THREE PHASE systems. Part 1: "Balanced" conditions

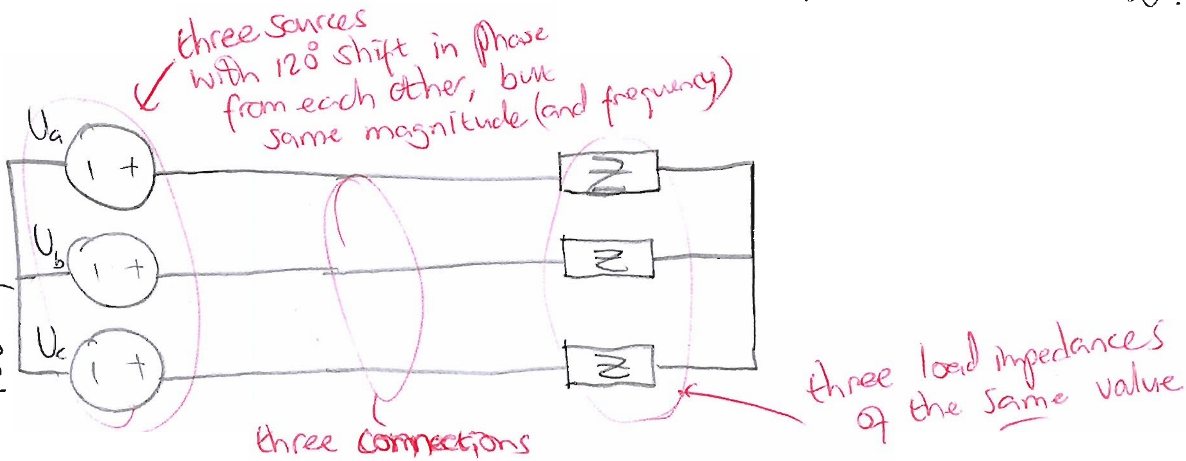
In a (balanced) three-phase system, the sources, loads and connections "come in threes" and the only difference between the things in each three is that they have voltages and currents phase shifted 120° from each other.

Example

$U_a = U\angle\alpha$

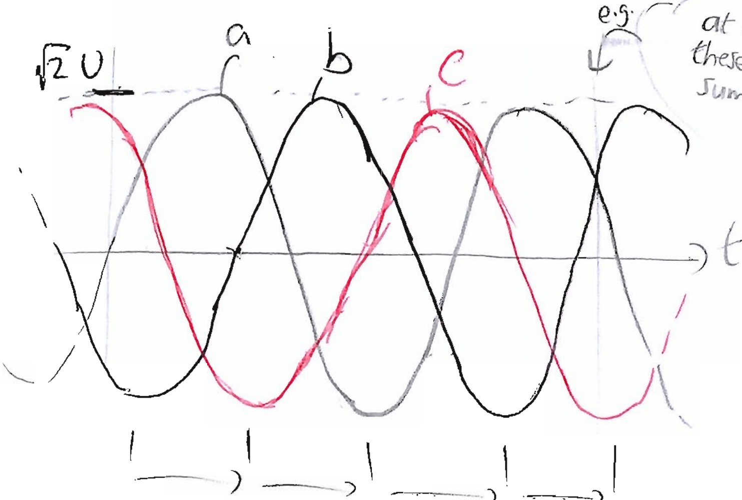
$U_b = U\angle\alpha - 120°$

$U_c = U\angle\alpha - 240°$



Quantities (voltage or current) with 120° phase shifts look like this:

in time



delays of 120° ($\frac{2\pi}{3}$ radian) in phase

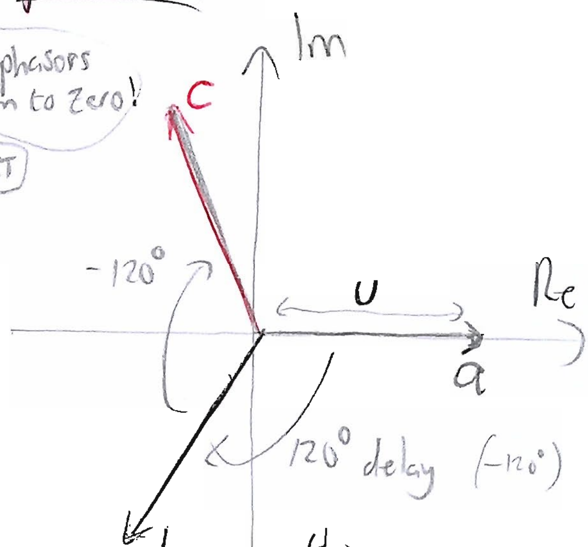
i.e. $T/3 = \frac{2\pi}{\omega} \cdot \frac{1}{3}$ in time

at any time, these curves sum to zero

the phasors sum to zero!

USEFUL FACT

as phasors



this assumes that α (see previous page) is zero — if not, shift all phasors by α.

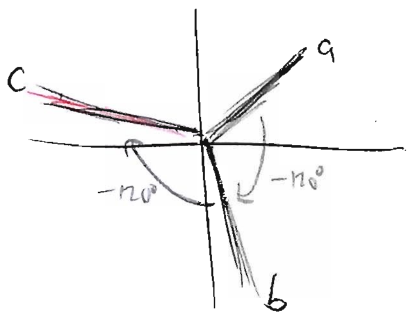
Note that $1\angle 120^\circ \equiv 1\angle -240^\circ \equiv 1\angle 2\pi/3$ etc.

We commonly write e.g. $U_b = |U_a|\angle -120^\circ$ to imply a delay ...but it's the same as $|U_a|\angle +240^\circ$!

"Phase rotation"

For many practical purposes we care only about voltage & current magnitudes, and power flows; these can be calculated quite easily in most balanced cases.

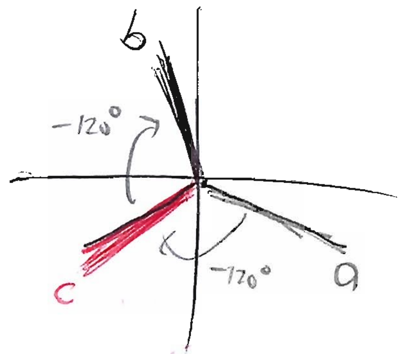
For some situations, particularly **unbalanced** systems, it's important to know the "phase rotation": ie does phase b or phase c follow more closely after phase a? (relative phase)



PHASE ROTATION:

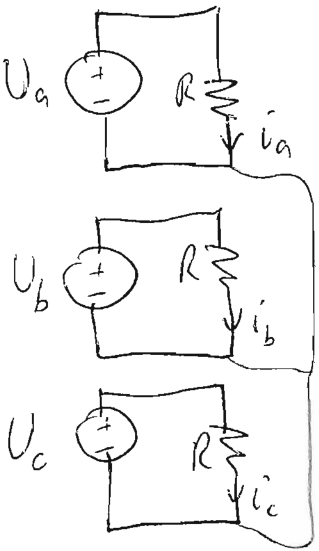
abc ←
or bca
or cab

acb →
or bac
or cba



Practically: "wrong" phase rotation can make motors turn backwards!

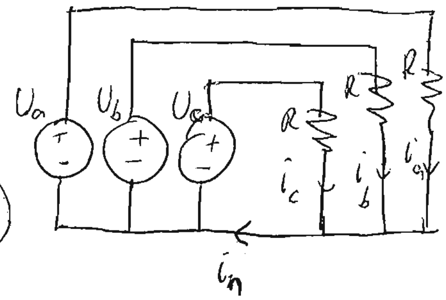
A closer look at 3phase connections.



three "separate" source-load pairs

a connection that doesn't change the u, i or power flow, (as it forms no loop)

$\Rightarrow$ redraw the same circuit adding some markings $\Rightarrow$



$U_a = U\angle 0$

$U_b = U\angle -120°$

$U_c = U\angle -240°$

By KVL: $\begin{cases} i_a = \dfrac{U_a}{R} \\ i_b = \dfrac{U_b}{R} \\ i_c = \dfrac{U_c}{R} \end{cases}$

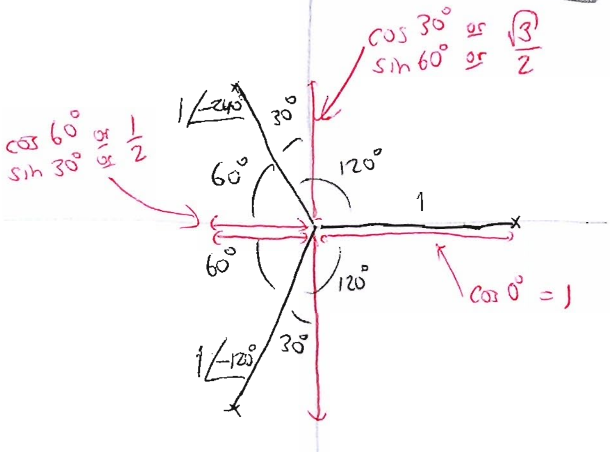
By KCL: $i_n = i_a + i_b + i_c$

$\Rightarrow$ next page!

