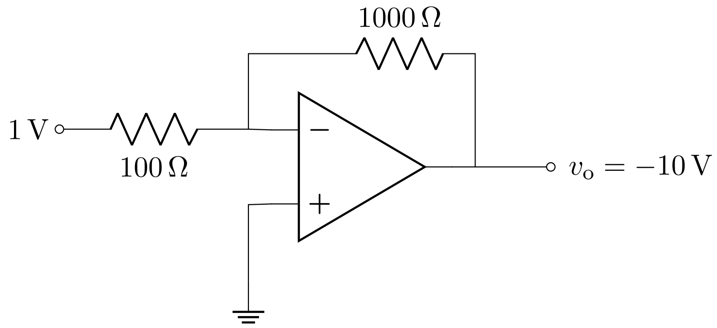
Solutions for Tutorial: Opamps

0. Warm-up.

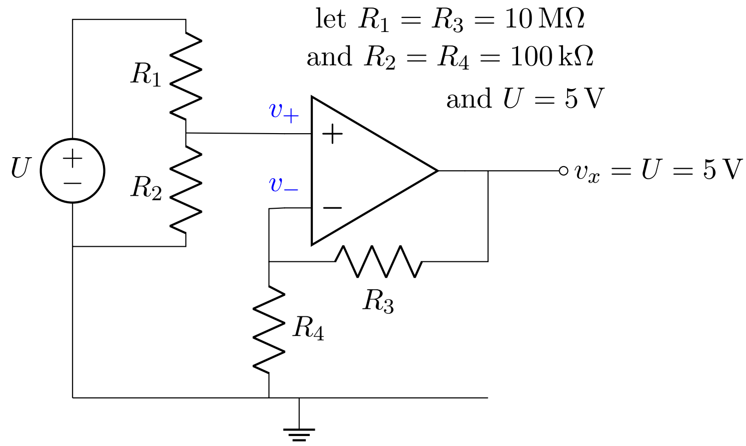
The first circuit is a classic inverting amplifier.

$$v_\mathrm{o} = -\frac{R_\mathrm{feedback}}{R_\mathrm{input}} \cdot v_\mathrm{input} = -\frac{1000\,\Omega}{100\,\Omega} \cdot 1\,\mathrm{V} = -10\,\mathrm{V}.$$

See the derivation of inverting amplifier gain in the notes.



The second circuit is a source and voltage divider (U, R_1, R_2) with its output connected to the input of a non-inverting amplifier (made from the opamp, R_3 and R_4).



The non-inverting amplifier's input connects only to one of the opamp inputs, so no current flows in it. Therefore, no current comes out from the divider, so we can consider R_1 and R_2 to be in series, meaning that simple voltage-division can be used to find v_+.

By voltage division of U, noting that the bottom of the divider has zero potential, $v_+ = \frac{R_2}{R_1+R_2}\,U$.

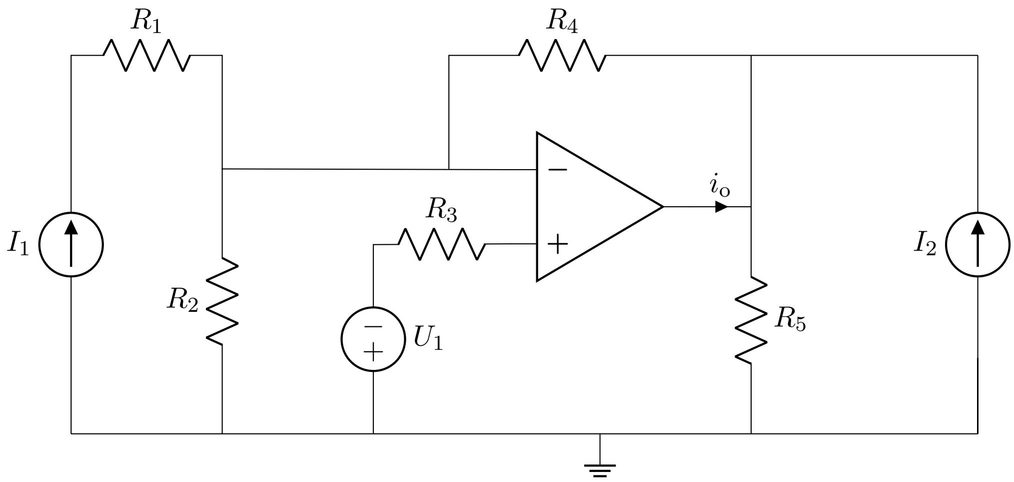
By voltage division of v_x, we find $v_- = \frac{R_4}{R_3+R_4}\,v_x$.

