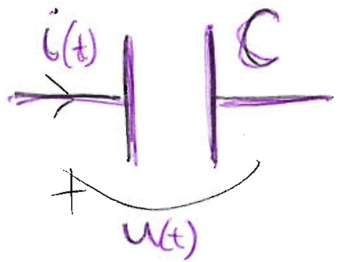
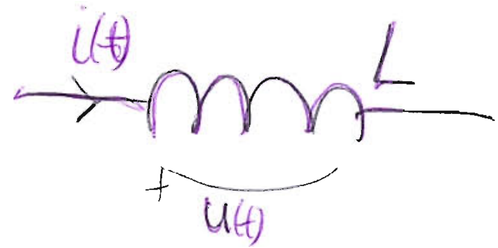
New components: CAPACITOR and INDUCTOR

kondensator — spole (coil)





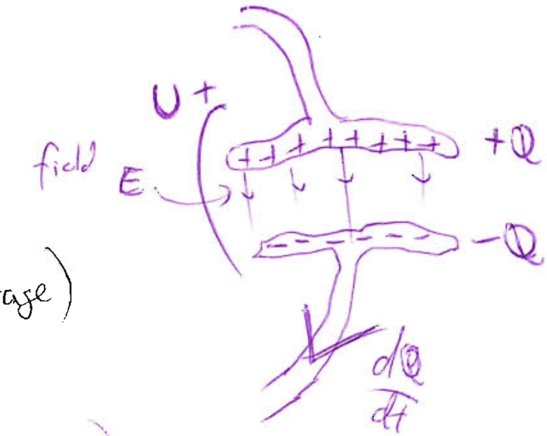
$$i(t) = C \frac{du(t)}{dt} \qquad u(t) = L \frac{di(t)}{dt}$$

These are the fundamental definitions ... a bit like a resistor, but with a derivative.

"Now you know all you need to know" (as long as you can do the maths...)

(again ... but we need to practise about how they behave)

Relation to earlier knowledge ...



Capacitor stores charge.

$$C = \frac{Q}{U}$$ (charge per unit voltage)

Thus $$\frac{d}{dt}(Q) = \frac{d}{dt}(C\,U) \Longrightarrow \underbrace{i}_{\frac{dQ}{dt}} = C\,\frac{du}{dt}$$

(for inductor, there isn't such an easy physical concept as "charge" ... but if you use a voltage × time product you can do the same sorts of calculations.)

Now our equations will be differential equations instead of just algebraic as in dc circuits with U, I, R.

These "reactive" components (C, L) store energy $\begin{cases} \text{electric field (in } C) \\ \text{magnetic field (in } L) \end{cases}$

This stored energy cannot be instantaneously changed.
... it takes time ... we see an inertia (tröghet)

The components (by their energy) have a "memory".

CAPACITOR → electric field energy → Voltage doesn't change easily
spänningströg

INDUCTOR → magnetic field energy → Current doesn't change easily
strömtrög

$i = C\frac{du}{dt}$

$$\int i(t)\,dt = \int C \frac{du(t)}{dt}\,dt \Rightarrow u(t) = \frac{1}{C}\int_{t_0}^{t} i(t)\,d\tau + u(t_0)$$

integrate both sides of the i,u relation

value now

the <u>change</u> between t_0 and now (t)

some <u>known</u> value at time t_0

this shows how the "continuous" (trog) ~~variab~~ quantity has a value depending on all of the past

Similarly for inductor;

$u = L\frac{di}{dt}$

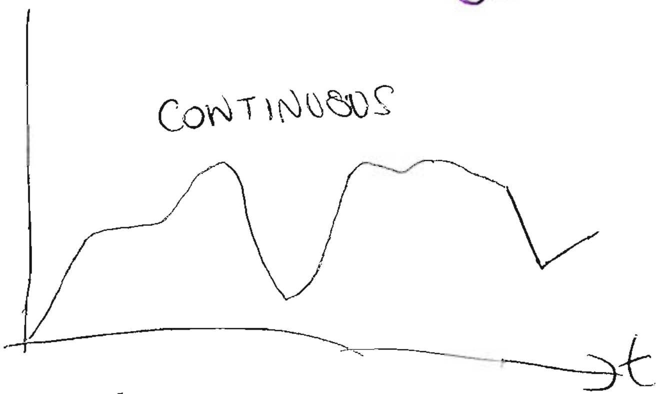
$$\int u(t)\,dt = \int L \frac{di(t)}{dt}\,dt \Rightarrow i(t) = \frac{1}{L}\int_{t_0}^{t} u(t)\,d\tau + i(t_0)$$