(continued)

$$i_n = i_a + i_b + i_c = \frac{U_a}{R} + \frac{U_b}{R} + \frac{U_c}{R} = \frac{U}{R}\underbrace{\left(1\angle 0^\circ + 1\angle -120^\circ + 1\angle -240^\circ\right)}_{\text{What is this?}}$$

Geometric solution.



$$\Rightarrow 1\angle 0^\circ + 1\angle -120^\circ + 1\angle -240^\circ$$

$$= 1 - \frac{1}{2} - \frac{1}{2} + j\frac{\sqrt{3}}{2} - j\frac{\sqrt{3}}{2}$$

$$= 0$$

(can guess this looking at the "balance" of the phasors.)

Algebraic solution.

$$1\angle 0^\circ + 1\angle -120^\circ + 1\angle -240^\circ = e^0 + e^{-j\frac{2\pi}{3}} + e^{-j\frac{4\pi}{3}}$$

(Recall: $e^{j\theta} = \cos\theta + j\sin\theta$)

$$\Rightarrow \cos 0 + j\sin 0 + \cos(-120^\circ) + j\sin(-120^\circ) + \cos(-240^\circ) + j\sin(-240^\circ)$$

$$= 1 + 0 - \left(-\frac{1}{2}\right) + j\left(-\frac{\sqrt{3}}{2}\right) + \left(-\frac{1}{2}\right) + j\left(\frac{\sqrt{3}}{2}\right)$$

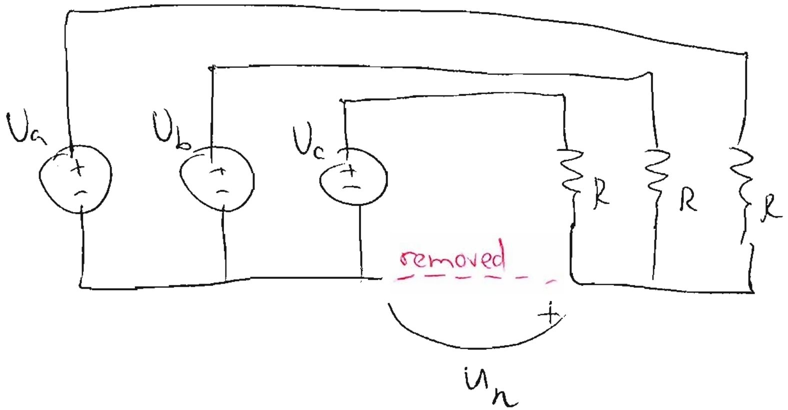
$$= 0$$

CONCLUSION

$$\boxed{\therefore\ i_n = 0}$$

So the balanced three phase currents "cancel" — no current is needed in the neutral.

As $i_n = 0$, why not remove this neutral conductor?



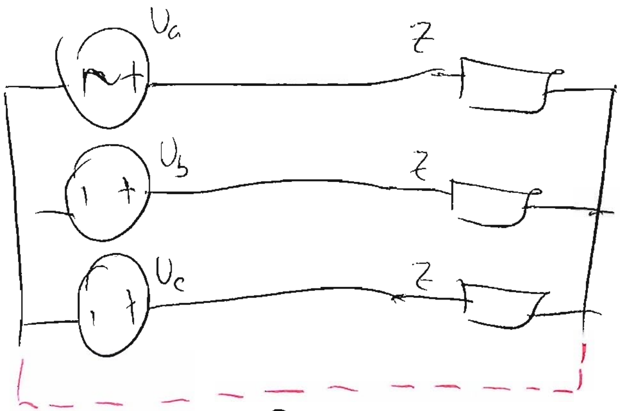
What is U_n in this case, with no conductor to force it to $U_n = 0$?
(short circuit)

KCL $\quad \dfrac{U_a - U_n}{R} + \dfrac{U_b - U_n}{R} + \dfrac{U_c - U_n}{R} = 0$

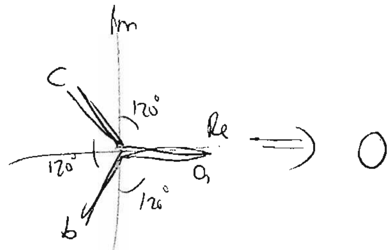
$$\therefore U_n = \frac{U_a + U_b + U_c}{3} = \frac{U}{3}\Big(\underbrace{1\angle 0^\circ + 1\angle{-120^\circ} + 1\angle{-240^\circ}}_{\text{recognise this } = 0 \text{ from the previous solution of } i_n}\Big) = 0$$

So we have confirmed that removing the 'neutral' made no difference.

We have now seen that the three sources and loads can be connected so that each load behaves as if connected to one specific source, but using only a total of three (not six!) wires.



the currents "cancel" here



this conductor ("neutral") has no significance if the system is balanced (i.e. all loads have the same impedance Z, and the sources have the same magnitude and 120° phase differences).

(BUT: In unbalanced cases the neutral can be useful to return the unbalance current.)

The following 5 pages are about "Why have three-phase systems" (instead of 1 or 2 or 4 etc).

You can safely skip them if not interested.

(But it's an interesting thought ...)

Good features of three-phase system.

smooth power delivery ... total power to a balanced load is the same at each point in time — the time variations of power flow in the different phases cancel to give a constant value.

- this is good for smooth torque in motors and generators and it avoids needing to store so much energy (capacitor) in ac-dc converters

rotating magnetic field ... three phase supply to coils at different angle positions in a motor makes a magnetic field that changes its direction like a rotation.

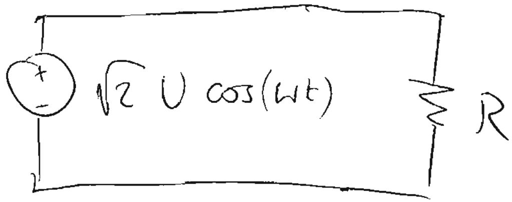
- this allows several useful types of motor to be made.

each source/load phase can connect by one wire, and their currents then cancel so no further wires are needed

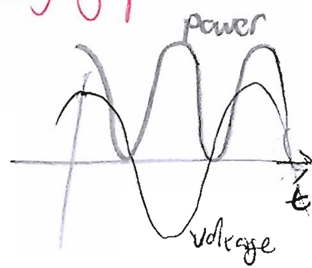
- efficient power transfer — less power loss in the resistance of the wires, as current goes through less length of wire.

Smooth power delivery – proof.

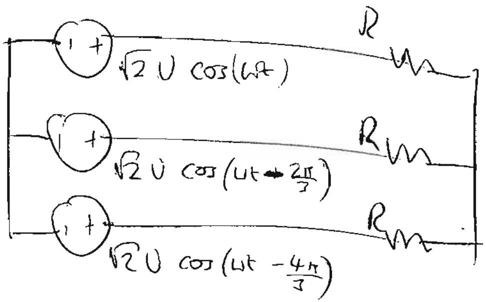
1-phase:



$$p(t) = \frac{\left(\sqrt{2}\, U \cos(\omega t)\right)^2}{R} = \frac{\cancel{2}\, U^2}{\cancel{2}\, R}\left(\overbrace{1}^{\text{average}} + \overbrace{\cos 2\omega t}^{\text{time varying part}}\right)$$



