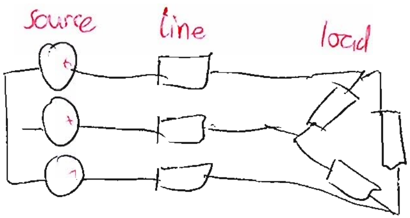
THREE PHASE systems: Part 2: UNBALANCED conditions.

When a three-phase system gets as big as { 1 source (3ph) / 1 load (3ph) / 1 line (with impedance) } | very basic.

we already have 6 impedance components
3 source components
7 or 6 potentials



With all the different angles (phases) this is a lot of calculation if done fully, by hand. (Computers are very useful ....)

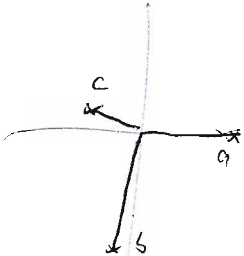
That is why we have taken lots of advantage of SYMMETRIES and of STANDARD FORMULAE like $|S| = \sqrt{3}\, I_L U_L$ to help get solutions, easily.

These were based on the assumption of balance ... which we now REMOVE.

What can make the system unbalanced?

source has different magnitudes on its phases

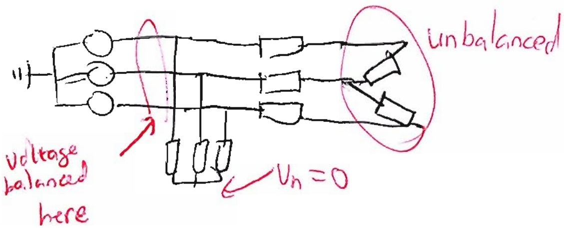
has not got 120° phase shift between all phases



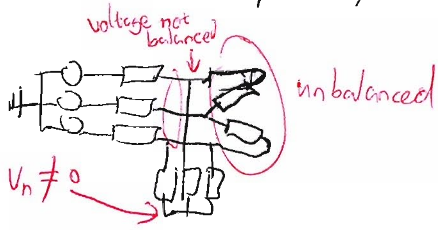
line or **load** has different impedances in phases

extreme case: only connected to one or two phases.

One unbalanced component <u>may</u> put the whole system out of balance.
(We might like to estimate if it's enough to matter, for the purpose of our study).



In these systems the unbalance only affects the voltage on the right of the line impedance.



Why is it relevant to study this type of situation?

1: Ok — many, many studies of three phase systems are based on balanced conditions, especially in high voltage parts where loads are well balanced (lots of load on each phase, averaging out).

But: * faults (short circuit, open circuit, etc) can happen and need to be analysed ... they are often between two lines or from "line to earth" (80% of HV faults is a popular statistic)

* and at the lower parts of the system — like your house — there are many single phase loads connected to the three phase system ... they are seldom balanced.

Methods for handling unbalanced calculations.

Brute force (sledgehammer)

often good if using a computer

- no reliance on symmetry or on standard formulae of balanced cases
- just calculate with each phase of each source, load or line treated as a separate component ... { phasor calculation of every node potential and every current
- We can do this for tiny systems
- bigger systems need computer help

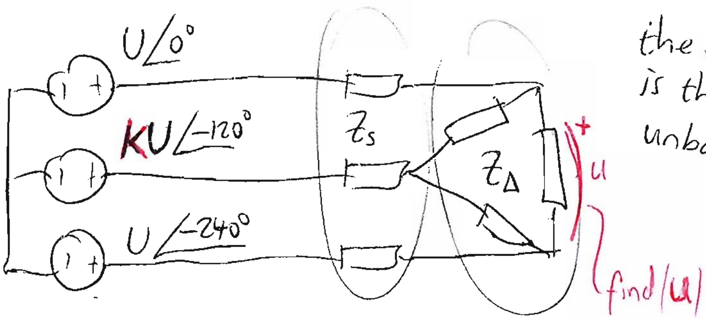
Hybrid

- identify the unbalanced parts in the system.
- try to find a way to solve "most" of the problem by balanced calculation with a smaller unbalanced calculation to take care of the unbalance
- not always possible or useful.