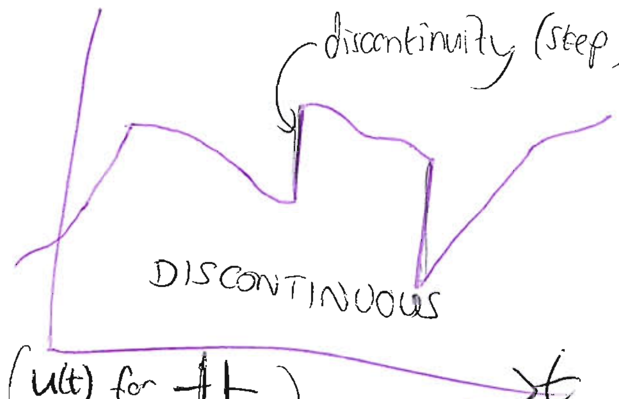
integrate both sides

We used the term "continuous quantity" for the energy-storing variable (the trog one)

kontinuerlig storhet

That means it doesn't 'jump' — it changes gradually, needing time for energy to move in or out.



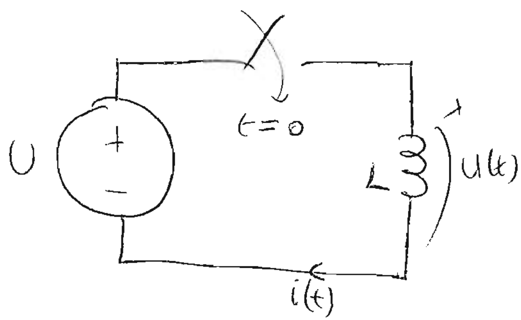


$u(t)$ for

$i(t)$ for

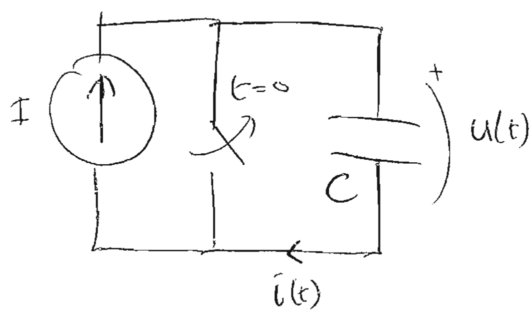
cannot look like this

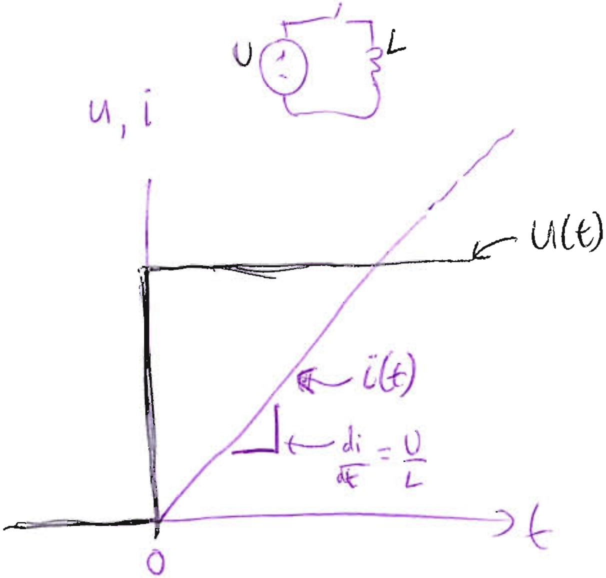
Let's test our confidence in the component definitions and in simple graphical calculus !