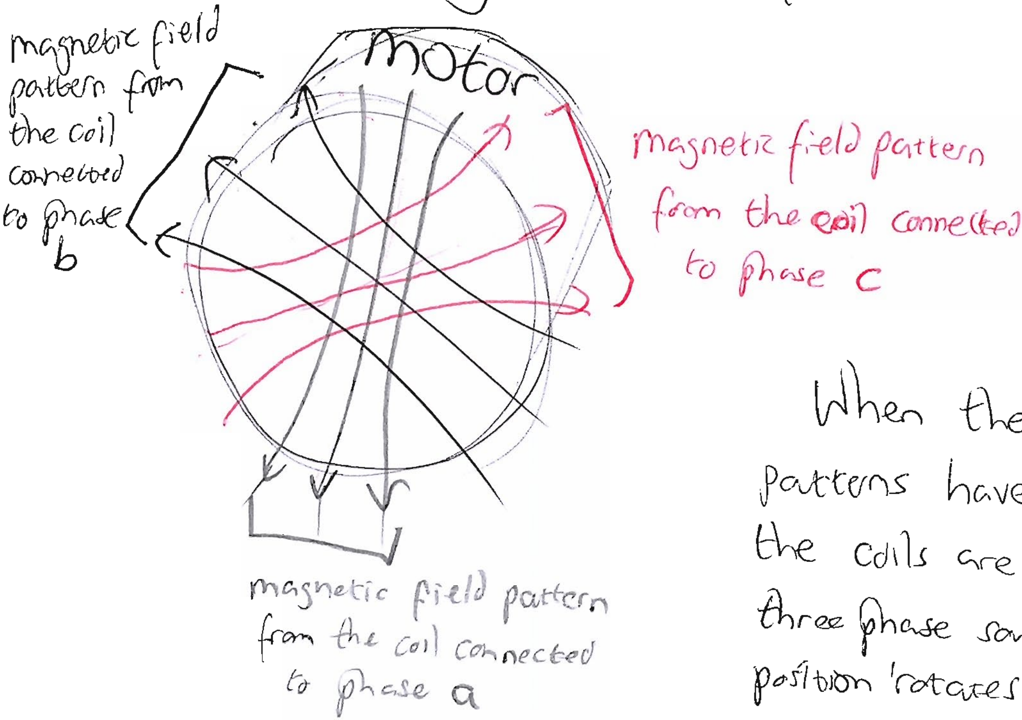
3-phase



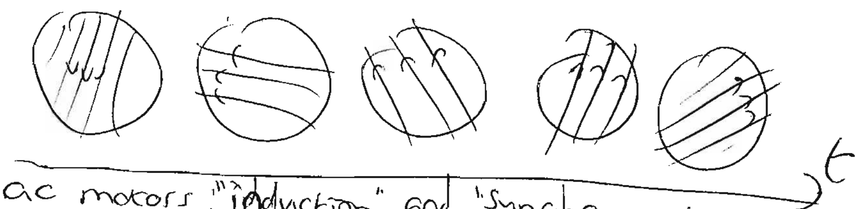
$$p(t) = \frac{\left(\sqrt{2}\, U \cos(\omega t)\right)^2}{R} + \frac{\left(\sqrt{2}\, U \cos(\omega t + \frac{2\pi}{3})\right)^2}{R} + \frac{\left(\sqrt{2}\, U \cos(\omega t - \frac{4\pi}{3})\right)^2}{R}$$

$$= \frac{U^2}{R}\left(1 + 1 + 1 + \underbrace{\cos(2\omega t) + \cos(2\omega t - \tfrac{4\pi}{3}) + \cos(2\omega t - \tfrac{8\pi}{3})}_{\text{cancel to zero}}\right) = \overbrace{\frac{3U^2}{R}}^{\text{Constant}}$$

"Rotating magnetic field"

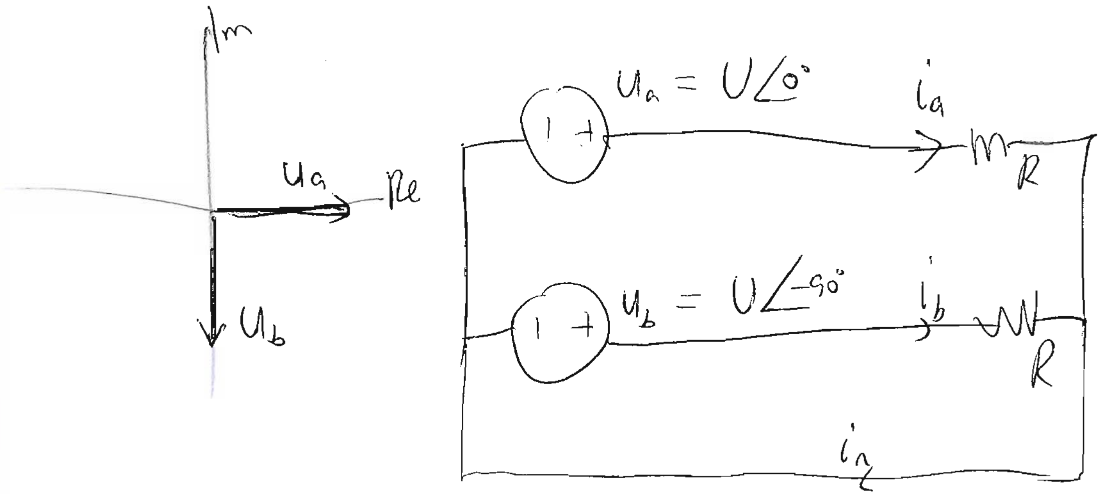


When the coil's magnetic field patterns have 120° "phase shifts" (looking at their position in the motor) and the coils are connected to a three phase source, the magnetic field position 'rotates' over time.



This is the principle behind most ac motors: "induction" and "synchronous".

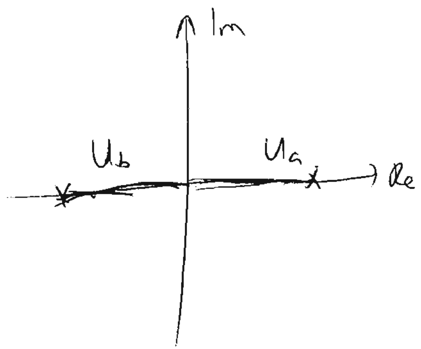
In fact, a two phase system can give some of these features.

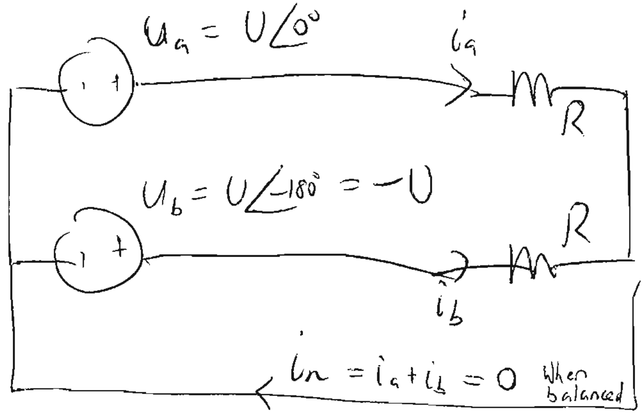


- The total power (both phases) is a constant — prove this!
- A "rotating field" can be produced, using coils with 90° difference in position.
- But $i_n = i_a + i_b \Rightarrow |i_n| = \sqrt{2}\,|i_a|$ or $\sqrt{2}\,|i_b|$

The currents don't cancel ... they sum to more than each one separately.

Another choice of 2-phase system is this:

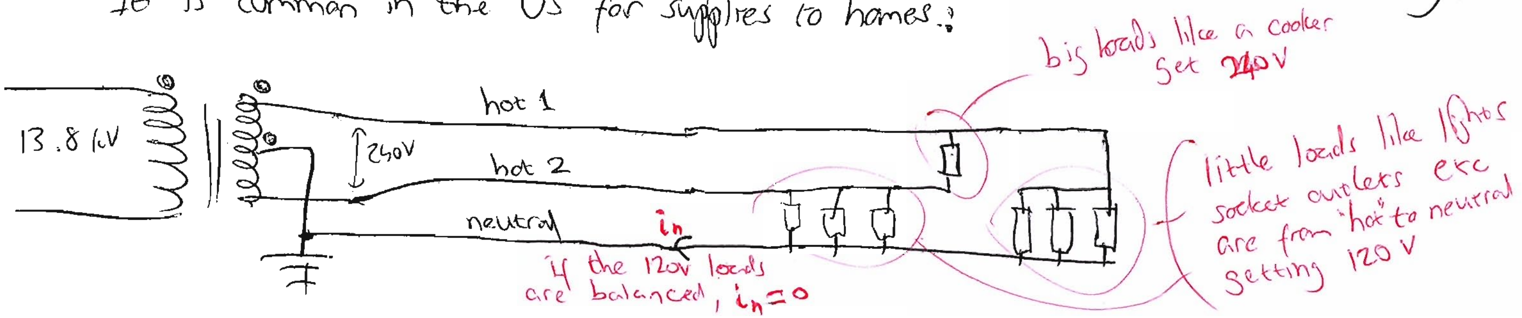




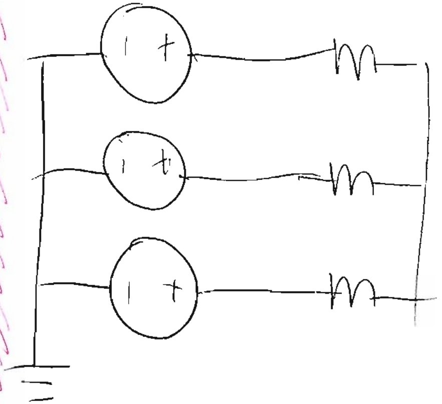
Some would argue it is not "2 phase" but just one phase and a reversal of the same, It is normally called "split phase".