Other

- famous traditional method "symmetric components" involves a transformation of phase – line quantities and impedances, which usually simplifies calculation (no more on that here ... less necessary in the days of computers, but a useful concept)

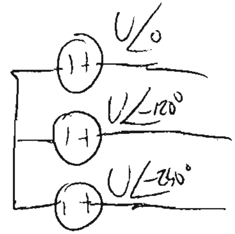
Examples of hybrid-type methods that do work.



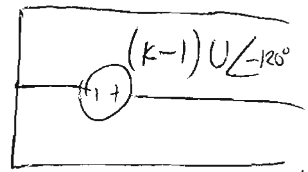
the source is the only unbalance

replace it with two superposition states!

Calculate each and add the results

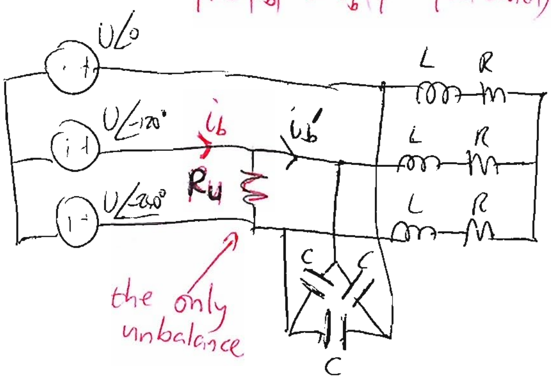


balanced



the unbalanced part

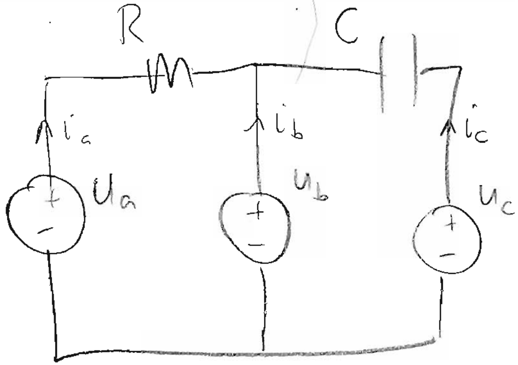
find $|i_b|$ or i_b (if we feel cruel)



The unbalanced part is connected to an ideal voltage source. So it doesn't affect the voltage applied to the other loads. We can do balanced calculation for i_b' then add (as phasor) the current through R_u.

Example

(next page)



$u_{(a,b,c)}$ are a balanced 3phase source with line voltage $\sqrt{3}\,U$ and phase rotation abc

Find the complex power in rectangular form, delivered by each of the components:

$S_a \quad S_b \quad S_c \quad S_R \quad S_C$

(The actual angles don't matter to the power as long as the relative angles are correct ... remember $S = u \cdot i^*$ so any shift of all angles in the circuit is cancelled by affecting u and i^* oppositely.)

Let's call phase a's angle 0.

Then $u_a = U\angle 0°$ $\quad u_b = U\angle -120°$ $\quad u_c = U\angle -240°$

If we find the three currents, we can find the complex powers.

$$i_a = \frac{U_a - U_b}{R} = \frac{U\left(1\angle 0 - 1\angle{-120^\circ}\right)}{R} = \frac{\sqrt{3}\,U\angle 30^\circ}{R}$$

The complex power into the resistor is $S = |i_a|^2 R = \frac{3U^2}{R}$,
But we were asked for complex power delivered by each component.

$$\boxed{S_R = -\frac{3U^2}{R}}$$

The complex power from source U_a is $S_a = U_a \cdot i_a^*$ (check the directions of the voltage and current)

$$S_a = U\angle 0 \cdot \frac{\sqrt{3}\,U\angle{-30^\circ}}{R} = \frac{\sqrt{3}\,U^2}{R}\angle{-30^\circ}$$

$$= \frac{\sqrt{3}\,U^2}{R}\left(\cos(-30^\circ) + j\sin(-30^\circ)\right) = \frac{\sqrt{3}}{2}\frac{\sqrt{3}\,U^2}{R} + j\,\frac{-1}{2}\cdot\frac{\sqrt{3}\,U^2}{R}$$

$$\boxed{S_a = \frac{3U^2}{2R} - j\,\frac{\sqrt{3}\,U^2}{2R}}$$

Let's take i_c next (it's simpler than i_b).

