TIME FUNCTIONS

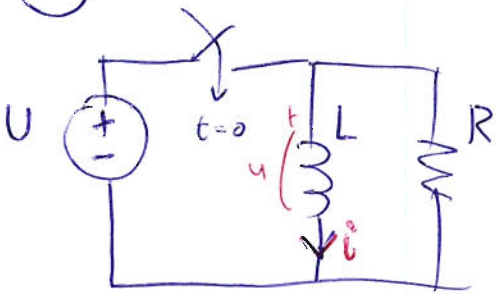
We've seen the basics of C and L components,
and **equilibrium** and **continuity** in larger circuits (many C and L)

Now, we look at complete time functions of u or i for any circuit with **one C or L component** and **constant-valued sources** (after a step/switch).

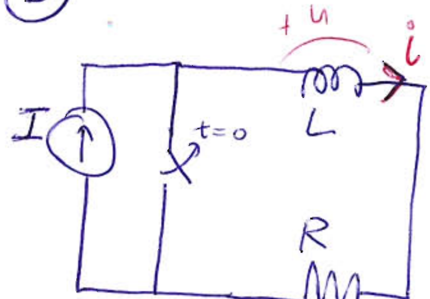
(Similar principles could be used for non-constant sources, or more C&L components, but the ODEs would become harder to solve.)

COMPARISON

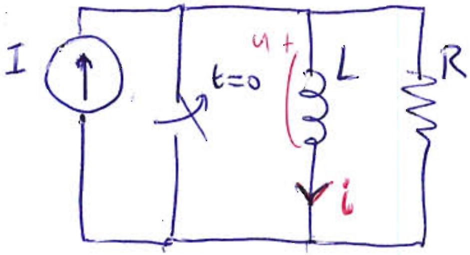
(a)



(b)



(c)



After $t=0$:

Three very different situations!

(a) u constant, i rises (forever)

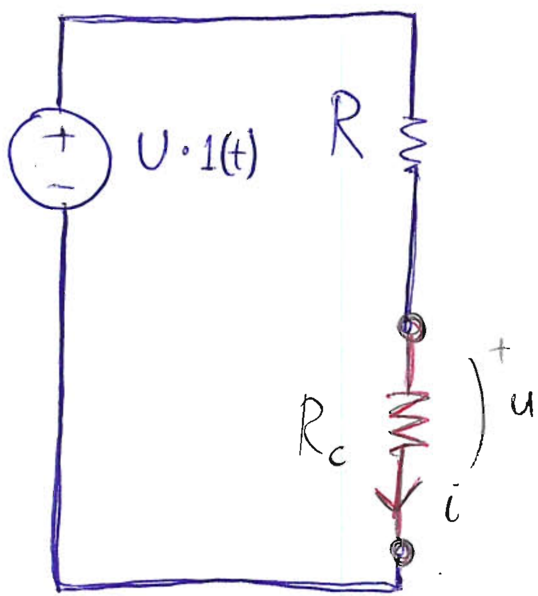
(b) u has a big spike, then zero; i quickly becomes constant

(c) u gradually falls towards zero, slower and slower
i gradually rises towards I, slower and slower

It is cases like (c) that we're considering.
(we already know how to handle (a) and (b) from earlier)

An introduction to including C or L in a circuit

What is the function $u(t)$?



The circuit is static (no stored energy)

Before the step, everything is zero (no independent source is active).

After the step the blue part of the circuit tells us:

$$U - u(t) = i(t) R$$

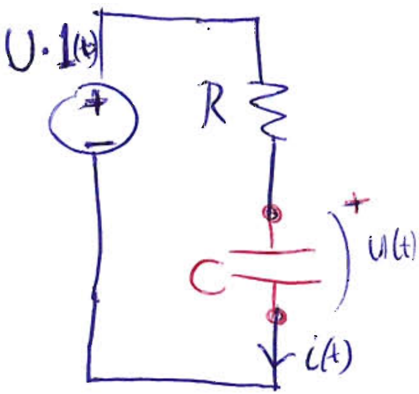
and the red part tells us:

$$i(t) = \frac{u(t)}{R_c}$$

Thus $U - u(t) = u(t)\frac{R}{R_c}$

$$\Rightarrow \boxed{u(t) = \frac{R_c}{R+R_c} U}$$ (as expected from voltage division)

That seemed silly. Let's see the point... change R_c to C!