→ This gives current cancellation in the neutral, but not smooth power or rotating field.

It is common in the US for supplies to homes:



Summary of three-phase benefits



A three phase system gives all the advantages of:

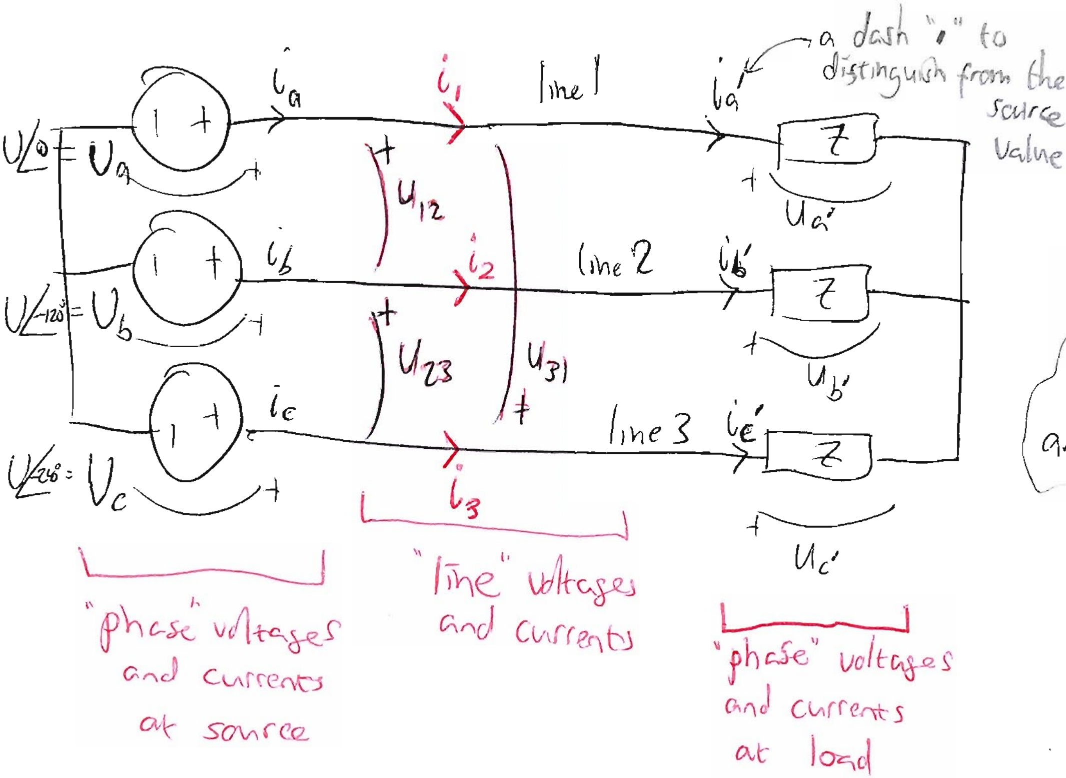
- Smooth power transfer
- "rotating field" for motors
- cancellation of balanced currents (no "return" neutral needed)

-- which cannot be got from 1 phase, and not all together from a 2phase system.

One can consider larger numbers of phases, but the construction complexity goes up without, usually, obvious benefit.

Often it is a benefit that the voltage "to earth" can be lower than the voltage available between the lines ... this is studied in the following pages ... it is no longer true for large numbers like "7-phase".

Back to this! Let's do calculations.



The individual (single phase) sources and loads that make up the three phase load/source are called the "phases".

Sometimes, this is said even for the "phases of a line" to describe the separate conductors.

Its very clear (KCL) that $i_a = i_1 = i_a'$, and similarly for i_b, i_c.
But what about the relation of U_{12} to U_a etc.?

By KVL: $U_{12} = U_a - U_b$

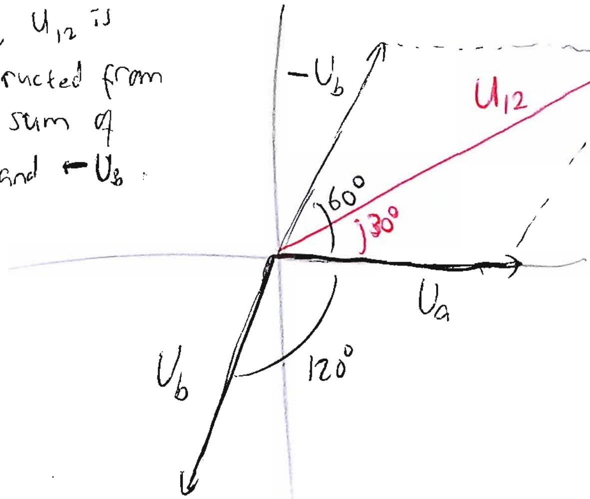
line voltage between lines 1 e 2

phase voltages of phases a e b of the source

(By symmetry of the balanced threephase system, we expect a similar result for U_{23} e U_{31} but with further angle shifts of $-120°$ e $-240°$)

Geometrically

Here, U_{12} is constructed from the sum of U_a and $-U_b$.



Algebraically: $U_{12} = U\left(1\angle 0 - 1\angle -120°\right)$

$= U\left(\cos 0 + j\sin 0 - \cos(-120°) - j\sin(-120°)\right)$

$= U\left(1 + 0 - \left(-\frac{1}{2}\right) - \left(-j\frac{\sqrt{3}}{2}\right)\right) = U\left(\frac{3}{2} + j\frac{\sqrt{3}}{2}\right)$

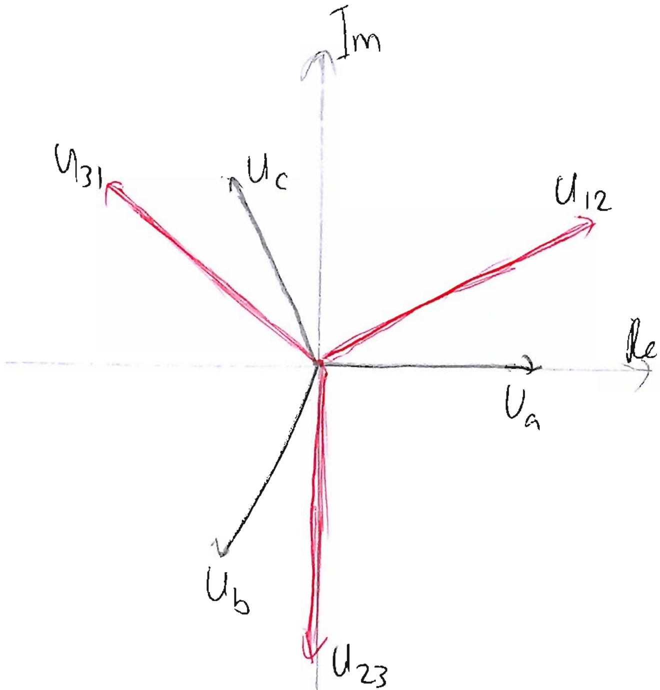
So $|U_{12}| = U\sqrt{\left(\frac{3}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2} = U\sqrt{\frac{12}{4}} = \sqrt{3}\,U$

