \quad v_+ = u_i$$

we derive the gain as

$$K = \frac{u_o}{u_i} = \frac{R_1 + R_2}{R_1} = 1 + \frac{R_2}{R_1}.$$

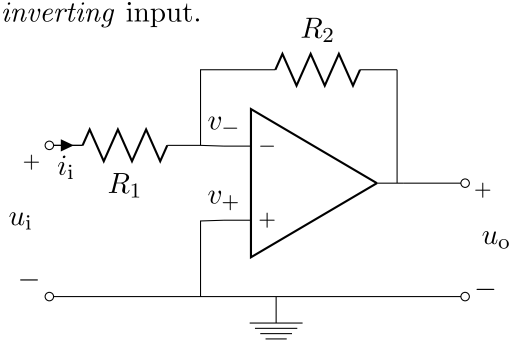
Assuming $\{R_1, R_2\} > 0$, gain must be $K > 1$.

As the amplifier's input connects only to one of the opamp inputs, no current flows into the amplifier,

$$R_{in} = \infty.$$

Inverting amplifier

The *inverting amplifier* in its simplest form is an input resistor that meets a feedback resistor at the *inverting* input.



By negative feedback and ideal opamp behaviour, the opamp's output adjusts to be whatever is needed to obtain $v_- = v_+$.

In this simple example, the non-inverting input is connected to earth so $v_+ = 0$.

The feedback therefore forces $v_- = 0$. We call it a *virtual earth*: it is *not* part of the earth node, although it has the same potential; a *separate* KCL can be written at node v_-.

Analysis:

KCL at the inverting input v_-,

$$\frac{u_i - v_-}{R_1} + \frac{u_o - v_-}{R_2} = 0,$$

but we've already claimed $v_- = v_+ = 0$, so

$$\frac{u_i}{R_1} + \frac{u_o}{R_2} = 0,$$

which gives the gain as

$$K = \frac{u_o}{u_i} = \frac{-R_2}{R_1}.$$

Again assuming $\{R_1, R_2\} > 0$, we see $K < 0$. This amplifier always changes the signal's sign. It can reduce the magnitude ($-1 < K < 0$) or increase it ($K < -1$) or just invert (negate) the input ($K = -1$).

The input resistance is seen from the resistor between the input and virtual earth ($v_- = 0$),

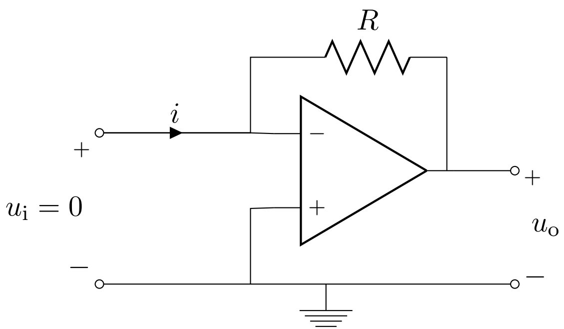
$$R_{in} = \frac{u_i}{i_i} = \frac{u_i}{\frac{u_i - 0}{R_1}} = R_1.$$

Current-input amplifier

Up to now, we've considered amplifiers where the input and output are both *potentials*.

Another choice is an output voltage proportional to an input *current*.

This can be achieved by removing the input resistor on the inverting amplifier, leaving just a feedback resistor R.



Here, the input is held to a constant 0 V (virtual ground) by the feedback.

Analysis:

By KCL and Ohm's law, the gain is

$$\frac{u_\mathrm{o}}{i} = -R.$$

This is a *transimpedance*, meaning a ratio of output voltage to input current (dimension: resistance).

A circuit designed for measuring current should ideally have very low input resistance (an ideal ammeter is like a short-circuit). Then it accepts the measured current without creating a voltage that affects how much current flows.

This amplifier has the desired property: its input voltage is held to zero, so

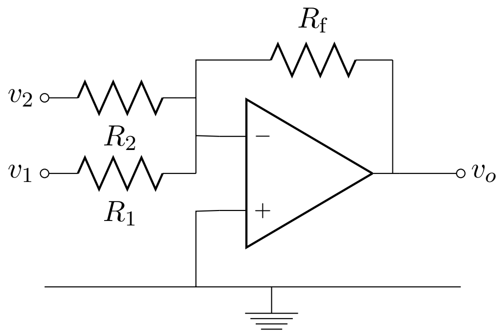
$$R_\mathrm{i} = \Delta u_i / \Delta i = 0.$$

Practical detail:

We use basically this circuit for measuring femtoampere currents in insulation materials in the high-voltage lab.