Equating $v_+ = v_-$ (the usual assumption), $\frac{R_4}{R_3+R_4}\,v_x = \frac{R_2}{R_1+R_2}\,U$.

Rearranging this, and inserting the given values,

$$v_x \quad = \quad \frac{R_3+R_4}{R_4}\,\frac{R_2}{R_1+R_2}\,U \quad = \quad \frac{101}{1}\,\frac{1}{101}\,5\,\mathrm{V} \quad = \quad 5\,\mathrm{V}.$$

1. This was in 2016-02_EM_ks1.



a) Power supplied by the source U_1: 0 (opamp input has zero current)

b) Power absorbed by R_1: $I_1^2 R_1$ (R_1 is in series with current source I_1)

c) Power absorbed by R_5: $\dfrac{\left(I_1 R_4 + U_1\left(1+\frac{R_4}{R_2}\right)\right)^2}{R_5}$

To find this, find the opamp's output potential (let's call it v_o), then use it to find $\frac{v_o^2}{R_5}$.
So, how can we find v_o? The potential at the inverting input is $-U_1$: this is found from seeing that with no current through R_3, the potential at the *non-inverting* input must be $-U_1$; we then assume the *inverting* input is held to this value by the negative feedback.
KCL at the inverting input then gives an equation where v_o is the only unknown. A possible simplification is to notice that R_1 is irrelevant (in series with a current source), so I_1 and R_2 can be source-transformed into a Thevenin source with voltage $I_1 R_2$ and resistance R_2. But straight nodal analysis is probably just as good or better:

$$\frac{-U_1 - v_o}{R_4} + \frac{-U_1}{R_2} - I = 0 \quad \Longrightarrow \quad v_o = -I_1 R_4 - U_1\left(1 + \frac{R_4}{R_2}\right).$$

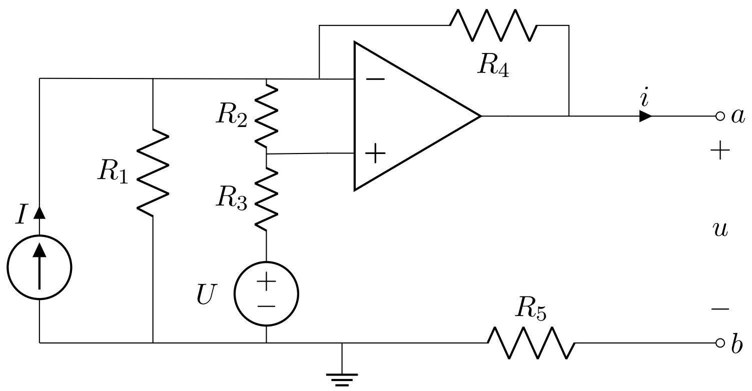
d) The opamp output current: $i_o = -\dfrac{R_2+R_4+R_5}{R_2 R_5} U_1 - \left(1+\dfrac{R_4}{R_5}\right) I_1 - I_2$

Method: using v_o from before, the currents in R_4 and R_5 can be found, then KCL can be applied to the node of the opamp output to obtain

$$i_o = \frac{v_o}{R_5} + \frac{v_o - (-U_1)}{R_4} - I_2.$$

Substituting the known expression for v_o (see part 'c') then rearranging into coefficients of U_1, I_1 and I_2, the final expression for i_o is found. It's not obvious what is the simplest way to express this, so many variations (rearrangements) of the solution would be acceptable.