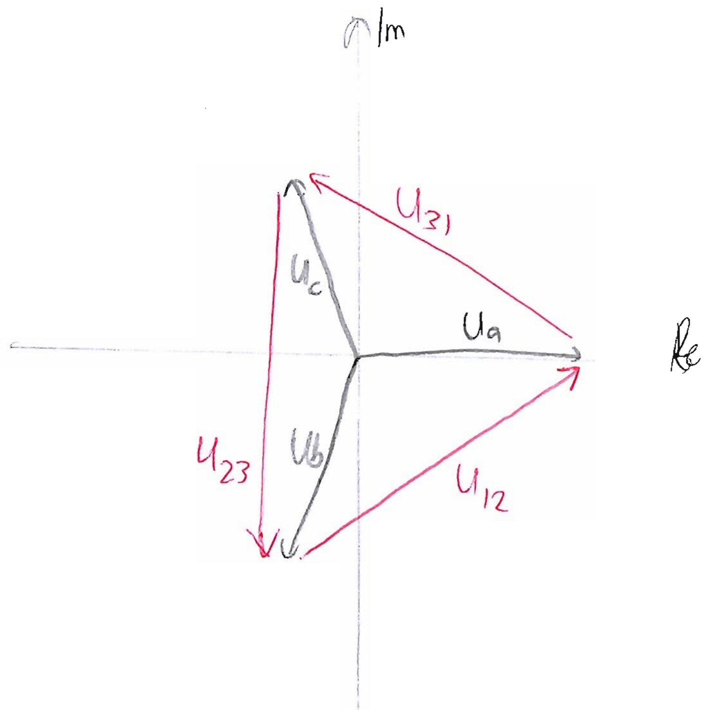
Line voltage magnitude is $\sqrt{3}$ of phase voltage

($\sqrt{3} \simeq 1{,}73$)

Angle of U_{12} is $\tan^{-1}\frac{\sqrt{3}/2}{3/2} = \tan^{-1}\frac{1}{\sqrt{3}} = 30°$

We can draw the phasors for all the line voltages: U_{12} U_{23} U_{31}





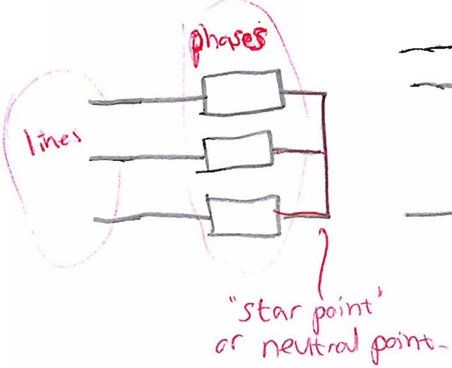
Style 1: phasors start at origin

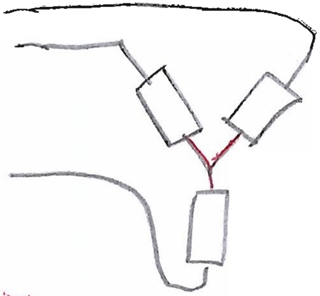
Style 2: phasors show how they are related to phase voltages.
(and that the line voltages sum to zero!)

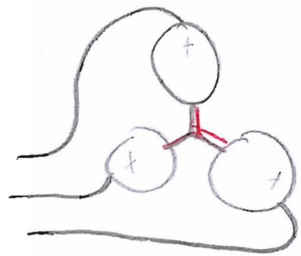
Up to now we have used just one way to connect the phases of a source or load, to the lines that connect sources and loads.

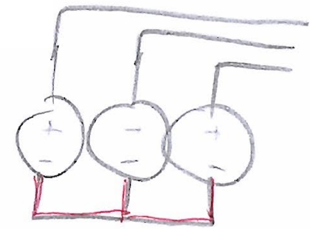
This is called STAR or WYE (letter Y)

the examples below are all the same connection topology.

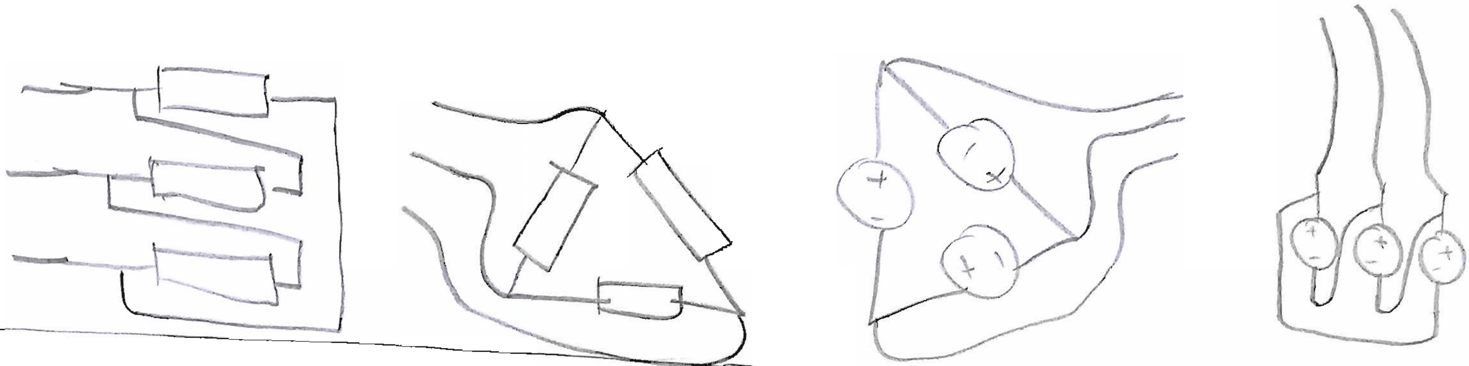




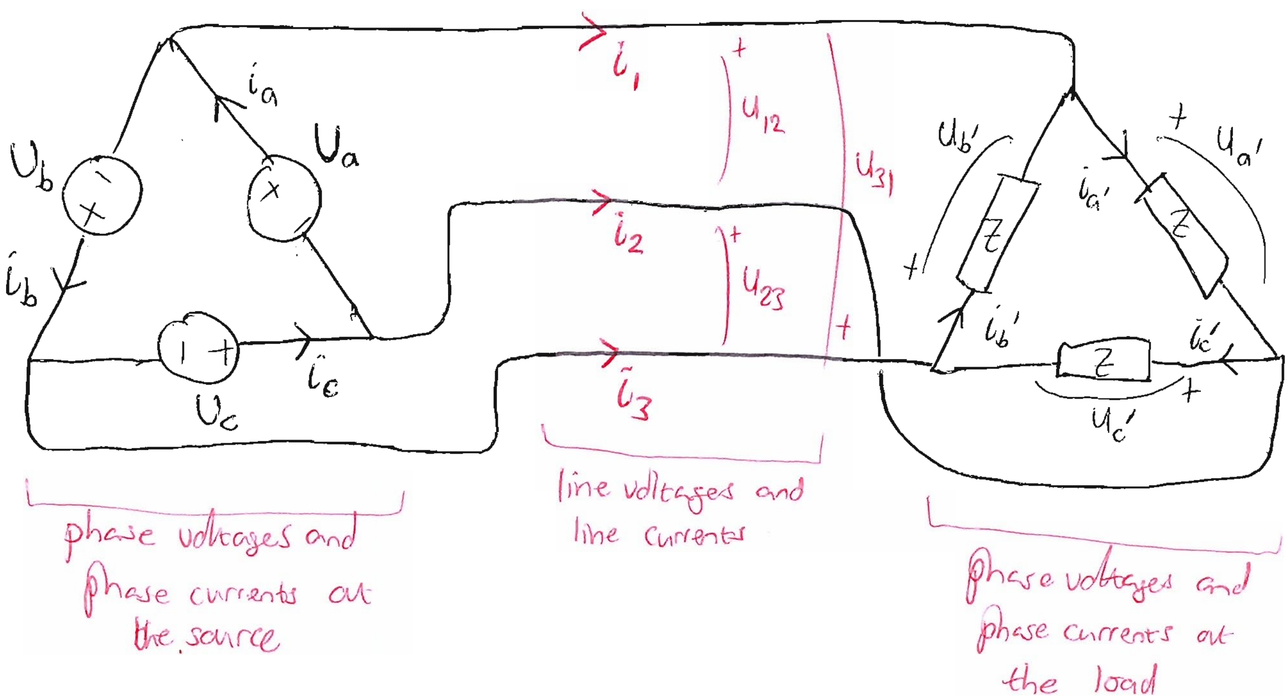




Another way is DELTA (Δ) or 'MESH'



Now we study phase and line relations in a Δ-connected system.