KVL $\quad i_c = \dfrac{U_c - U_b}{\frac{1}{j\omega C}} = j\omega C U \left(1\angle{-240^\circ} - 1\angle{-120^\circ}\right)$

$$= j\omega C\sqrt{3}U\angle 90^\circ$$

$$= \omega C\sqrt{3}U\angle 180^\circ$$

complex power into capacitor $= |i_c|^2 Z_c = |i_c|^2 \dfrac{1}{j\omega C}$

$$= \frac{3\omega^2C^2U^2}{j\omega C} = -j3U^2\omega C$$

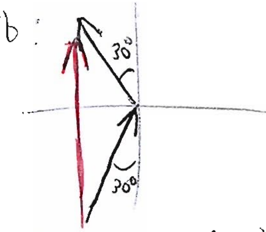
$\therefore$ complex power from capacitor:

$$\boxed{S_c = +j3U^2\omega C}$$

big C

Simplifying $1\angle{-240^\circ} - 1\angle{-120^\circ}$

You could draw ib:



Or write as $\cos(-240^\circ) + j\sin(-240^\circ)$

$\quad - \cos(-120^\circ) - j\sin(-120^\circ)$

Or use our rule that for phasors of magnitude 1 and relative angle of 120°,

$$1\angle\alpha - 1\angle{\alpha - 120^\circ} \Rightarrow \sqrt{3}\angle{\alpha + 30^\circ}$$

$$1\angle\alpha - 1\angle{\alpha + 120^\circ} \Rightarrow \sqrt{3}\angle{\alpha - 30^\circ}$$

from which, if $\alpha = -240^\circ$ (or $+120^\circ$)

$$1\angle\alpha - 1\angle{\alpha + 120^\circ} = \sqrt{3}\angle{\alpha - 30^\circ}$$

$$1\angle{-240^\circ} - 1\angle{-120^\circ} = \sqrt{3}\angle{-270^\circ} = \sqrt{3}\angle 90^\circ$$

Delivered by source c,

no imaginary part, so $i_c^* = i_c$

$$S_c = u_c \cdot i_c^* = U\angle{-240^\circ} \cdot \omega C\sqrt{3}U \angle{180^\circ} = \sqrt{3}U^2\omega C \angle{-60^\circ}$$

little c

$$S_c = \sqrt{3}U^2\omega C\left(\cos(-60^\circ) + j\sin(-60^\circ)\right) = \sqrt{3}U^2\omega C\left(\frac{1}{2} - j\frac{\sqrt{3}}{2}\right)$$

$$\boxed{S_c = \frac{\sqrt{3}}{2}U^2\omega C - j\frac{3}{2}U^2\omega C}$$

Now to source b: by KCL, $i_b = -i_a - i_c = -\frac{\sqrt{3}U}{R}\angle{30^\circ} - \omega C\sqrt{3}U\angle{180^\circ}$

$$= \frac{\sqrt{3}U}{R}\angle{-150^\circ} + \omega C\sqrt{3}U\angle{0^\circ}$$

So $S_b = u_b \cdot i_b^*$.

We must be careful with what steps to take, to get a solution not too full of $\tan^{-1}()$ etc!

$$S_b = U\angle 120^\circ \cdot \left(\frac{\sqrt{3}U}{R}\angle -150^\circ + \omega C\sqrt{3}U\angle 0^\circ\right)^*$$

$$= U\angle -120^\circ \cdot \left(\frac{\sqrt{3}U}{R}\angle +150^\circ + \omega C\sqrt{3}U\angle 0^\circ\right)$$

move complex conjugates inside

$$= \sqrt{3}U^2\left(\frac{1}{R}\angle 30^\circ + \omega C\angle -120^\circ\right)$$ ... then split into cos + jsin components!

$$= \sqrt{3}U^2\left(\frac{\sqrt{3}}{2}\frac{1}{R} + j\frac{1}{2}\frac{1}{R} - \frac{1}{2}\omega C - j\frac{\sqrt{3}}{2}\omega C\right) = \frac{\sqrt{3}}{2}U^2\left(\frac{\sqrt{3}}{R} - \omega C + j\left(\frac{1}{R} - \sqrt{3}\omega C\right)\right)$$

$$\boxed{S_b = \frac{\sqrt{3}}{2}U^2\left(\frac{\sqrt{3}}{R} - \omega C\right) + j\frac{\sqrt{3}}{2}U^2\left(\frac{1}{R} - \sqrt{3}\omega C\right)}$$

Because we had a balanced system, and phase U_a was at angle zero, and the capacitor's current was real, our angles were convenient multiples of 30°. In other cases it might not be so easy! (Leave with cos, tan⁻¹ etc.)