<u>at $t \geq 0$</u>

$$U - u(t) = R\,i(t) \quad \text{(rest of circuit)}$$

$$i(t) = C\frac{du(t)}{dt} \quad \text{(capacitor)}$$

$$U - u(t) = CR\frac{du(t)}{dt}$$

This can be arranged to a standard-looking non-homogeneous ODE,

$$\frac{du(t)}{dt} + \frac{1}{CR}u(t) = \frac{U}{CR}$$

$$(\; y' + ay = b \;)$$

which has solution:

$$u(t) = \frac{b}{a} + ke^{-at} = \frac{\frac{U}{\cancel{CR}}}{\frac{1}{\cancel{CR}}} + ke^{-t/CR}$$

What is k? Think of the initial condition: $u(0^+) = u(0^-) = 0$

(equilibrium, continuity)

$$u(0) = 0 = U + ke^0 \Rightarrow k = -U$$

So $u(t) = U - U e^{-t/CR} = U\left(1 - e^{-t/CR}\right)$

(valid for $t \geq 0$)

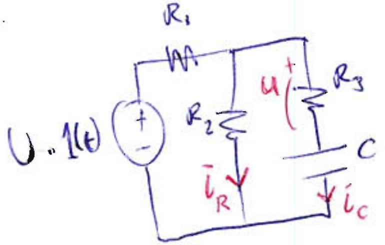
We often write "CR" as the "time constant": $\tau = CR$. (more later about that)

In this example we were finding the continuous quantity (of the only energy storing component in the circuit)

This meant that the initial condition was easily found by $u(0^+) = u(0^-)$.

More generally we might want to find other quantities

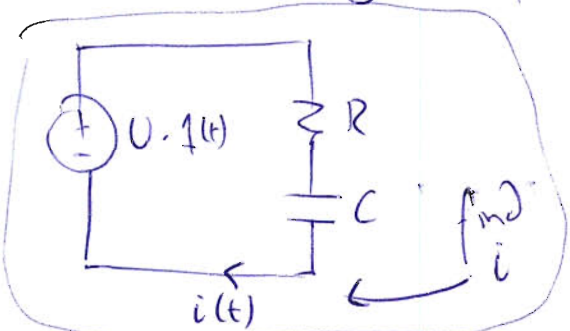
E.g.



$$\left\{ \begin{array}{l} u(t) \\ i_R(t) \\ i_c(t) \\ \text{or power in } R_1 : P(t) \\ \vdots \end{array} \right.$$

Probably the most convenient (simple to follow) way to handle the general case is to find the continuous quantity then find other quantities based on this

Let's try finding a different quantity.



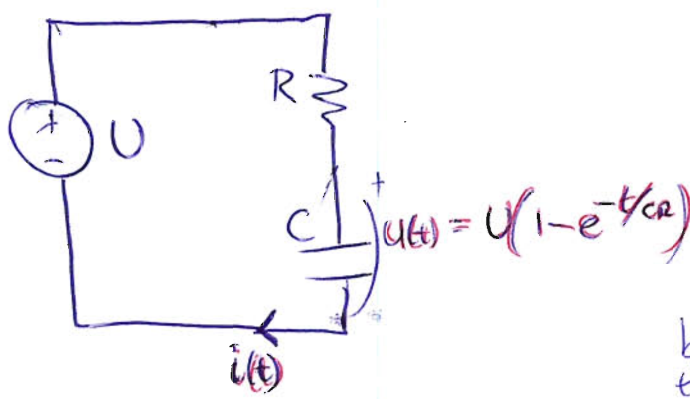
We can do:

based on the source and resistor

$$i(t) = \frac{U - u(t)}{R} = \frac{U - U(1-e^{-t/CR})}{R}$$

$$\boxed{= \frac{U}{R} e^{-t/CR}}$$

at $t \geq 0$, this is:



or

based on the capacitor

$$i(t) = C\frac{du(t)}{dt} = UC\frac{d}{dt}\left(1-e^{-t/CR}\right)$$

$$= UC\left(\frac{-1}{CR}\right)\left(-e^{-t/CR}\right)$$

$$\boxed{= \frac{U}{R} e^{-t/CR}}$$

Time constants

$$e^{-t/CR} \quad \text{or} \quad e^{-t/\tau}$$

neater way of expressing

$$e^{-tR/L} \quad \text{or} \quad e^{-t/\tau}$$

$$\tau = CR = \underbrace{[A][s][V^{-1}]}_{Q/U} \cdot [V][A^{-1}] = [s]$$

$$\tau = L/R = \frac{\overbrace{[V][s][A^{-1}]}^{V/\frac{di}{dt}}}{[V][A^{-1}]} = [s]$$

When $t = \tau$, we have $e^{-t/\tau} = e^{-1} \simeq 0,37$ (37%).

("after one time constant")

i.e. 63% of the total change has already happened

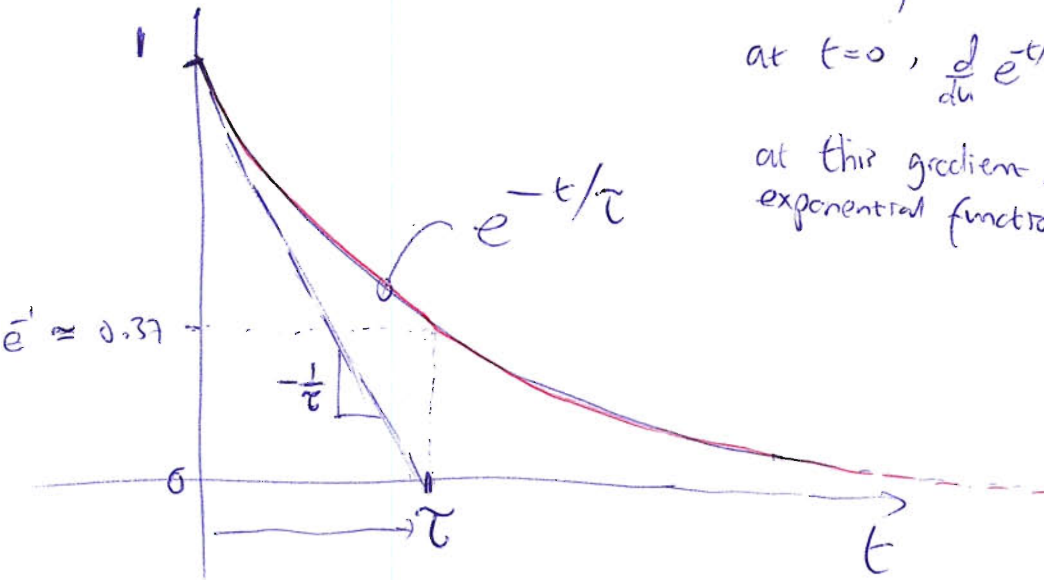
Another way of "describing" a time constant:

"the time to reach the final value, if the initial <u>rate of change</u> were to continue".

gradient of $e^{-t/\tau}$ is $\frac{d}{dt}e^{-t/\tau} = \frac{-1}{\tau}e^{-t/\tau}$

at $t=0$, $\frac{d}{dt}e^{-t/\tau} = \frac{-1}{\tau}e^{0} = \frac{-1}{\tau}$

at this gradient, we reach the exponential function's asymptote at $t=\tau$.



(Remember this when estimating time constants on the oscilloscope.)