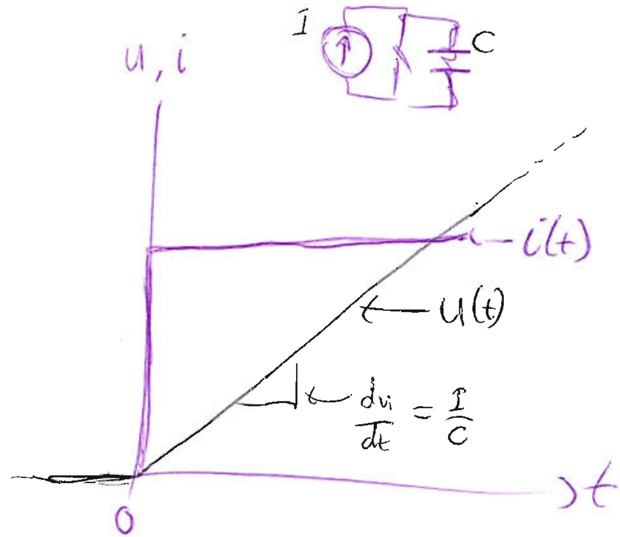


plot $u(t)$ e $i(t)$ for each case.

assume $u(0) = i(0) = 0$





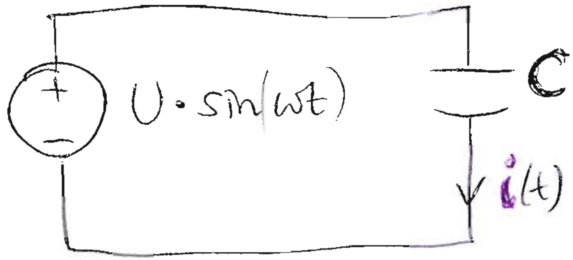


So these cases keep pumping more and more energy in!
(Not of practical interest ... resistance becomes significant!)

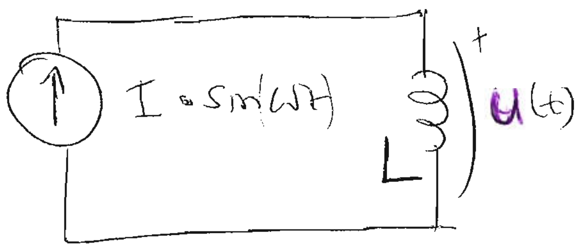
Note the contrast of continuous / noncontinuous quantities.

How about these? <u>time-varying</u> sources.

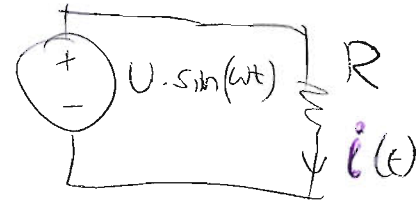
(a)



(b)

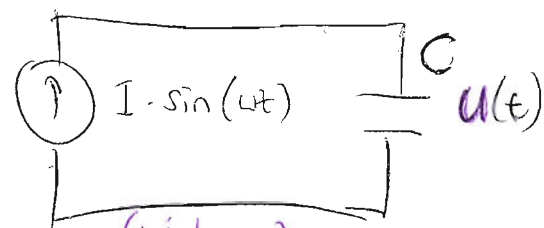


(c)



find $i(t)$ or $u(t)$ in each case!

(d)



(tricky...)

Answers :

(a) $i = C\frac{dv}{dt} = C\frac{d}{dt}\left(V\sin(\omega t)\right) = \omega C V\cos(\omega t)$

(b) "Same, by duality" (swap $\{i, v\}$, $\{L, C\}$)

$v = L\frac{di}{dt} = L\frac{d}{dt}\left(I \cdot \sin(\omega t)\right) = \omega L I\cos(\omega t)$

(c) $i(t) = \frac{v(t)}{R} = \frac{V \cdot \sin\omega t}{R} = \frac{V}{R}\sin(\omega t)$ ← easy — no "dynamics" ie. no memory.

(d) Trickier! $i(t) = I\sin(\omega t) = C\frac{dv(t)}{dt}$ integrate both sides

$v(t) = \frac{I}{C}\int \sin(\omega t)\,dt + \underbrace{v_k}_{\text{integration constant}} = -\frac{I}{\omega C}\cos(\omega t) + v_k$

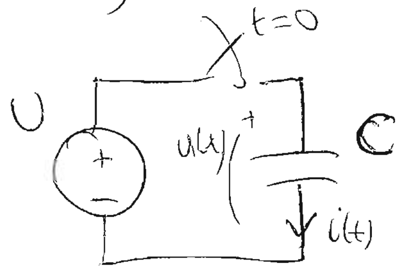
We weren't given enough information to find this!

Here we'd need more information to solve completely.