Adder **(översättning: huggorm)**

The ‘virtual earth’ in an inverting amplifier is convenient! We can have multiple input resistors, each with a current proportional to the potential applied to it: v_1 and v_2 in this circuit.



The output voltage is whatever is needed to allow all this input current to flow through the feedback resistor R_f.

Analysis:

By KCL at the inverting input,

$$v_\mathrm{o} = -R_\mathrm{f}\left(\frac{v_1}{R_1} + \frac{v_2}{R_2}\right).$$

If $R_1 = R_2 = R_\mathrm{f}$, the output potential is the sum of the input potentials. Varied resistances permit ‘weighted’ sums and amplification or attenuation (reduction).

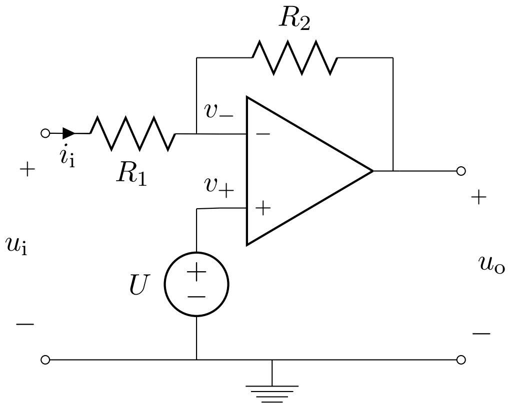
You’d have trouble getting this convenient arithmetic without an opamp!

The input resistances are (as with a one-input inverting amplifier) equal to the resistances on the respective inputs: R_1 or R_2.

Non-zero references / sources

An amplifier circuit (containing an opamp) is not limited to only resistors.

Below is an example where the non-inverting input is held to U instead of 0, in an inverting amplifier. Other sources could also be included in the circuit.



The source introduces constants in the equations for e.g. u_o and i_{in}. Then it **cannot** be assumed that $\frac{\Delta u_o}{\Delta u_i} = \frac{u_o}{u_i}$.

Analysis:

As before, KCL at the inverting input v_- gives

$$\frac{u_i - v_-}{R_1} + \frac{u_o - v_-}{R_2} = 0,$$

but now we have $v_- = v_+ = U$, so

$$\frac{u_o}{R_2} = U\left(\frac{1}{R_1} + \frac{1}{R_2}\right) - \frac{u_i}{R_1},$$

which gives the output voltage as

$$u_o = \frac{U(R_1 + R_2)}{R_1} - \frac{R_2}{R_1} u_i.$$

The gain is how the input affects the output,

$$K = \frac{\Delta u_o}{\Delta u_i} = \frac{-R_2}{R_1},$$

The input current is

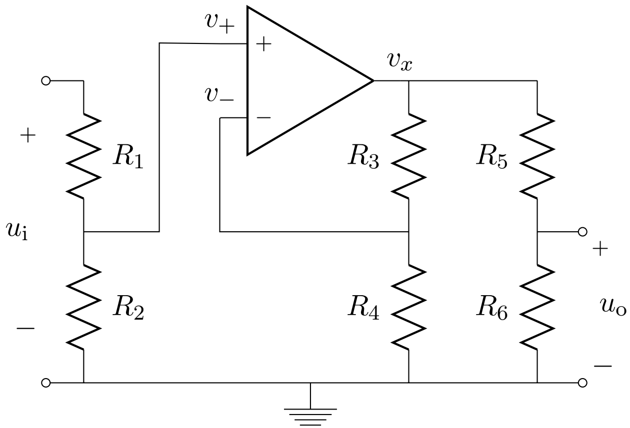
$$i_i = \frac{u_i - U}{R_1} = \frac{u_i}{R_1} - \frac{U}{R_1},$$

from which input resistance is

$$R_{in} = \frac{\Delta u_i}{\Delta i_i} = R_1.$$

More about input and output impedance

Here's another *non-inverting amplifier*, with further dividers on the input and the output. A bit artificial, but it's for pedagogical purposes!



