AC power

(intended as unusually "low text-content" notes
--- see the chapter if you want more!)

DC circuit: u, i constant (& real)

Power into resistor is

$$\boxed{P = ui}$$

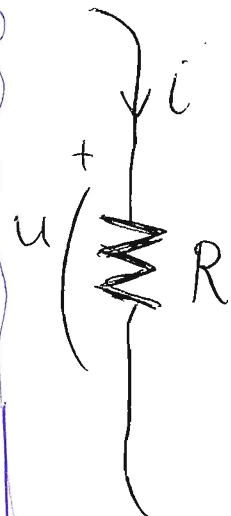
or

$$\boxed{P = \frac{u^2}{R}}$$

Easy!

or

$$\boxed{P = i^2 R}$$



AC circuit

Eg.

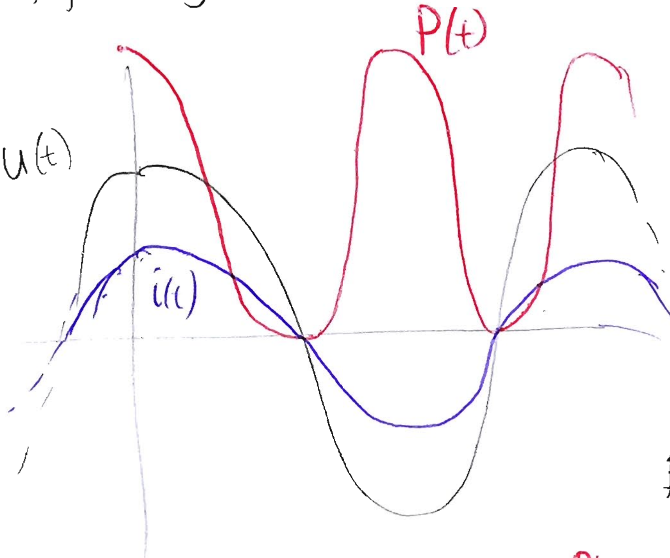
$$\boxed{\begin{aligned} u(t) &= \hat{U} \cos(\omega t) \\ i(t) &= \frac{\hat{U}}{R} \cos(\omega t) \\ &= \hat{I} \cos(\omega t) \end{aligned}}$$

$$P(t) = u(t) \cdot i(t)$$

$$= \hat{U}\hat{I} \cos^2(\omega t)$$

$$= \frac{\hat{U}^2}{R} \cos^2(\omega t)$$

So, plotting:



← Observation: P oscillates at 2ω between a peak and zero (always non-negative --- a resistor doesn't make power)

take product $U(t)\, i(t)$ at each time to get $P(t)$

By symbols we see this as:

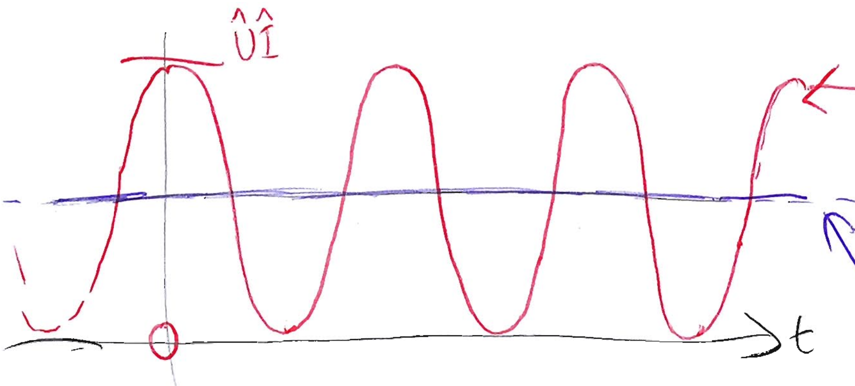
$$P(t) = \hat{U}\hat{I}\cos^2(\omega t) = \frac{\hat{U}\hat{I}}{2}\Big(\underbrace{\cos 2\omega t}_{\text{sinusoid}} + \underbrace{\cancel{\cos 0}^{\,1}}_{\text{constant}}\Big)$$

this is from the CONVENIENT RELATION:

$$\cos\alpha \cdot \cos\beta = \frac{1}{2}\left[\cos(\alpha+\beta) + \cos(\alpha-\beta)\right]$$

Often it is the MEAN VALUE of this oscillating power $p(t)$ that we care most about.