2. This was in 2015-09_E_ks1.



a) In the open-circuit condition, no current flows in R_5: the node b is therefore at zero potential, and so the marked voltage u is equal to the opamp's output potential.

This potential can be found by a step-by-step approach or by formal nodal analysis.

The nodal analysis could be done in the following way. Define the opamp's output potential as v_o. Define the potential of the opamp inputs: we only need to define one symbol v_i, as we assume the input potentials are equal for an ideal opamp with negative feedback.

KCL can be written for the nodes at the two opamp inputs. The opamp output and the point above source U can be treated as fixed potentials where we don't care about the current in the voltage sources (the supernode type of approach). Remember: we ultimately only want to find v_o.

$$\mathrm{KCL}(+)_{(\mathrm{out})}: \quad 0 = \frac{v_\mathrm{i} - v_\mathrm{i}}{R_2} + \frac{v_\mathrm{i} - U}{R_3} \tag{1}$$

$$\mathrm{KCL}(-)_{(\mathrm{out})}: \quad 0 = \frac{v_\mathrm{i} - v_\mathrm{i}}{R_2} + \frac{v_\mathrm{i}}{R_1} - I + \frac{v_\mathrm{i} - v_\mathrm{o}}{R_4} \tag{2}$$

After solving for v_o, which was shown above to the equal to u for the open-circuit case, the result is

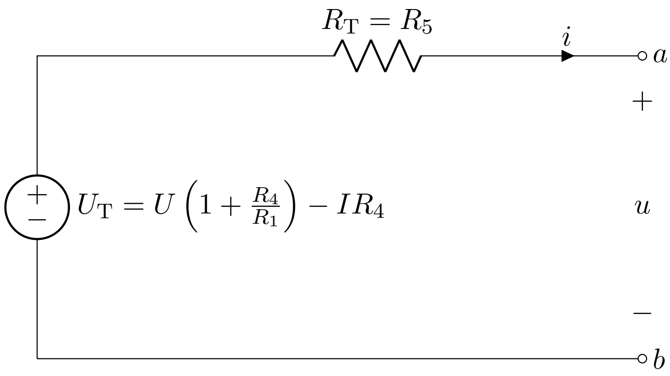
$$u = v_\mathrm{o} = U\left(1 + \frac{R_4}{R_1}\right) - IR_4$$

The less formal, step-by-step method is to notice that no current can flow in R_2 if the opamp inputs have equal potential, and thus that no current can flow in R_3 either, as the opamp input has no current. Thus, both inputs are at potential U. KCL can then be written for the inverting input, using potential U; the only unknown is the sought potential v_o.

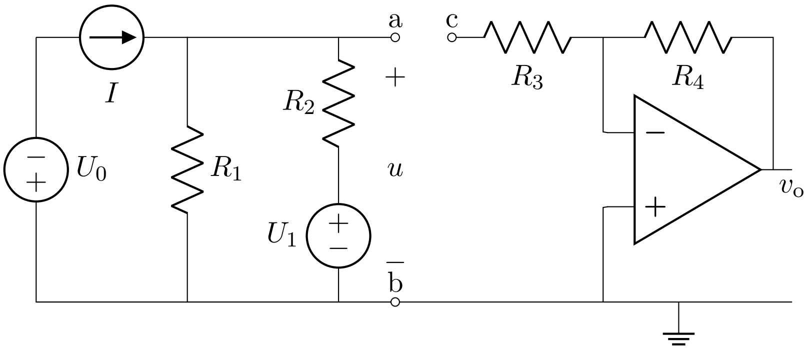
b) The ideal opamp's output can be treated as an ideal voltage-source with its other side connected to earth. The Thevenin resistance between the opamp's output and earth is therefore zero.