In the earlier circuits we didn't discuss *output resistance*. In fact, those amplifiers' outputs behaved as ideal voltage sources, driven directly by the ideal opamp output: $R_\mathrm{o} = 0$.

Analysis:

Let's now consider all of K, R_in, R_out.

From the previous non-inverting amplifier's analysis, we see that

$$v_x = v_+ \frac{R_3 + R_4}{R_4}.$$

Including the effect of the input divider v_+/u_i and the output divider u_o/v_x,

$$K = \frac{u_\mathrm{o}}{u_\mathrm{i}} = \frac{R_6}{R_5 + R_6} \cdot \frac{R_3 + R_4}{R_4} \cdot \frac{R_2}{R_1 + R_2}.$$

No current goes into the opamp inputs, so the circuit's input appears as two resistors to earth:

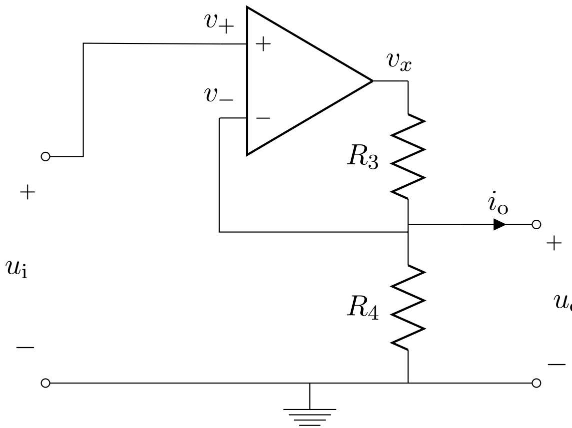
$$R_\mathrm{i} = R_1 + R_2.$$

The *opamp's* output v_x is 'stiff' (ideal voltage source). But the complete amplifier circuit's output is through a divider of Thevenin resistance

$$R_\mathrm{o} = R_5 \parallel R_6 = \frac{R_5 R_6}{R_5 + R_6}.$$

Still more input and output impedance

Here we've 'simplified' by taking the output from the feedback divider, and omitting the input divider. It *looks* close to the original non-inverting amplifier circuit.



How differently does this behave from the previous circuit, in which $R_o \neq 0$?

Analysis:

From the previous non-inverting amplifier's analysis, we see that

$$v_x = v_+ \frac{R_3 + R_4}{R_4}.$$

But we have taken u_o here from the point connected to v_-, so we can expect that $u_o = v_- = v_+ = u_i$ ... it is just a buffer,

$$K = \frac{u_o}{u_i} = 1 = \frac{R_3 + R_4}{R_4} \cdot \frac{R_4}{R_3 + R_4}.$$

No current goes into the opamp inputs, so

$$R_i = \infty.$$

The opamp output v_x has zero output resistance. But we take our output via the divider: shouldn't that mean we have non-zero output resistance? No! The feedback has to keep $v_- = v_+$. So v_x will adjust to force $u_o = u_i$ always,

$$R_o = \frac{\Delta u_o}{\Delta i_o} = 0.$$

Summary

The key assumptions used for analysis of "ideal opamp with negative feedback" are:

- **Input potentials are equal:** $v_- = v_+$**.**
- **No current goes into the opamp inputs.**
- The opamp's output potential will become whatever is needed in order to push v_- to match v_+. Define an unknown potential if the output potential is not already defined.
- The opamp's output can supply any current that's needed to achieve this and to supply any loads connected to the output. Remember that the three-terminal opamp symbol is a trick: its output is "taking current from a hidden earth connection".

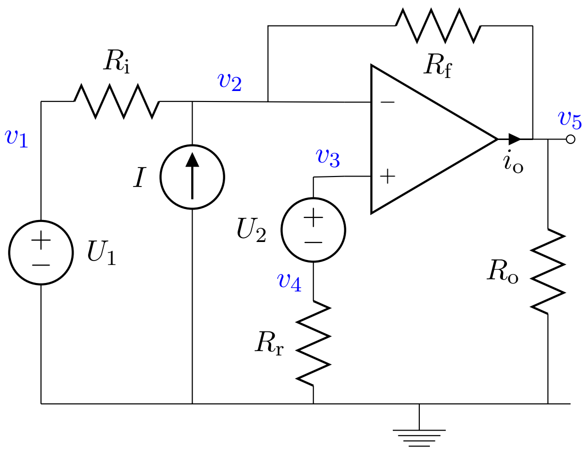
Simplifications-based analysis of opamps

One way to analyse a circuit that contains opamps is a step-by-step method, with perhaps a mixture of simplification, KVL, KCL, source-transformation, superposition, etc.