[Eg. a heater or light --- a variation at 2ω (eg. 100 Hz for a 50 Hz voltage) is not even noticed --- we see just the average.]



$$p(t) = \frac{\hat{U}\hat{I}}{2}\left(\underbrace{\cos 2\omega t}_{\text{averages (integrated) to zero over each period}} + \underbrace{1}_{\text{constant}}\right)$$

$$P_{mean} = \frac{\hat{U}\hat{I}}{2}$$

So if we have a resistor and know the **peak** voltage or current in an ac circuit:

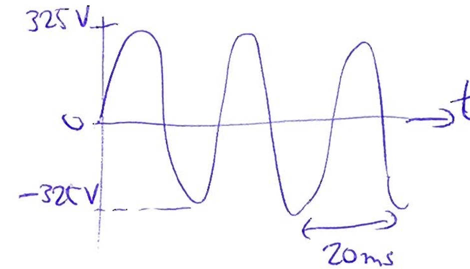
$$\boxed{P_{mean} = \frac{1}{2}\frac{\hat{U}^2}{R} = \frac{1}{2}\hat{I}^2 R = \frac{\hat{U}\hat{I}}{2}}$$

"Effektivvärde" or RMS (root-mean-square)

What if we choose to describe all our voltages and currents by a value that is $\frac{1}{\sqrt{2}}$ of the peak value?

what you get from a normal socket 

e.g. $\boxed{u(t) = (325\,V) \cdot \sin(2\pi \cdot 50\,Hz \cdot t)}$



would be called a "230 V" source

or 230 V (rms)

... usually in ac power contexts it is assumed that everything is in rms., i.e. peak$/\sqrt{2}$

Then if we take the rms values we can calculate power without extra factors of 2:

$$P_{(mean)} = \frac{U_{RMS}^2}{R} = \left(\frac{\hat{U}}{\sqrt{2}}\right)^2 / R = \frac{\hat{U}^2}{2R}$$

← as found previously

And

$$P_{mean} = I_{RMS}^2 R$$

$$P_{mean} = U_{RMS} I_{RMS}$$

By describing AC voltages and currents as RMS values, we get convenient power calculation to resistors, and a resistor that should give eg. 1 kW of heat on 250 V dc will also give that heat on 250 V ac (rms).

$\Longrightarrow$ normal practice in power subjects is that phasors have magnitude equal not to the sinusoid's amplitude ("peak") but to $\frac{1}{\sqrt{2}}$ of this.

E.g. $u(t) = \hat{U} \sin(\omega t) \xleftrightarrow[\text{rms value}]{\text{sine reference}} U(\omega) = \frac{\hat{U}}{\sqrt{2}} \angle 0°$

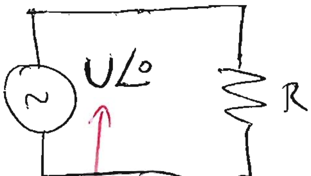
time

frequency

or $U_{rms} \angle 0°$

gives it a convenient name

So if we wanted to find power into a resistor in an AC circuit when working with phasor calculations, it could be done like this:



rms value

$$P = \frac{U^2}{R}$$

mean energy transferred per unit time

But be careful! We can't just take the phasors:

$$\frac{(U\angle\alpha)^2}{R} = \frac{U^2}{R}\angle 2\alpha \neq P$$

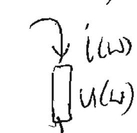
unless $\alpha = 0$

power transferred should not depend on our choice of reference angle

don't expect "energy/second" to be complex!

$\Longrightarrow$ need to use just magnitudes, not angles

Later we see an alternative:



$$P = \mathrm{Re}\left(u(\omega)\, i^*(\omega)\right)$$

real

In case interested in further details:

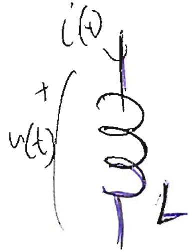
RMS is more general a concept than "$\frac{1}{\sqrt{2}}$" $\times$peak
— that value is just for sinusoid's RMS value.

"effective value" could be more general than RMS ...
RMS is the case that's relevant for a linear resistor

See "Chapter" for more on this.

Not essential for course examination.