Eg. "assume $v(0) = 0$" (at $t=0$) which tells us that $v_k = \frac{I}{\omega C}$

$v(0) = 0 = -\frac{I}{\omega C}\cos 0 + v_k$

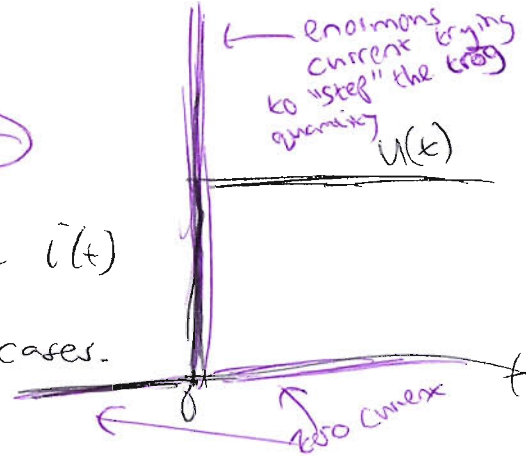
finally ...

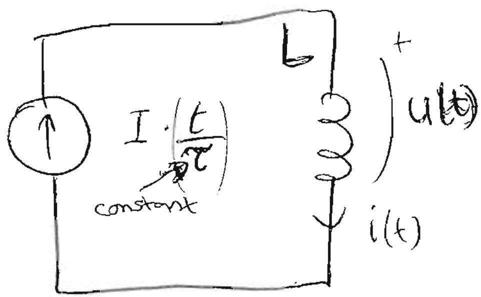


let $u(0) = 0$

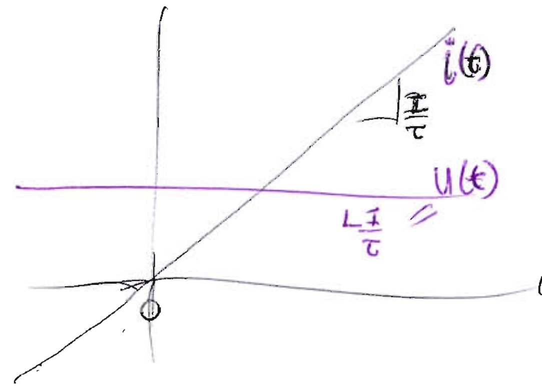
plot $u(t)$ e $i(t)$

in both cases.

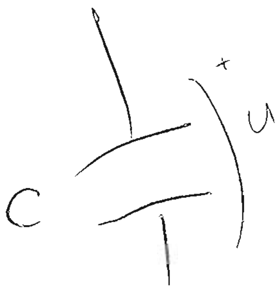


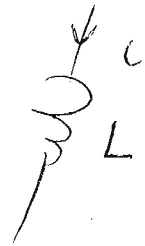


let $i(0) = 0$



Energy in capacitor and inductor ... depends on the continuous quantity (not a mere coincidence!)





$$W = \frac{1}{2}Li^2$$

$$W = \frac{1}{2}Cu^2$$

"work" or energy
(we often use this letter, as E is used for electric field)

Reminded of anything from mechanics?

how to derive the energy ...

take an uncharged capacitor ... $q = 0$, so $U = 0$, no energy.

keep pushing charges dq into it !

the work needed to push each one on is $\underbrace{dW}_{\text{energy}} = \underbrace{U}_{\text{voltage}} \underbrace{dq}_{\text{charge}}$

do this until we reach charge Q

total energy to do this is $\int 1\, dW = \int U \, dq = \int \frac{q}{C} \, dq$

$$\Downarrow$$

$$W = \left(\frac{1}{C} \frac{1}{2} q^2 \right|_0^Q = \frac{1}{2} \frac{1}{C} Q^2 + \underbrace{\text{const}}_{\text{zero as we started at zero energy}}$$

sloppy but shows the priciple !

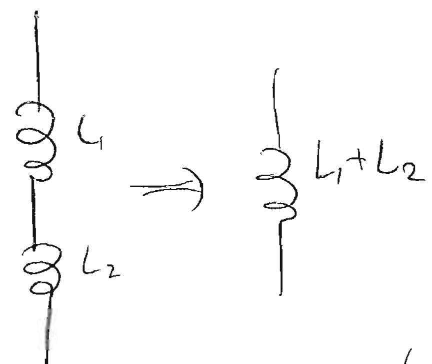
Now substitute the final voltage : $U = \frac{Q}{C}$ i.e. $Q = UC$