(Now we can check that

$$S_a + S_b + S_c + S_R + S_C = 0$$

— "conservation" of reactive power ...)

Example

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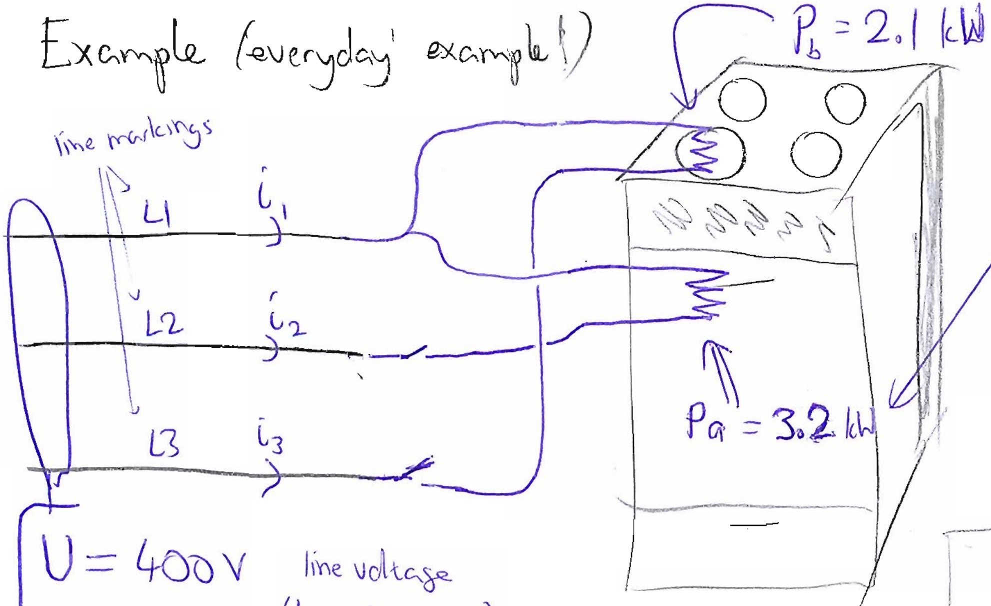
Example ('everyday' example!)



$U = 400$ V line voltage (huvudspänning)

3ph supply

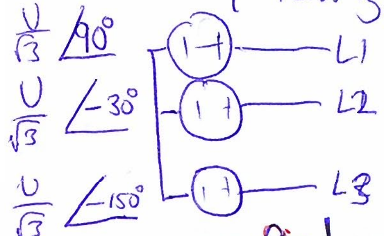
The powers show the power at "rated voltage" 400 V.

The elements can be treated as resistors.

This shows an electric cooker in which two 'elements' are switched on. They are connected between different pairs of lines: 'a' is L1→L2 & 'b' is L1→L3.

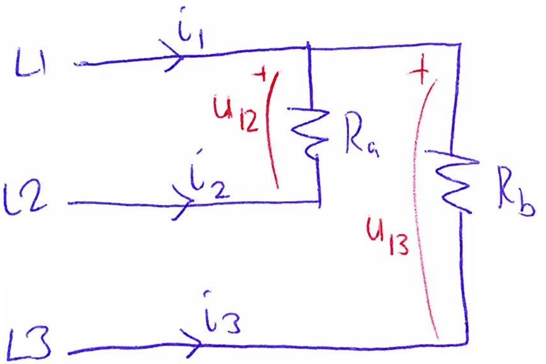
(a) find $|i_b|$, $|i_c|$, $|i_a|$

(b) assume the source is the following:



find $\angle i_a$

first, write the situation more like an "idealised circuit":



$$R_a = \frac{U^2}{P_a} \quad , \quad R_b = \frac{U^2}{P_b}$$

$$|u_{12}| = |u_{13}| = U = 400\,V$$

Without thinking about angles, we can find two of the current magnitudes, based on knowing the voltage magnitudes across the resistors:

$$|i_2| = \frac{|u_{12}|}{R_a} = \frac{U}{R_a} = \frac{U\,P_a}{U^2} = \frac{P_a}{U}$$

$$|i_3| = \frac{P_b}{U} \text{ (by the same method)}$$

Obvious if we think a little: $P = |u||i|$ for a resistor.