What about L & C ?



$$i(t) = \hat{I}\sin(\omega t)$$

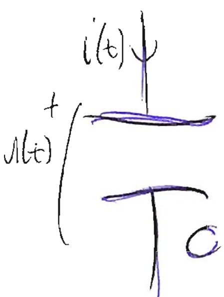
$$\Rightarrow u(t) = \omega L\hat{I}\cos(\omega t)$$

or call this $\hat{U}$

$$p(t) = \omega L\hat{I}^2\left(\cos(\omega t)\cdot\underbrace{\cos(\omega t - \pi/2)}_{\sin(\omega t)}\right)$$

$$= \frac{\omega L\hat{I}^2}{2}\left(\underbrace{\cos(2\omega t - \pi/2)}_{\text{zero mean}} + \underbrace{\cos(-\pi/2)}_{0}\right)$$

<u>SUMMARY</u>: <u>no</u> mean power during each cycle.
power goes in and out ... temporary storage.!



$$u(t) = \hat{U}\sin(\omega t)$$

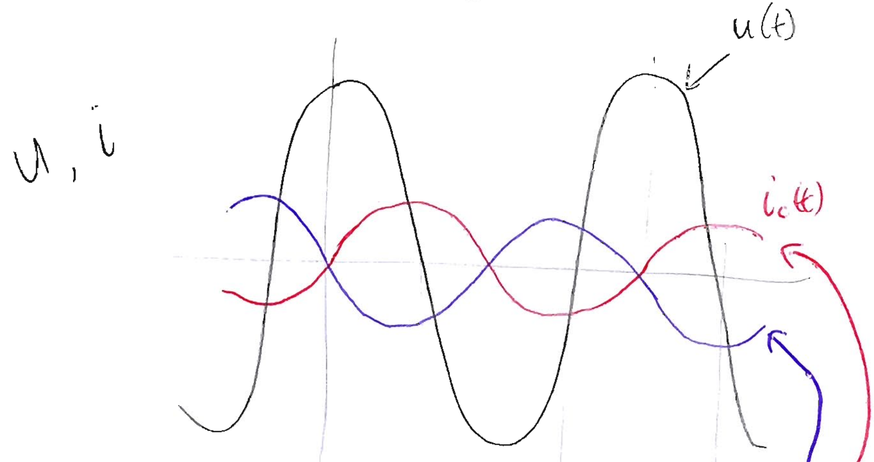
$$\Rightarrow i(t) = \omega C\hat{U}\cos(\omega t)$$

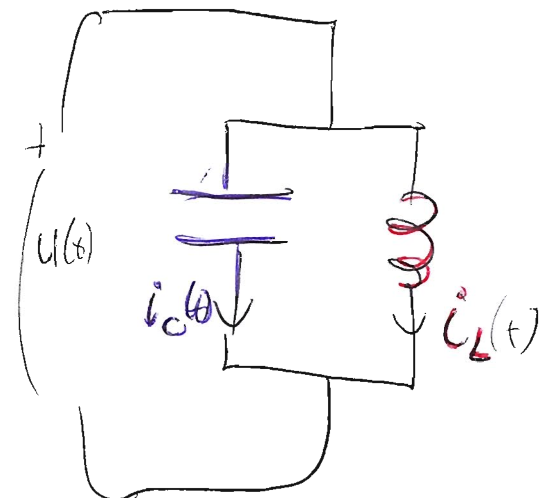
or call this $\hat{I}$

$$p(t) = \omega C\hat{U}^2\left(\cos(\omega t)\cdot\cos(\omega t - \pi/2)\right)$$

$$= \frac{\omega C\hat{U}^2}{2}\left(\underbrace{\cos\left(2\omega t - \frac{\pi}{2}\right)}_{\text{zero mean}} + \underbrace{\cos(-\pi/2)}_{0}\right)$$

Graphically for L & C





inductor : current "lags" voltage 90°
capacitor: current "leads" voltage 90°

frequency ω

⇓

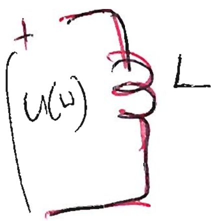
frequency 2ω

When power goes into one, it's going out of the other

$$\frac{1}{\omega C} = \omega L \Rightarrow \text{resonance}$$

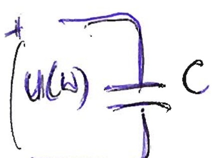
When dealing with capacitors or inductors we describe the voltage × current product as **REACTIVE POWER** . . .

there is current and voltage and (instantaneously) power flow, but the mean power transferred to the L or C component per cycle (period) is zero.