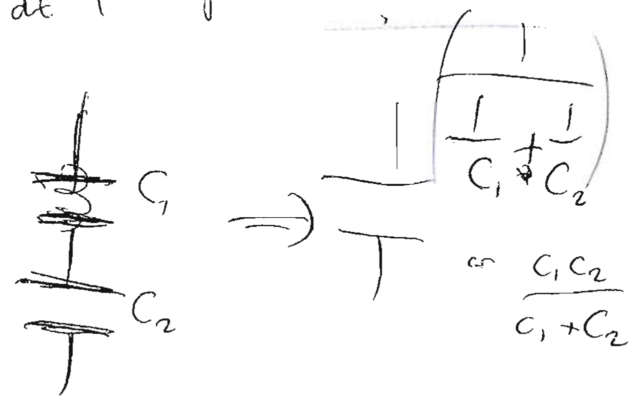
$$W = \frac{1}{2} \frac{1}{C} (Q)^2 = \frac{1}{2} \frac{1}{C} U^2 C^2 = \frac{1}{2} C U^2$$

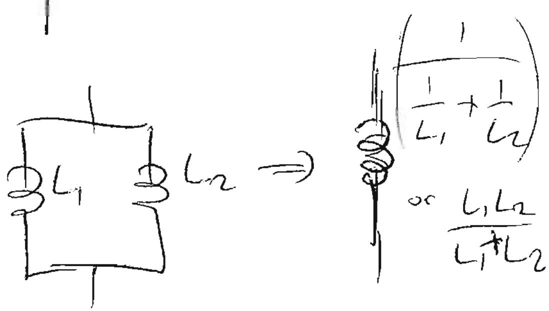
Similarly for inductor energy (but again we don't have a nice word/concept like "charge" for the $\int \dot{u}\,dt$ product)

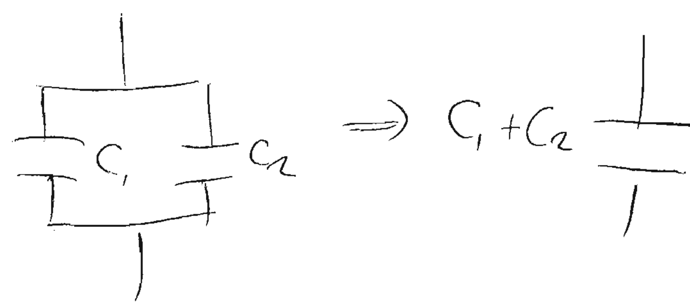
Parallel and series.

The same derivations can be used as for resistors, but with e.g. $\frac{di}{dt}$ for inductor (instead of i) or $\frac{dv}{dt}$ for capacitor









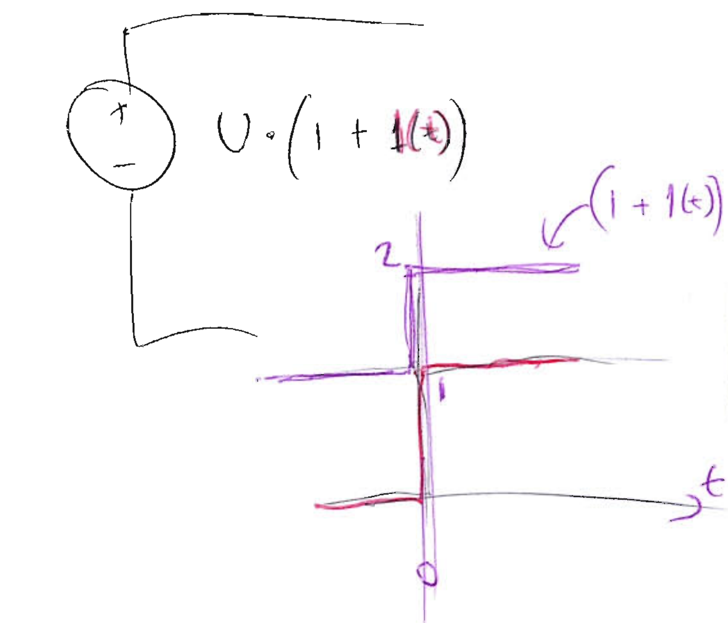
Unit step and switch

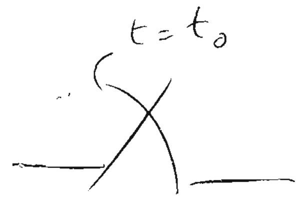
one

function $\mathbb{1}(t)$: 1 if $t \geq 0$
0 if $t < 0$

close at time 0

e.g.





open at time t_0