Compared to a systematic application of nodal analysis, the simplifications-based method might help us more to feel how the circuit behaves, and to avoid getting lots of equations to handle at once.

On the other hand, we might sometimes get stuck by not being able to see what step can be made next. And we'd like a method that scales for computer solution of big circuits. For this we'll look at systematic methods.

Example



Analysis (step by step)

The current in R_r must be zero, as the only route for this current is through an (ideal) opamp's input.

By Ohm's law, there can then be no voltage across this resistor:

$$v_4 = 0, \qquad \text{and so} \qquad v_3 = U_2.$$

By the usual opamp assumption, the negative feedback holds the two inputs to equal potentials:

$$v_2 = v_3 = U_2.$$

By KCL in the node marked v_2,

$$\frac{U_2 - U_1}{R_\mathrm{i}} - I + \frac{U_2 - v_5}{R_\mathrm{f}} = 0,$$

in which the output voltage v_5 (the only unknown) is found as

$$v_5 = R_\mathrm{f} \left(\left(\frac{1}{R_\mathrm{i}} + \frac{1}{R_\mathrm{f}} \right) U_2 - \frac{1}{R_\mathrm{i}} U_1 - I \right).$$

Nodal analysis (by systematic rules)

Only a few rules need to be added to what we already know about systematic nodal analysis, in order to include an opamp.

The opamp model that shows a dependent voltage source driving the output gives a good indication of how to treat the opamp output: a voltage source connected between the earth node and opamp output.

The trouble is that a normal dependent voltage source has some finite value: we can then define its output voltage as e.g. Hi_x. An ideal opamp has infinite gain, and from this we've inferred that the controlling variable $(v_+ - v_-)$ is zero! So we have a source with $0 \cdot \infty$, which is not directly helpful.

The important point is that we can define an unknown potential at the output, and the opamp gives us an *extra* equation of $v_+ = v_-$ which makes the equation system solvable. (The inputs provide two different KCL equations, but only one independent potential.)

So the rules for including an opamp are:

> Define the opamp output's potential, e.g. v_x, if not already defined.
>
> Extended nodal analysis: define output current e.g. i_{x}, with an arrow. (Supernode: don't need the current, as opamp output is part of 'earth supernode', for which KCL is not needed.)
>
> Write the further equation $v_- = v_+$.

Nodal analysis: write the equations

First, KCL at all nodes except the ground node. This is just the usual procedure in extended nodal analysis. We have to define the unknown currents in voltage sources: let's define i_α and i_β into the +-terminals of sources U_1 and U_2 respectively, and i_o out of the opamp as shown in the above diagram.

$$
\begin{aligned}
\text{KCL(1):}\quad 0 &= i_\alpha + \frac{v_1 - v_2}{R_\mathrm{i}} \\
\text{KCL(2):}\quad 0 &= \frac{v_2 - v_1}{R_\mathrm{i}} - I + \frac{v_2 - v_5}{R_\mathrm{f}} \\
\text{KCL(3):}\quad 0 &= i_\beta \\
\text{KCL(4):}\quad 0 &= -i_\beta + \frac{v_4}{R_\mathrm{r}} \\
\text{KCL(5):}\quad 0 &= \frac{v_5}{R_\mathrm{o}} + \frac{v_5 - v_2}{R_\mathrm{f}} - i_\mathrm{o}
\end{aligned}
$$

Then, each voltage source brings its own relation between voltages,

$$
\begin{aligned}
\text{VSRC(1):}\quad v_1 &= U_1 \\
\text{VSRC(2):}\quad v_3 - v_4 &= U_2,
\end{aligned}
$$

and the opamp is slightly different in that it relates the node potentials connected to its inputs,

$$\text{OPAMP:}\quad v_3 = v_2$$

There are no dependent sources with controlling variables that need to be defined, so we're now finished with writing the equations: there are 8 equations, and 8 unknowns (5 node-potentials and 3 currents).

Good luck with solving the above, for v_5!
Not recommended: waste of time!
Seeing the principle was the important thing.