if u is an rms-magnitude phasor,

← there is a reactive power @ $\boxed{\dfrac{|u(\omega)|^2}{\omega L} = |u(\omega)| \cdot \underbrace{\dfrac{|u(\omega)|}{\omega L}}_{|i|}}$ "flowing" here

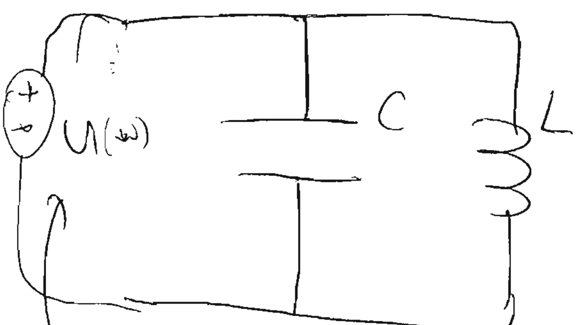


← and @ $\boxed{\underbrace{|u(\omega)|^2 \omega C}_{|u(\omega)| \cdot \underbrace{|u(\omega)| \omega C}_{|i|}}}$ here

(Compare to $\dfrac{|u(\omega)|^2}{R}$ for the ACTIVE power to a resistor.)

Because the "reactive power" of a capacitor and inductor <u>cancel</u> each other (have + and − peaks together) it is convenient to give them a <u>sign</u>.

We say a capacitor "generates" reactive power and an inductor "consumes" it. (It's an arbitrary definition.)



rms magnitude

Capacitor & inductor together consume reactive power of $\boxed{\frac{|U|^2}{\omega L} - |U|^2 \omega C}$

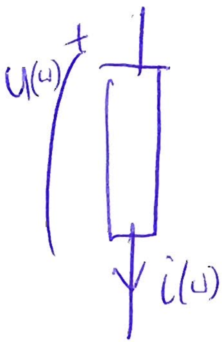
So if we choose

$$\omega L = \frac{1}{\omega C},$$

there is no reactive power from/to the source

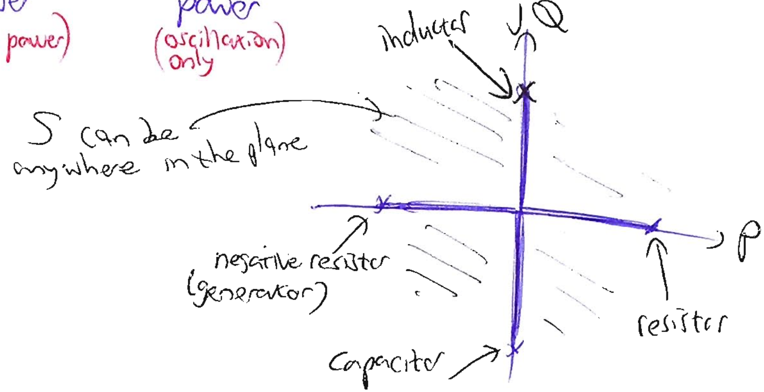
A <u>very useful</u> and utterly common definition is this:

for some two terminal component (or larger two-terminal circuit) in an ac circuit, the **COMPLEX POWER** into it is:



$$\underset{\text{complex power}}{S} = \underset{\substack{\text{active power} \\ \text{(mean power)}}}{P} + j\underset{\substack{\text{reactive power} \\ \text{(oscillation only)}}}{Q} = \overbrace{U(\omega)\cdot \bar{I}(\omega)^{*}}^{\text{rms-value phasors}} = \overbrace{\frac{U(\omega)\cdot I(\omega)^{*}}{2}}^{\text{if using peak value phasors}}$$

conjugate: $(|i|\angle\alpha)^* = |i|\angle{-\alpha}$

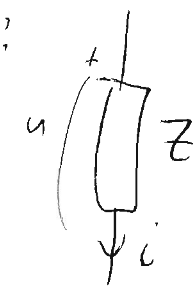


(Like impedance, complex power is <u>not</u> a phasor)

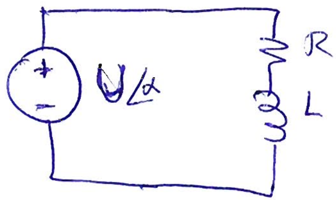
Useful results based on this:

- a typical 'load' such as a motor "consumes" <u>active</u> & reactive power
  (to run it) (in its coil's inductance)
- <u>One</u> calculation tells us all about the active and reactive power
- Shorter forms if we know <u>just</u> current or <u>just</u> voltage:

$$S = u \cdot i^* \Longrightarrow \begin{cases} S = u \cdot \left(\dfrac{u}{Z}\right)^* = \dfrac{u\,u^*}{Z^*} = \dfrac{|u|^2}{Z^*} \\ S = (iZ)\, i^* = i\, i^* Z = |i|^2 Z \end{cases}$$



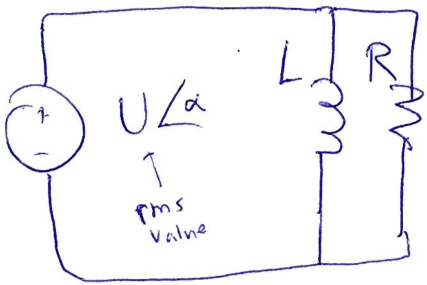
Example: find complex power delivered by source (i.e. into the R,L load)



Similar to the previous, but series connection. (We cannot treat each component separately as having a known voltage)

$$S = \frac{(U\angle\alpha)^2}{Z^*} = \frac{U^2}{R - j\omega L} = \boxed{\frac{U^2 (R + j\omega L)}{R^2 + \omega^2 L^2}} \Rightarrow \begin{cases} P = \dfrac{U^2 R}{R^2 + \omega^2 L^2} \\[2ex] Q = \dfrac{U^2 \omega L}{R^2 + \omega^2 L^2} \end{cases}$$

Example: find the complex power consumed by this (L, R) load.



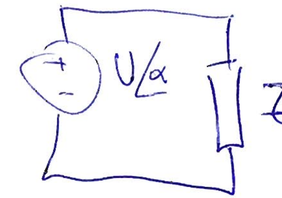
Method 1

for the inductor, $S_L = \dfrac{|U\angle\alpha|^2}{Z_L^*} = \dfrac{U^2}{-j\omega L} = j\dfrac{U^2}{\omega L}$

for the resistor, $S_R = \dfrac{|U\angle\alpha|^2}{Z_R^*} = \dfrac{U^2}{R}$

Sum: $\boxed{S = S_L + S_R = \dfrac{U^2}{R} + j\dfrac{U^2}{\omega L}}$

Method 2



$$S = \frac{|U\angle\alpha|^2}{Z^*} = \frac{U^2}{Z^*}$$

$$Z = \frac{1}{\frac{1}{R} + \frac{1}{j\omega L}} \quad \text{or} \quad \frac{j\omega L R}{R + j\omega L}$$

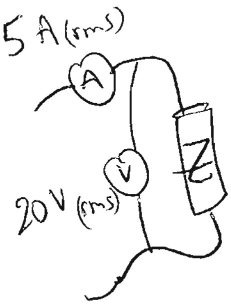
more convenient

$$\therefore S = U^2\left(\frac{1}{R} + \frac{1}{-j\omega L}\right)$$

$$= \frac{U^2}{R} + j\frac{U^2}{\omega L}$$

further definitions! Apparent power

$|S|$ is apparent power (Skenbar effekt) ... it's what you'd believe if you measure rms current and voltage and multiply them without knowing the phase of these quantities



$\Rightarrow |S| = 100$ **VA**

(if it's a resistor, then this is 100W.)

Depending on the nature of 'Z' it might be a capacitor, inductor, resistor, generator, or combination.

We write unit VA instead of W (for $|S|$ & S) to point out that we don't know what the power transfer actually is.

For reactive power we write VAr (r: reactive), for active power, W

Power factor

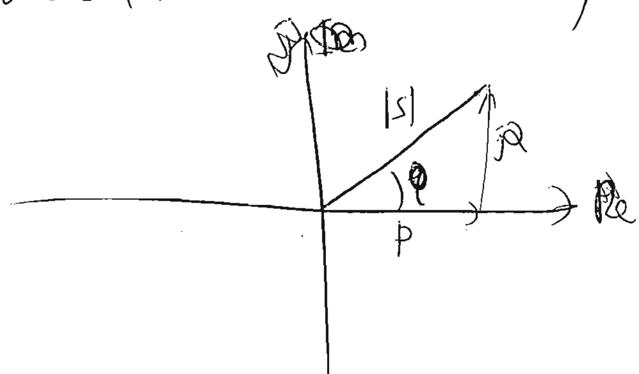
In practice, we want to transfer energy and <u>leave</u> it there!
We want ACTIVE power. (Usually).