By KCL, $i_1 = -i_2 - i_3$

$|i_1| = |i_2 + i_3|$
phasor sum is needed:

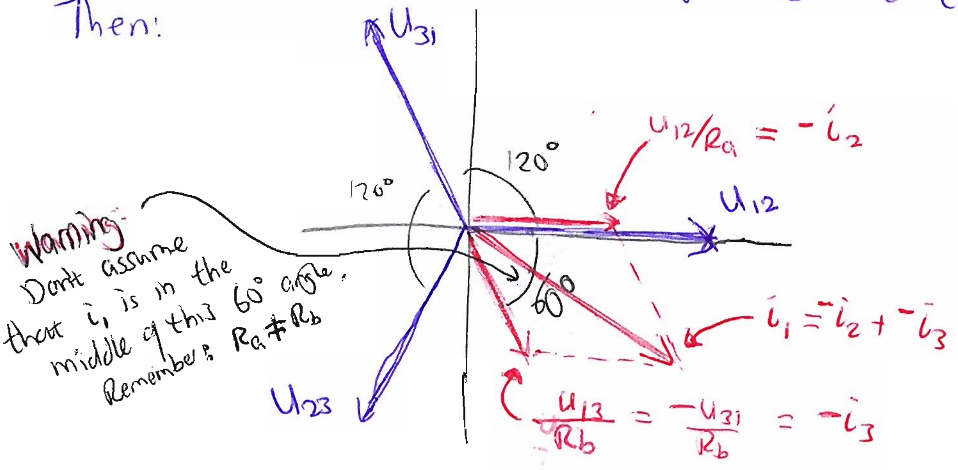
These currents have different angles; we cannot just add magnitudes: $|i_1| \neq |i_2| + |i_3|$

Task (b) defines specific angles. for task (a) let's ignore that, and make our own choice of the definition of reference angle; only the relative angles are important for finding the requested magnitudes.

We'll define line voltage u_{12} at zero angle: $\angle u_{12} = 0$.

Assume 'abc' (1,2,3) phase rotation (it doesn't actually affect the resulting magnitudes in our case, as the impedances are the same type, resistive.)

Then:



Having defined $\angle u_{12} = 0°$, the next line voltage is $\angle u_{23} = -120°$, then $\angle u_{31} = -240°$

We must be careful to draw the currents in the right directions: careful about the signs.

Our task is therefore to find:

$$i_1 = \frac{U}{R_a}\angle 0^\circ + \frac{U}{R_b}\angle{-60^\circ} = \frac{U}{R_a} + \frac{U}{R_b}\left(\cos(-60^\circ) + j\sin(-60^\circ)\right)$$

$$= \frac{U}{R_a} + \frac{U}{2R_b} - j\frac{\sqrt{3}\,U}{2R_b}$$

from which $|i_1| = U\sqrt{\left(\frac{1}{R_a} + \frac{1}{2R_b}\right)^2 + \left(\frac{\sqrt{3}}{2R_b}\right)^2}$

Putting in the numbers:

$|i_1| = 11.6$ A

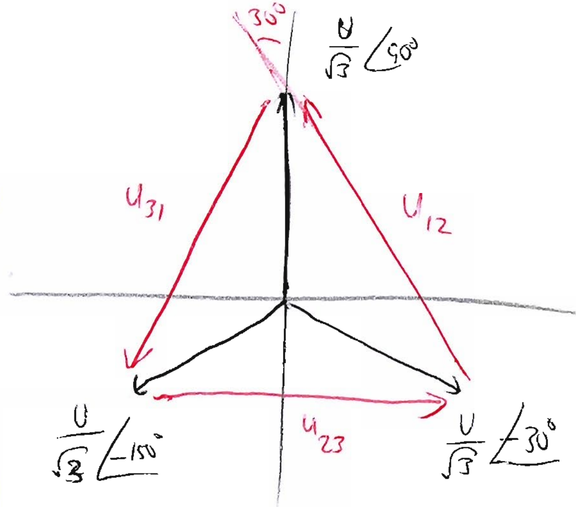
$|i_2| = 8.0$ A

$|i_3| = 5.3$ A

Easier by computer:

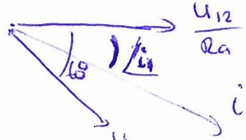
```
U=400, Pa = 3.2e3, Pb = 2.1e3, Ra = U^2 /Pa, Rb = U^2/Pb
ua = U/sqrt(3), ub = ua/exp(1j*2*pi/3), uc = ub/exp(1j*2*pi/3)
iamag = abs((ua-ub)/Ra + (ua-uc)/Rb)
```

(b) Now we find $\angle i_1$. In this part, (b), the angles in the circuit have been specified by defining the phase voltage phasors of a Y connected source. $\Rightarrow$



from this we see that $\angle u_{12} = +120°$.

In part (a) we chose a different definition, $\angle u_{12} = 0°$. Lets initially use that, to find $\angle i_1$ from the phasor diagram in part (a). Then we can shift the angle by $+120°$ to fit the definition used here in part (b). (It's convenient to have the current on the real axis.)



$\frac{u_{12}}{R_a} = \frac{U\angle 0}{R_a}$

$i_1 = ?$

$\frac{u_{13}}{R_b} = \frac{U\angle -60°}{R_b}$

$$\angle\left(\frac{u_{12}}{R_a} + \frac{u_{13}}{R_b}\right) = \angle\left(\frac{1}{R_a} + \frac{1}{R_b}\angle -60°\right)$$

$$= \tan^{-1}\left(\frac{\frac{\sqrt{3}}{2R_b}}{\frac{1}{R_a} + \frac{1}{2R_b}}\right) = -\tan^{-1}\left(\frac{\sqrt{3}R_a}{R_a + 2R_b}\right)$$ $\leftarrow$ in our arbitrary choice of angle

So;

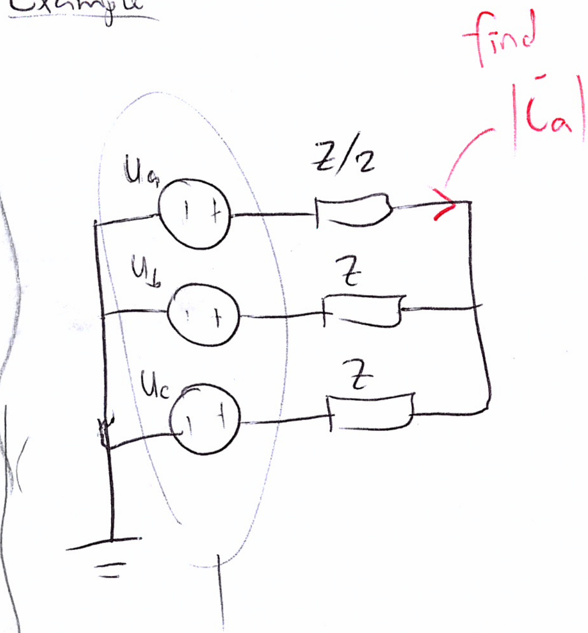
$$\boxed{\angle i_1 = \sim 97° = 120° - \tan^{-1}\left(\frac{\sqrt{3}R_a}{R_a + 2R_b}\right)}$$

in the angle reference specified in part b.

Example

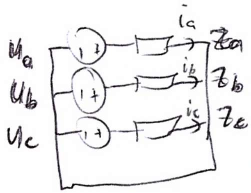
(next page)

Example



balanced 3ph source
with phase voltage U

If there were a neutral conductor (of no impedance) this would be easy:



$$i_a = \frac{U_a}{Z_a}$$

But there isn't!
The unbalanced load means that the load's 'neutral' point has not the same potential as the source's neutral.

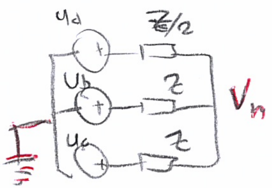
We can assume any angle reference for the three-phase source. e.g. let's say $U_a = U\angle\alpha$.
Then assuming the default phase rotation abc,
$U_b = U\angle\alpha - 120^\circ$ and $U_c = U\angle\alpha + 120^\circ$ (or $U\angle\alpha - 240^\circ$)

One method. Direct solution of the circuit

(Not starting with 3ph simplifications or separating balance/unbalance)

Call the potential V_n at the load-neutral (the "star-point" – node at the right).

