KTH, Electric Circuit Analysis, EI1120 VT-2021

Brief Solutions for Tutorial 01 (Basics)

Numbering is by page numbers in the PDF-file of questions.

Solutions are brief: more detailed solutions to selected problems are done at the tutorial (övning).

The files of 'exercises' (for self-study or group-study) typically have more detailed solutions.

1.

The voltage source *produces* the most power (all the other components are consuming power). See the list of consumed powers given below: we're looking for the *most negative* of these, if we want to find the highest produced power.

The current source *consumes* the most power. Notice that voltage or current sources can produce or consume power, depending on what the rest of the circuit causes the source's undetermined quantity (e.g. a current source's voltage) to be; but a resistor of positive resistance can only consume power, as its current always falls in potential.

Consumed powers:
Sources (left to right): $-20\,\mathrm{W}$, $50\,\mathrm{W}$.
Resistors (left to right): $20\,\mathrm{W}$, $40\,\mathrm{W}$, $30\,\mathrm{W}$.

2.

There are lots of ways to approach this. We could argue about charges going up or down in energy. We could change negative values for positive ones by swapping the direction of current-arrows or voltage-'+' markings. The result should of course be the same, if done correctly by any method.

Here, I'll get my solutions by forcing each case to follow the 'passive convention' (so that the current is defined going into the terminal where the voltage '+' is marked). If the original markings are not already like this, then either the currents or the voltage will be changed by swapping both its sign and its direction. The results are:

a) $R = 5\,\mathrm{V}\,/\,3\,\mathrm{A} = 1.67\,\Omega$

b) $R = 3\,\mathrm{V}\,/\,-2\,\mathrm{A} = -1.5\,\Omega$ (the strange case where it's a negative value)

c) $R = 7\,\mathrm{V}\,/\,4\,\mathrm{A} = 1.75\,\Omega$

d) $R = 12\,\mathrm{V}\,/\,8.33\,\mathrm{A} = 1.44\,\Omega$.

Component (b) has a negative resistance, so it is not a simple passive resistor! (It will supply power to the circuit.) It might be a model of an electronic circuit where dependent sources provide energy to the current.

3.

$i_1 = 3\,\mathrm{A}$
Use KVL in the left loop, and Ohm's law in the $1\,\Omega$ resistor.

$i_2 = -i_1 = -3\,\mathrm{A}$
You perhaps can see anyway that this whole left part of the circuit has only one path for the current. For a more formal proof, do KCL in each node of the left loop.

$u_3 = 1\,\mathrm{V}$
KVL around the right loop.

$P = 5\,\mathrm{W}$ (supplied by $2\,\mathrm{V}$ source)
One way: find the current leaving the '+' terminal of the $2\,\mathrm{V}$ source; then the current and the marked voltage follow the 'active convention', so their product is the power supplied by this source.
The current is $-i_2$ into the left branch, and $-u_3\,/\,2\,\Omega$ into the right branch: this gives $3\,\mathrm{A} - 0.5\,\mathrm{A}$ out of the voltage-source. So the power supplied by this $2\,\mathrm{V}$ source is $2\,\mathrm{V} \cdot 2.5\,\mathrm{A}$.

4.

$i_x = I_2 - I_3$
From KCL at the bottom node ($i_x + I_3 - I_2 = 0$).
Or from the node above R_2, KCL is ($-i_x + I_1 - I_1 - I_3 + I_2 = 0$).

$u_y = -R_1 I_1$
KCL in the top node, Ohm's law in R_1.

$P = R_3 I_3^2$
KCL in the right node gives the current in R_3. By Ohm's law and the power equation the power into this resistor can be found. Simpler is the expression $i^2 R$ for the power consumed by resistor R when current i passes in it (notice, the i^2 is positive regardless of the sign/direction of the current – convenient).

$u_1 = -u_y = R_1 I_1$
By KVL in the top loop (we already know u_y from earlier).

$P = I_3^2(R_2 + R_3) - I_2 I_3 R_2$ (can be expressed several ways).
This is more difficult than the others!
To find the power supplied by source I_3 we need to find its voltage: we already know its current by its definition as a current-source. Let's define a voltage u_3 across this current source, following the 'active convention' (i.e. the '+' of the voltage-marking is where the source's current-arrow points out), so that $u_3 I_3$ will give the power supplied by this source.
Then we need to find a loop for applying KVL to this voltage u and to other voltages that we know or easily can calculate; we try to avoid other current sources, as we don't know their voltage. We take the loop around I_3, R_2, R_3. First use KCL to find currents in resistors, and Ohm's law to calculate voltages: R_3 has voltage $I_3 R_3$, and R_2 has voltage $(I_3 - I_2)R_2$ (if the voltage '+' is defined to the left of each).
Then by KVL, $u = (I_3 - I_2)R_2 + I_3 R_3$, so the power is $((I_3 - I_2)R_2 + I_3 R_3)\, I_3$.

5.

$v_1 = -U_1$.
Follow the potential ('potentialvandra') from the earth node (jordnod, bottom right) to the node marked v_1. We go down in potential by U_1.

$v_4 = U_2 - U_1$.
Continue your hike (fortsatt potentialvandring) up by voltage U_2.

$v_2 = U_2 - U_1 - U_5 + (R_1 + R_2)I_2$.
Find a path without current sources, between a known potential and the unknown. KCL says that current I_2 passes through R_2 and R_1, so Ohm's law tells us their voltages (careful about the sign and direction). Then it's just more potentialvandring.

6.

$u_x = -I_1 R_1$.
This is by Ohm's law in R_1 (and KCL if you want to prove that the current down the resistor equals I_1).

$u_y = -G u_x R_2 = G I_1 R_1 R_2$.
The first part of this solution is by applying the same KCL + Ohm as when finding u_x, but now on the second loop.
Then substitute the previous expression for u_x, and simplify the double-negative.

7.

$i_a = \frac{10\,\mathrm{V}}{2\,\Omega} = 5\,\mathrm{A}$
Ohm's law. Use KVL if wanting to prove that the $2\,\Omega$ resistor in the left loop has $10\,\mathrm{V}$ across it. With practice, you'll probably stop thinking about it being KVL, and just say "because they're in parallel".

$v_b = 3 \cdot i_a \cdot 1\,\Omega = 15\,\mathrm{V}$
The current from the dependent current-source $3\,i_a$ goes round the middle loop and down the $1\,\Omega$ resistor between the v_b node and earth node. Ohm's law in that resistor thus solves the potential v_b. You can prove that no current 'escapes' from the middle loop, by looking at KCL for each of the other loops by itself: with only one connection, the only solution of $\sum i_{\mathrm{out}} = 0$ is that this current is zero (a single connection between different parts of a circuit does not provide a loop for current to flow in).

$u_c = 2 \cdot i_a \cdot 3\,\Omega = 30\,\mathrm{V}$
Similarly to the above, a dependent current-source causes current $2\,i_a$ down the $3\,\Omega$ resistor, giving u_c by Ohm's law. Then we just have to substitute the previously found value of i_a.

$v_d = v_b - 3v_b - u_c = -60\,\mathrm{V}$
Follow the potential along the path from v_b to v_d. We go up by $-3\,v_b$, then down by u_c. Then substitute the unknown quantities (u_c, v_b) with their solutions that we found earlier.