Reactive power is often an <u>unwanted</u> effect of ~~accidental~~ "parasitic" (unintended) L & C,

E.g. inductance of motors and transformers.

Reactive power "transfer" $\Longrightarrow$ more current in cables $\Rightarrow$ more power loss.

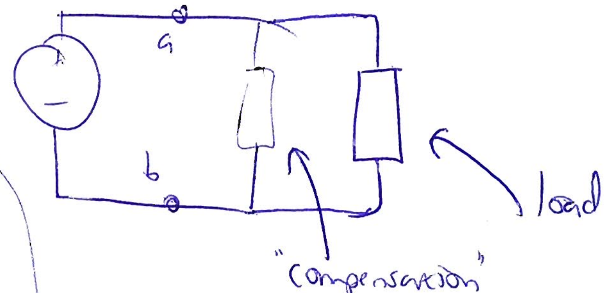
POWER FACTOR is the ratio of ACTIVE / APPARENT power.



$$\text{"p.f."} = \frac{P}{|S|} = \cos\varphi = \frac{P}{\sqrt{P^2+Q^2}}$$

(power factor)

Power factor example



The load consumes:

$$S = (40 + j30)\ \text{kVA}$$

What are P, Q, pf, $|S|$, φ ?

$$P = \mathrm{Re}(S) = 40\ \text{kW}$$

$$Q = \mathrm{Im}(S) = 30\ \text{k}\underbrace{\text{var}}_{\text{"VA}_r\text{"}}$$

$$|S| = \sqrt{30^2 + 40^2}\ \text{kVA} = 50\ \text{kVA}$$

$$pf = \frac{P}{|S|} = \frac{40\ \text{kW}}{50\ \text{kVA}} = 0.8\ \text{(lagging)}$$

because it is inductive: (Q is positive)

$$\varphi = \cos^{-1}\frac{P}{|S|} = \tan^{-1}\frac{Q}{P} \approx 37^\circ$$

Power companies don't want to transfer reactive power.
No gain ... power loss ... overheating ... etc

They try to <u>compensate</u> it and to encourage users to do so too.

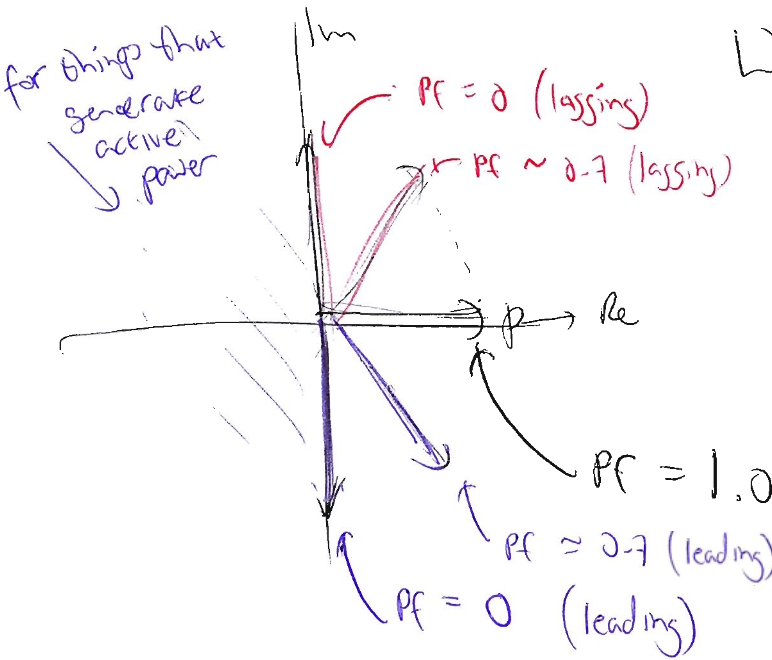
⇒ if your load is like an inductor and resistor (eg motor)
then it consumes active power <u>and</u> consumes reactive power.

⇒ → <u>So</u> install a capacitor in parallel or ~~series~~
to <u>generate</u> reactive power.

usually like this for a motor

This can be called POWER FACTOR COMPENSATION.
full compensation → <u>no</u> reactive power from the source → PF = 1 (resonance!)
Or else, partial compensation, eg. "make PF ≥ 0.9"

Sometimes we want to distinguish a pf of eg. 0.9 due to capacitance or due to inductance.