Then <u>KCL gives</u>:



$$\frac{U_a - V_n}{Z/2} + \frac{U_b - V_n}{Z} + \frac{U_c - V_n}{Z} = 0$$

$$\frac{2U_a}{\cancel{Z}} + \frac{U_b}{\cancel{Z}} + \frac{U_c}{\cancel{Z}} = \frac{4V_n}{\cancel{Z}}$$

$$V_n = \frac{U_a}{4} + \underbrace{\cancel{\frac{1}{4}\left(U_a + U_b + U_c\right)}}_{=0 \text{ as it's balanced 3phase}}$$

<u>Ohm's law</u>:

$$i_a = \frac{U_a - V_n}{Z/2} = \frac{U_a - \frac{U_a}{4}}{Z/2} = \frac{\frac{3}{4}U_a}{Z/2} = \frac{3}{2}\cdot\frac{U\angle\alpha}{Z}$$

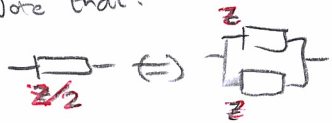
arbitrarily chosen angle of phase a voltage

$$\boxed{\text{So } \quad |i_a| = \frac{3U}{2Z}}$$

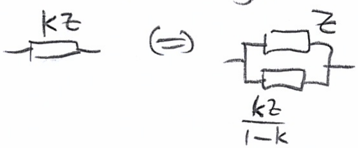
Another method.

being cunning (listig, slug)

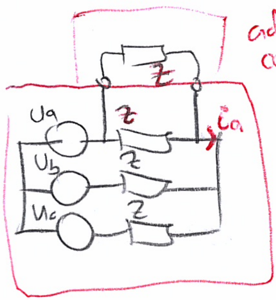
Note that:



or more generally,



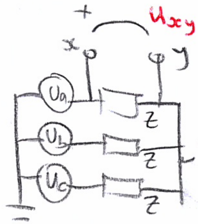
We can draw our circuit as:



additional unbalance connected at two terminals

balanced 3ph circuit

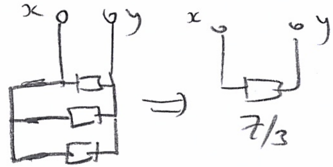
Find the Thevenin equivalent of the balanced part



$u_{xy} = u_a$ in open-circuit condition

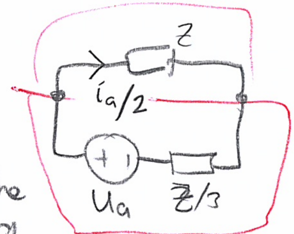
$\Rightarrow U_T = u_a = U\angle\alpha$

$V_n = 0$ because it's balanced, so the load and source neutrals have the same potential



$\Rightarrow Z_T = \frac{Z}{3}$

So adding the unbalance to the balanced Thevenin:



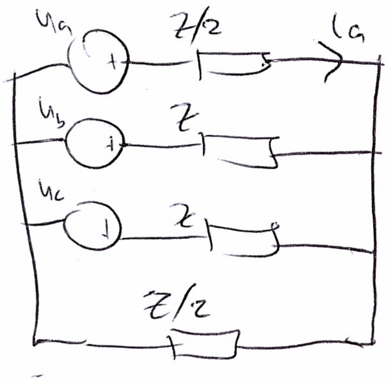
Notice that the current in the extra impedance Z is half of the current in the original circuit, as that current is in two parallel impedances Z.

$$\frac{i_a}{2} = \frac{u_a}{Z + Z/3} \quad \therefore \quad i_a = \frac{2u_a}{\frac{4}{3}Z} = \frac{3u_a}{2Z} = \frac{3U\angle\alpha}{2Z}$$

$$\boxed{|i_a| = \frac{3U}{2Z}}$$

The first ("direct") method probably looked easier.
But the second method may make it easier to see how a change will affect the solution.

E.g. change the question by adding a neutral connection between the source and load star points, with impedance $Z/2$.



By the "Thevenin & balanced-system" method, we see easily that U_T is unchanged and Z_T becomes $Z/5$ instead of $Z/3$, as the neutral adds in parallel.

So $i_a = \dfrac{2u_a}{\frac{6}{5}Z} = \dfrac{10\,u_a}{6\,Z}$