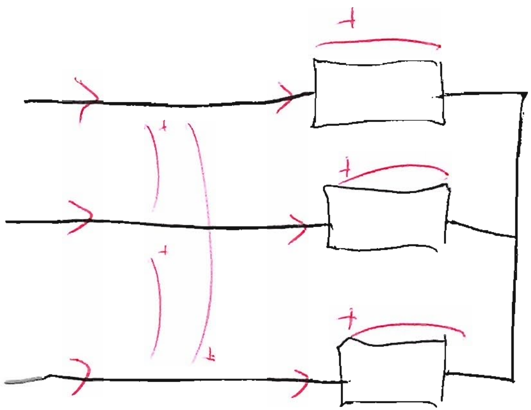
KVL $\quad u_{12} = U_a = u_a'$ So line- and phase voltages have same magnitude

KCL $\quad i_1 = i_a - i_b$ so line currents have $\sqrt{3} \times$ phase current magnitude

(this can be seen from the previous example with $u_{12} = u_a - u_b$)

Summary of Y and Δ connection regarding relation of phase and line quantities.

Y

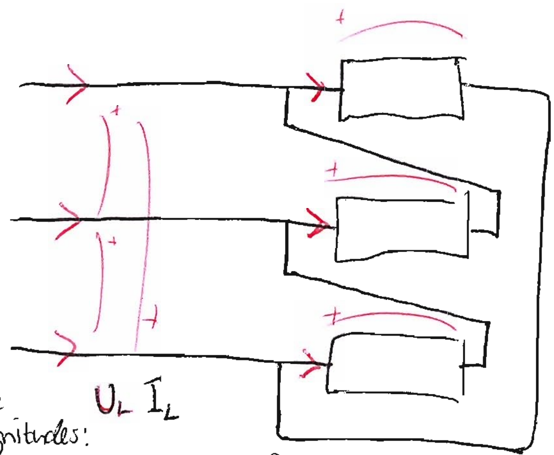


line magnitudes: U_L, I_L

phase magnitudes U_p, I_p

$$\boxed{\begin{aligned} U_L &= \sqrt{3}\, U_p \\ I_L &= I_p \end{aligned}}$$

Δ

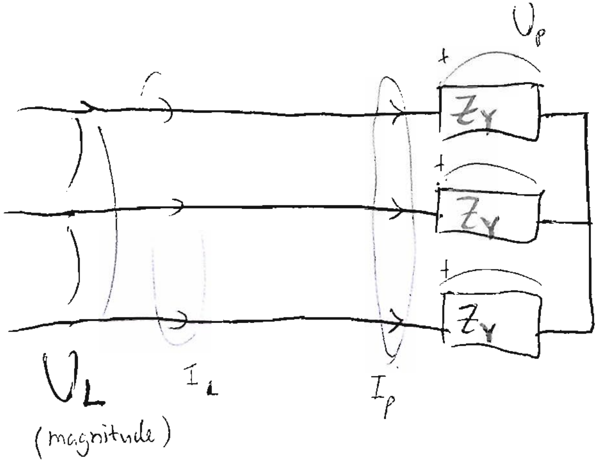


line magnitudes: U_L I_L

phase magnitudes: U_p I_p

$$\boxed{\begin{aligned} U_L &= U_p \\ I_L &= \sqrt{3}\, I_p \end{aligned}}$$

Complex power into (balanced) impedance load from (balanced) three phase source giving line voltage U_L.

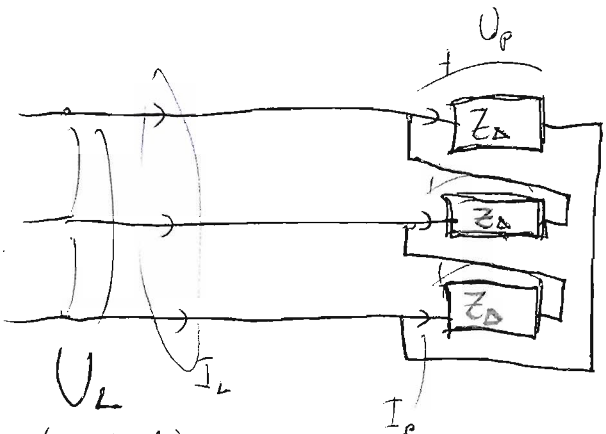


three phases

voltage magnitude across each impedance Z_Y

$$S_Y = \underset{\text{three phases}}{3} \cdot \frac{U_p^2}{Z_Y^*} = \frac{3\left(U_L/\sqrt{3}\right)^2}{Z_Y^*} = \frac{U_L^2}{Z_Y^*}$$

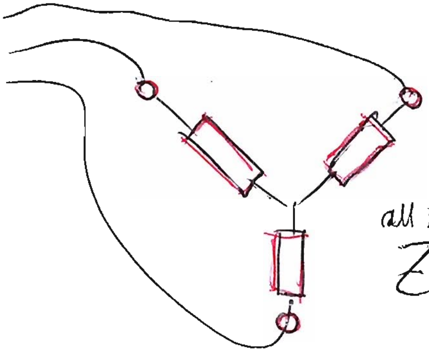
Notice: We only need know the impedance and the line voltage (or current) magnitude then the assumption "balanced three phase" lets us find the power.



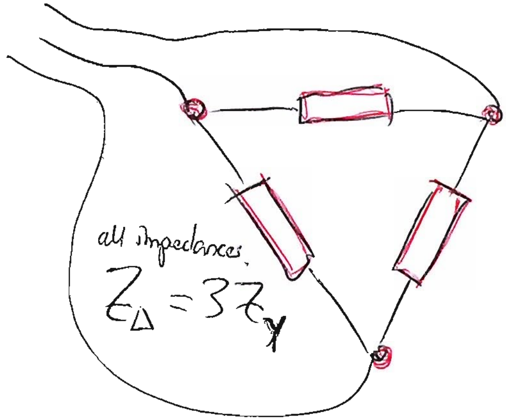
$$S_\Delta = 3\frac{U_p^2}{Z_\Delta^*} = \frac{3U_L^2}{Z_\Delta^*}$$

Notice: If $Z_\Delta = 3Z_Y$ then $S_Y = S_\Delta$

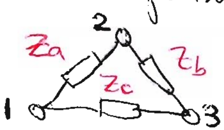
This relation, that $S_Y = S_\Delta$ if $Z_\Delta = 3Z_Y$ is useful, since it gives us a way to convert between Δ and Y impedance loads (balanced!) that are equivalent when seen at the terminals. This is useful in some circuit solutions.

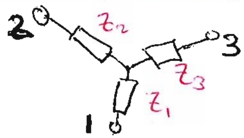


all impedances are Z_Y



(Point of interest: there is a more general transformation for $Y \Leftrightarrow \Delta$ that will work even for different values of impedance; but it has quite long expressions, and our special case of balanced loads makes it much simpler.



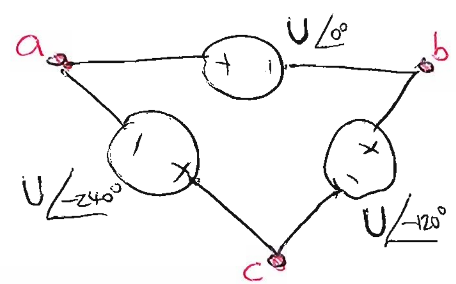


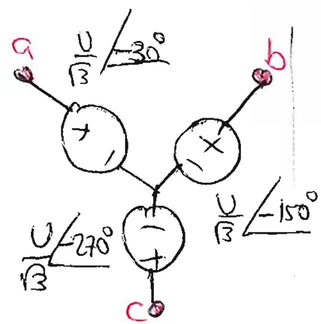
eg. $Z_1 = \dfrac{Z_a Z_c}{Z_a + Z_b + Z_c}$ $\qquad Z_a = \dfrac{Z_1 Z_2 + Z_2 Z_3 + Z_3 Z_1}{Z_3}$)

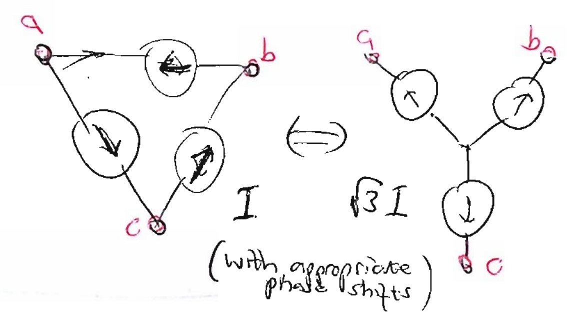
This type of $\Delta \leftrightarrow Y$ conversion can also be done for sources, so that the two forms "behave the same when seen by things connected outside their terminals".

(But a Y connected source with neutral connection (four terminals) can only be converted to a Δ-connected source if everything is balanced so that the neutral can be omitted!)