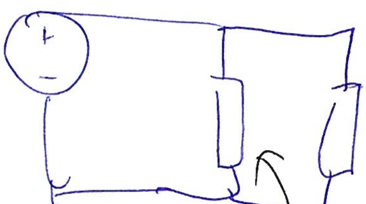
We use "lag" (inductive) and "lead" (capacitive) referring to the current relative to the voltage into a load.

Power factor compensation.

What should the "compensation" consume to make the source supply a total pf of 1.0? or 0.9 lagging?

(We assume the compensation is purely reactive.)

$pf = 1.0 \Rightarrow$ no ~~re~~reactive power from source



Consumes $(40 + j30)$ kVA

Consumes $(-j30)$ kVA

choose a capacitor that "generates" 30 kvar

total consumption is

$40 \text{ kW} \cancel{+ j30 \text{ kvar} - j30 \text{ kvar}}$

$pf = 1$

for $pf = 0.9$ (lagging)

We still expect $P = 40\,kW$ to be supplied from the source.

$pf = 0.9 \rightarrow |S| = \frac{40\,kW}{0.9} = 44.4\,kVA$

$P = 40\,kW$

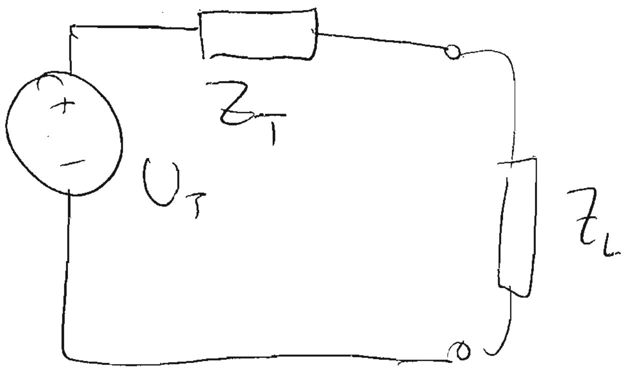
$Q = \sqrt{|S|^2 - P^2} = 19.4\,kVar$ ← this is positive (inductive) consumption as we know "lagging"

We want the total reactive power consumed to be 19.4 kVar but our load consumes 30 kvar. (inductive)

So choose a compensation that generates (capacitive) $(30-19.4)\,kVar$

$10.6\,kVar$

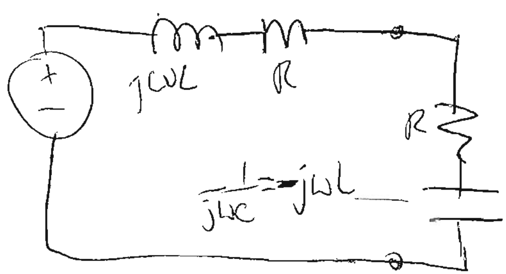
Maximum power in AC.



What should Z_L be to maximize the (active) power from the Thevenin source?

Answer: $\boxed{Z_L = Z_T^*}$

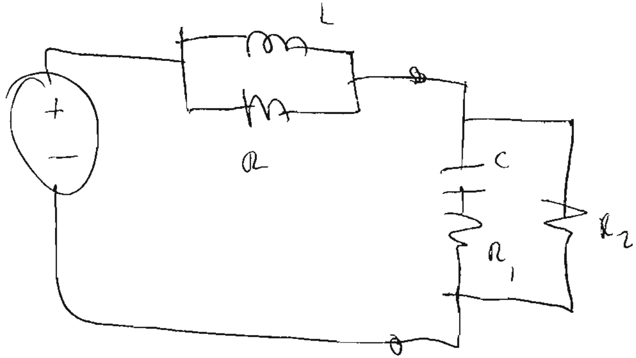
Rationale



If source has C then "resonate" against this with L, or vice versa. Then they have zero impedance.

Then it is like the dc case.

The main complexity in maximum power calculations is that we may have impedances in series/parallel combinations.



Try writing as combined impedances, one for source, one for load.

Then find the real and imaginary parts and make them fit $Z_L = Z_T^*$.

Sometimes it will be easier to use admittances $Y = \frac{1}{Z}$ instead.

Power superposition --- see "Chapter".

Mainly intended as a concept, not for required solving

(It can be helpful but is not unavoidable!)