KTH, Electric Circuit Analysis, EI1120 VT-2021

Solutions for Tutorial 02 (Simplifications)

1. (warm-up)

a) $u = 8\,\mathrm{V}$, by voltage-division.

b) $i = 1\,\mathrm{A}$, by current-division.

c) $i = -6.7\,\mathrm{A}$ and $u = -33.3\,\mathrm{V}$.
The first is directly by current-divison, treating the upper resistors as an equivalent. The second is by current-division and Ohm's law.

d) $i = 0.16\,\mathrm{A}$ and $u = -4\,\mathrm{V}$.
The first is by equivalent resistor and Ohm's law. The second could be by voltage division, or by current-division of i followed by Ohm's law.

2.

a) one resistor, $\frac{R_1(R_2+R_3+R_4)}{R_1+R_2+R_3+R_4}$.

b) current source I, pointing to terminal x.

c) voltage source U, with + towards terminal x.

d) resistor of $2\,\Omega$.

e) current source of $3\,\mathrm{A}$, pointing to terminal x.

f) resistor of $10\,\Omega$ (because voltage sources cancel).

g) voltage source of $6\,\mathrm{V}$, with + towards terminal x (might help to re-draw or use KVL).

But now it gets harder! No way to reduce circuits h and i to single components ...
There's also no obvious single answer: a Norton source is not obviously more or less simple than a Thevenin source, so either could be the answer.

h) current source I (pointing to terminal x) parallel with resistor R.
Component in series with the current source can be shorted away: irrelevant to what happens at x-y.

i) voltage source U (+ towards terminal x) with series resistor R_1.
Components in parallel with the voltage source can be ignored: irrelevant to what happens at x-y.

3.

a) $P_{\mathrm{R1}} = \left(\frac{U-IR_2}{R_1+R_2}\right)^2 R_1$.

We're told to do Thevenin-Norton source-transformation. The only Thevenin source in this diagram is formed by U and R_2: convert these to a resistor R_2 and current source U/R_2 (pointing up) in parallel.
Then a total current of $U/R_2 - I$ is passing down through a total resistance of $R_1 \parallel R_2$. Probably the easiest choice here is to find the voltage: $u = (U/R_2 - I) \cdot \frac{R_1 R_2}{R_1+R_2}$, from which the power in resistor R_1 can be found by u^2/R_1. Substitute the expression for u, and simplify a bit, to get the above solution.

b) $P_{\mathrm{R2}} = \left(\frac{U+IR_1}{R_1+R_2}\right)^2 R_2$.

Here we're told to use Norton-Thevenin source-transformation. Again, there's not much choice: we transform I and R_1 into a series voltage source IR_1 (with +-terminal down) and R_1, forming a single loop together with the U-R_2 components! The current in this loop (let's call it i, anticlockwise) is $i = \frac{U+IR_1}{R_1+R_2}$. The relation $i^2 R_2$ can be used to find the power consumed in R_2: as usual, these squared relations don't care about the direction, as $i^2 = (-i)^2$.

c) It's possible to get the same answers by transforming the opposite sources (e.g. doing the Norton-Thevenin transformation to find the power in R_1). But there's a danger: the component called R_1 in the Thevenin-source equivalent doesn't necessarily have the same power consumption as the component called R_1 in the original Norton-source. We are trying to study a quantity *inside* the part of the circuit that we've transformed: an "equivalent" behaves the same when seen by the rest of the circuit, but does not necessarily have the same internal details as the thing it replaces. The only reliable way is to use the transformed circuit to calculate some quantity outside the transformed source (e.g. the current at the equivalent source's terminals), then put back this quantity in the original circuit to find the desired quantity of the power in the resistor.

4. (lower down the third page of questions!)

$u = -U/2$.
The series-pair of resistors at the left is connected across the voltage source: KVL tells us the pair must have voltage U. So we use voltage division between the (identical) resistors, being careful to notice the directions of U and u.

$i = I/2$.
The two resistors at the bottom right are in parallel, carrying together the current source's current: thus, current division between identical resistors.

$v = U$.
Simplify the earth node. Notice that the two resistors at the left of the voltage source can be ignored when we're seeking v. Also, the two resistors below the current source, and the one resistor above it, can be ignored: the current source forces I into the node marked v, regardless of these resistors' values.
Now we have a simpler circuit that looks "horribly familiar" (pleasantly/boringly familiar?) as the classic "simplest non-trivial circuit with voltage source, current source and resistors" – another example is the diagram in theprevious question!
One solution method is source transformation: make the series U and R into a parallel U/R and R; or make the parallel I and R into series IR and R. In the former case the total current $2I$ passes through two parallel resistors R, so the voltage across each resistor is IR, which is defined as equal to U in the question. In the latter case, we have two identical Thevenin sources of voltage U connected in parallel: no current flows, because they push equally against each other; their terminal voltages are then equal to their source voltages, U.
Unfortunately, this solution of $v = U$ could easily have been found by accident, using a wrong reasoning ... so do think carefully about whether even a right answer was based on the right method!

5. (the non-hand-written question)

See VT17 Homework 02 solution when it is published!

(It was decided to use this question as the homework . . . sorry if that was a bit confusing; it would probably be better to have removed it from the tutorial list, but I thought it might be good to get started with it if there was time remaining in the tutorial.)