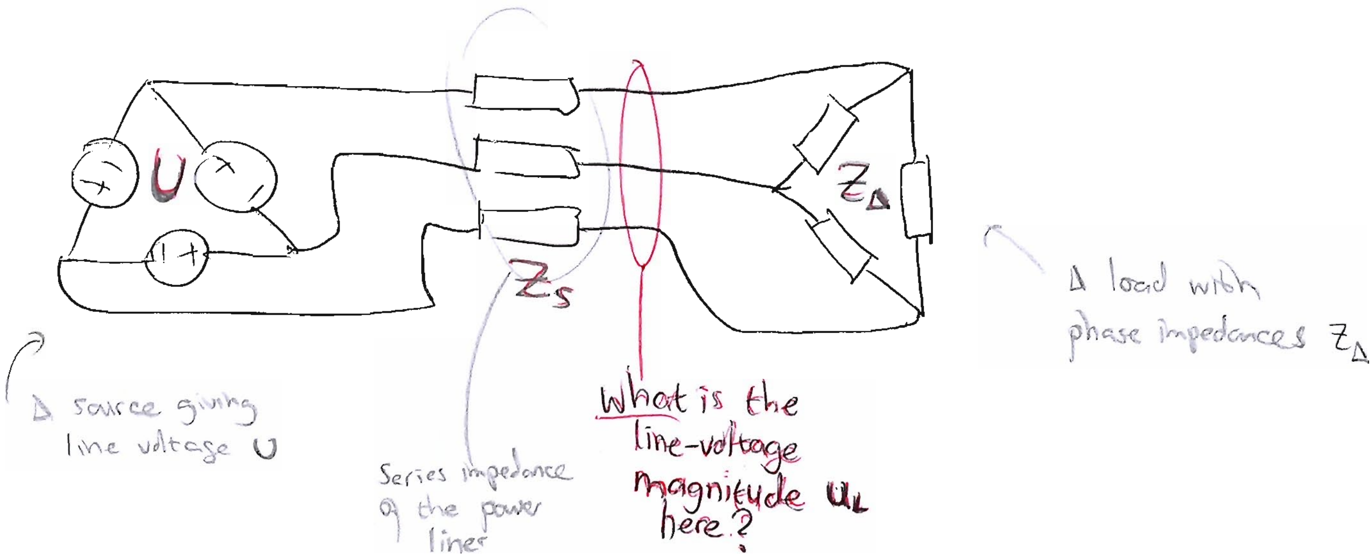
Often we are happy just to write the source magnitudes and know that each three phase source has 120° phase shifts between the three single phase sources it is made of. But if the exact phase angles are important for our solution, we must take more care! Δ-Y conversion will need a 30° phase shift.







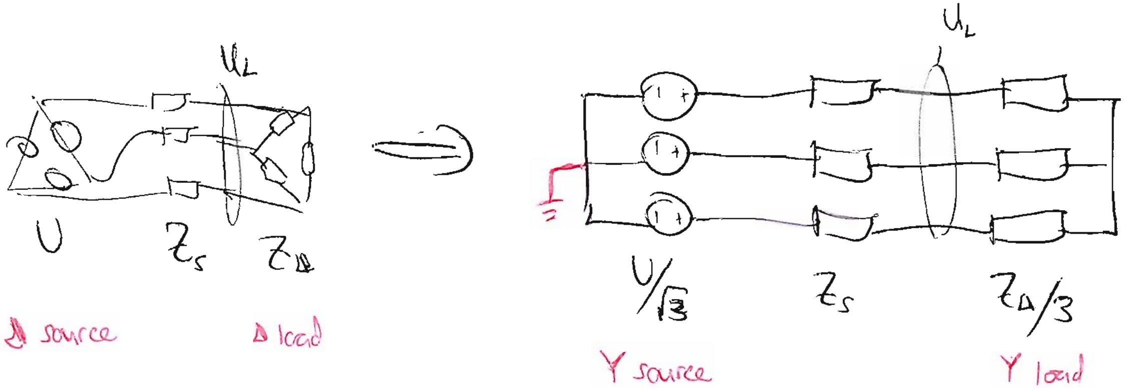
Example: $\Delta \rightarrow Y$ transformation in a solution.



This is a fairly typical case where magnitudes are to be found in a balanced three phase system. We don't want lots of detail of individual phase and line quantities and their angles.

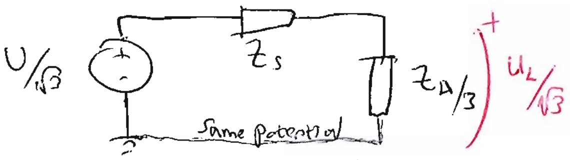
It is rather nasty looking, as the current in each line goes in two of the phases of the Δ-connected load and source, and the voltage across Z_s affects the voltage at the load.

So, we convert the Δ to Y at the source and load.



by symmetry in 3phase system, this point is same potential as the source neutral.

Then, due to the symmetry between phases (just an angle shift) we can analyse just one phase of the Y-connected system, looking at potentials relative to the neutral point



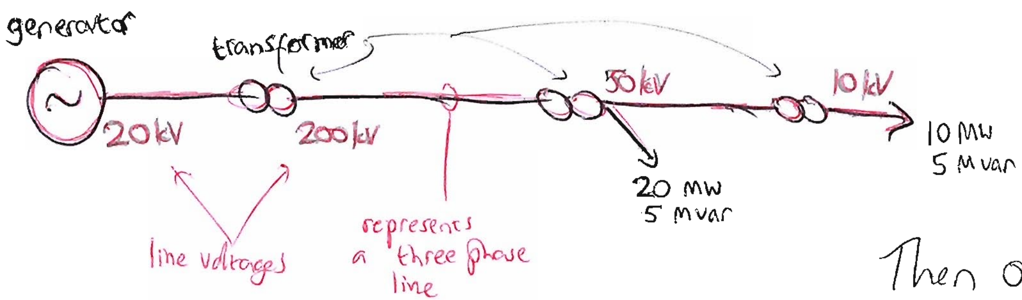
Voltage division $\quad \dfrac{U_L}{\sqrt{3}} = \dfrac{U}{\sqrt{3}}\left|\dfrac{Z_\Delta/3}{Z_s + Z_\Delta/3}\right|$

$$\boxed{\therefore U_L = U\left|\frac{Z_\Delta}{Z_\Delta + 3Z_s}\right|}$$

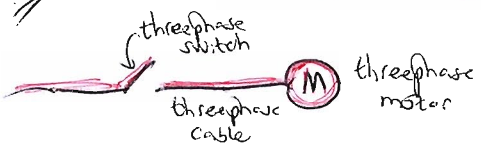
What we have done there is a "single phase equivalent" calculation.

The simplicity was partly from looking at just one phase, and partly from not caring about phase angles in the single phase being related to angles in the original circuit, as only a magnitude was needed.

Often a balanced three phase system is just represented as a single-line diagram! A single 'line', '—', represents all three phases.

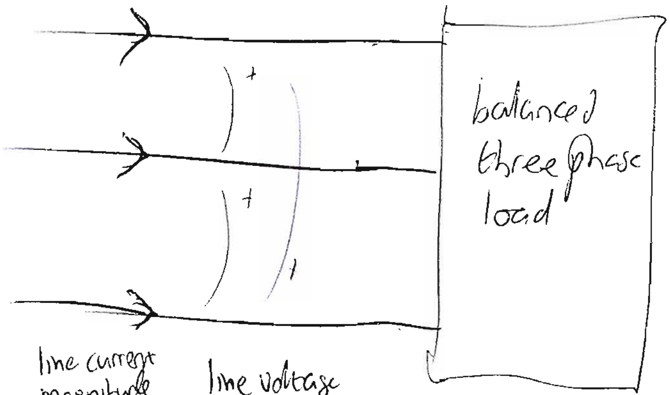


or



Then one thinks in terms of "powerflows" and voltage and current magnitudes.

It is convenient to have an expression relating the power flow to the line voltage and current! (magnitudes)



line current magnitude I_L

line voltage magnitude U_L

these are what we can measure easily with a voltmeter or ammeter --- we would need eg. an oscilloscope to see the phase between the voltage and current

$$\boxed{|S| = \sqrt{3}\, U_L I_L}$$

and if we know powerfactor, we can find P and Q from |S|.

Verification of the above:

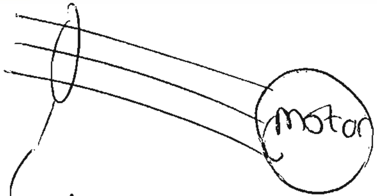
eg. Y connected impedances: $S = \dfrac{U_L^2}{Z_Y^*} \Rightarrow |S| = U_L \cdot \dfrac{U_L}{|Z_Y|}$

but $\dfrac{U_L}{|Z_Y|} = \sqrt{3}\,\dfrac{U_p}{|Z_Y|} = \sqrt{3}\, I_p = \sqrt{3}\, I_L$

so $|S| = \sqrt{3}\, U_L I_L$

you can try similarly with Δ connected impedances

Examples using $|S| = \sqrt{3}\, U_L I_L$



line voltage 400 V

line current 12 A

power factor 0.9 lagging

What are $P, Q, |S|$ into motor?

$$|S| = \sqrt{3} \cdot 400\,\text{V} \cdot 12\,\text{A} = 8.31\ \text{kVA}$$

$$P = pf \cdot |S| = 0.9 \cdot 8.31\ \text{kVA} = 7.48\ \text{kW}$$

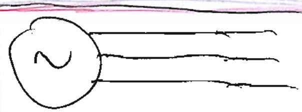
$$Q = \sqrt{|S|^2 - P^2} = |S| \cdot \sqrt{1 - pf^2} = 3.62\ \text{kvar}$$

check pf is lagging (inductive) so it makes sense that the reactive power in is positive.

400 kV line carrying 800 MVA (apparent power)

What is the line current (magnitude)?

$$I_L = \frac{800\ \text{MVA}}{\sqrt{3} \cdot 400\ \text{kV}} \approx 1155\ \text{A}$$



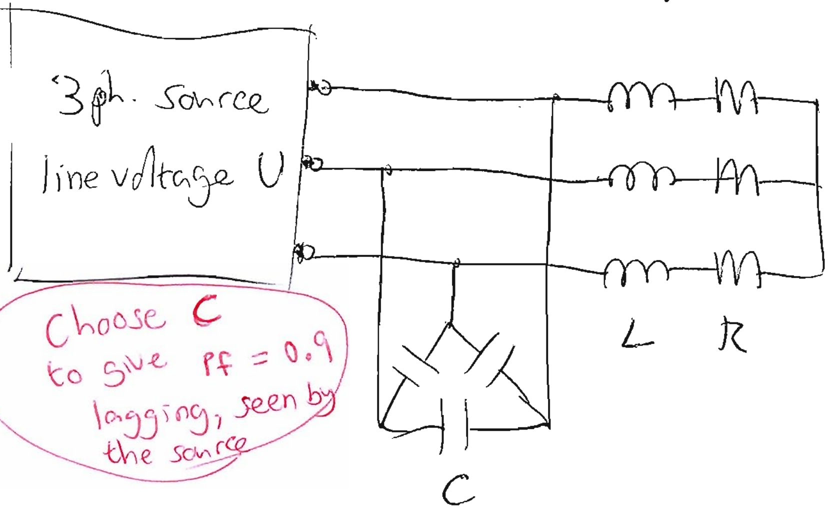
generator producing 10 MW and 3 Mvar line voltage 11 kV.

What is the line current (magnitude)?

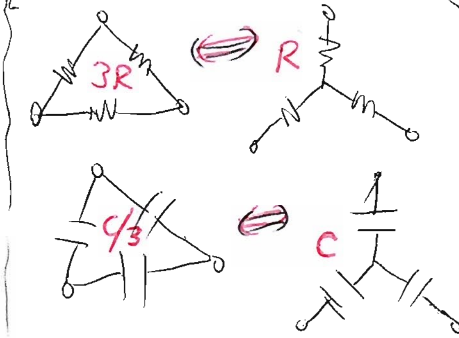
$$|S| = \sqrt{10^2 + 3^2}\ \text{MVA} = 10.44\ \text{MVA}.$$

$$I_L = \frac{S}{\sqrt{3}\, U_L} = \frac{10.44\ \text{MVA}}{\sqrt{3} \cdot 11\ \text{kV}} = 548\ \text{A}$$

Example of reactive power compensation.



Choose C to give pf = 0.9 lagging, seen by the source



Scaling of capacitors Y↔Δ is opposite from resistors or inductors, as impedance is inversely proportional to capacitance.

We could use single phase equivalent.

But as we're just considering power (not current or voltage between impedances) it's quite easy to work with power equations on the whole system.

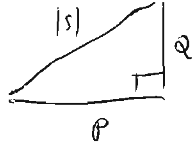
$$S = S_c + S_{RL} = \frac{3U^2}{Z_c^*} + \frac{U^2}{Z_{LC}^*} = U^2\left(-j\omega 3C + \frac{1}{R-j\omega L}\right)$$

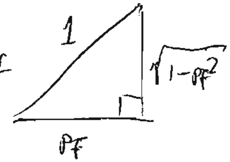
from source

delta connected

Continued →

So $$S = V^2\left(-j\omega 3C + \frac{1}{R - j\omega L}\right) = V^2\left(\underbrace{\frac{R}{R^2+\omega^2L^2}}_{\text{P part}} + j\omega\underbrace{\left(\frac{L}{R^2+\omega^2L^2} - 3C\right)}_{\text{Q part}}\right)$$

Consider 

or 

$$pf = 0.9 \text{ lagging} \Rightarrow \frac{Q}{P} = \frac{\sqrt{1-pf^2}}{pf} = 0.48$$

So we want:

$$\frac{\cancel{V^2}\,\omega\left(\frac{L}{R^2+\omega^2L^2} - 3C\right)}{\cancel{V^2}\,\frac{R}{R^2+\omega^2L^2}} = 0.48 \Rightarrow \boxed{C = \frac{\omega L - 0.48R}{3(R^2+\omega^2L^2)\omega}}$$

(Note being able to bring the pf to 0.9 lagging (by adding capacitors) implies that the pf of the load should have been a lower, lagging value without the capacitors! Else the required 'C' will turn out negative. This is seen from the $\omega L - 0.48R$ term.)

SUMMARY of THREE-PHASE CALCULATION ESSENTIALS

Important geometric relations!

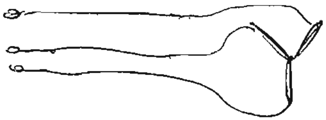
$$1\angle\alpha + 1\angle\alpha-120^\circ + 1\angle\alpha-240^\circ = 0$$

eg. KVL around a Δ
KCL in a Y

$$1\angle\alpha - 1\angle\alpha-120^\circ = \sqrt{3}\angle\alpha+30^\circ$$

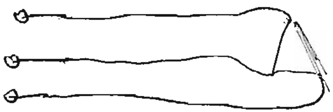
eg. phase voltages in Y to find line voltage
or phase currents in Δ to find line current

features of Y and Δ connections:



$I_L = I_P$
$U_L = \sqrt{3}\, U_P$
(magnitudes)

$$S_Y = \frac{U_L^2}{Z_Y^*}$$



$I_L = \sqrt{3}\, I_P$
$U_L = U_P$

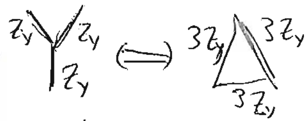
$$S_\Delta = \frac{3U_L^2}{Z_\Delta^*}$$

In balanced conditions: "neutral" connection makes no difference.

Power in relation to line quantities:

$$|S| = \sqrt{3}\, U_L I_L$$

Conversion $\Delta \leftrightarrow Y$ (equivalent)



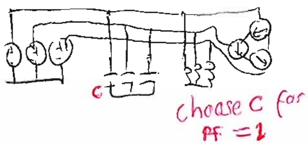
(impedances)

behave the same "at the terminals"

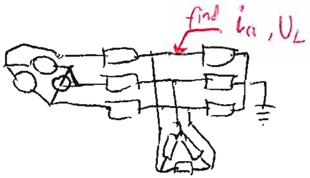
Summary (continued)

Common methods; taking advantage of symmetry (balanced) and formulae like $S_Y = \frac{U_L^2}{Z_Y^*}$

POWER-BASED: focus on magnitudes of u & i, together with complex impedance to calculate complex power --- often useful in power-factor questions, or for finding a line current magnitude



SINGLE PHASE EQUIVALENT (using the symmetry of a balanced threephase system):



Often useful for calculating when there are line impedances in series with loads, or angles as well as magnitudes are needed.

familiar methods of voltage division, equivalent impedance etc are easily applied.

(Not so traditional for exams)

SIMPLER SINGLE-LINE DIAGRAM: mainly an "accounting method" for power flows and current magnitudes.

(see example in exercises)