However, in the shown circuit, the terminals a-b are not exactly connected to the opamp output and earth ... resistor R_5 is in series with the output current. Thus the Thevenin resistance between a-b is $R_T = R_5$. The Thevenin voltage is the open-circuit voltage of the circuit, which was found in part 'a'.

An equivalent circuit should be shown as a diagram, to make clear the direction of the voltage relative to the marked terminals.



3. This was in 2015-02_EM_ks1.



a) The source U_0 is irrelevant, as it's in series with a current source and we don't want to find any quantities *within* this branch.

Nodal analysis (KCL at a single node) gives, for the open-circuit situation,

$$-I + \frac{u}{R_1} + \frac{u - U_1}{R_2} = 0,$$

whence

$$u_{\text{oc}} = U_{\text{T}} = \frac{(U_1 + IR_2)\, R_1}{R_1 + R_2}.$$

The Thevenin resistance (source resistance) can be found by short-circuit current, or by setting the sources I and U_1 (and U_0 if we haven't noticed that it has no influence) to zero and calculating the equivalent resistance of the circuit. Using the latter approach, we have R_1 and R_2 in parallel, so

$$R_{\text{T}} = \frac{R_1 R_2}{R_1 + R_2}.$$

You really *should* then draw the diagram showing the Thevenin voltage and resistance with the right direction of voltage with respect to the marked terminals!

b) The usual inverting-amplifier formula can easily be derived for the relation $\frac{v_{\mathrm{o}}}{v_{\mathrm{c}}} = \frac{-R_4}{R_3}$. But we can't just use this together with the open-circuit potential $v_a = U_{\mathrm{T}}$, to find v_{o}. In this circuit, $v_{\mathrm{o}} \neq \frac{-R_4}{R_3} U_{\mathrm{T}}$. That's because this opamp circuit does not have an infinite input resistance at terminal 'c', and the earlier circuit (left) does not have a zero output resistance at point 'a'.

If a potential v is put on terminal 'c', then a current $\frac{v}{R_3}$ will flow from 'c' to the virtual earth node of the opamp's inverting input, as the non-inverting input is connected to the earth node (current into the virtual earth node leaves it through R_4 ... the opamp's input terminal has no current). And, if a current is drawn from terminal 'a', the voltage u will change compared to its open-circuit value. So when the terminals 'a'-'c' are connected (shorted together) a current will flow between them, and the voltage u will depend on the Thevenin voltage and resistance of the circuit on the left *and* on the input resistance of the opamp circuit on the right.

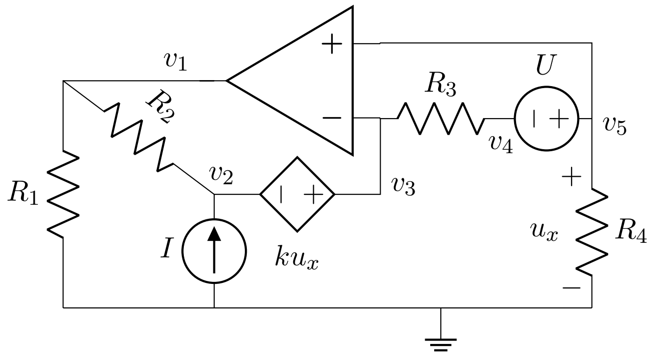
With 'a'-'c' connected, we find (by using the Thevenin equivalent and voltage division between R_{T} and R_3) that

$$u_{\mathrm{ab}} = a_{\mathrm{cb}} = v_{\mathrm{c}} = U_{\mathrm{T}} \frac{R_3}{R_{\mathrm{T}} + R_3}.$$

The inverting amplifier gain $\frac{-R_4}{R_3}$ *can* be used if we've correctly worked out what v_{c} is for the complete circuit:

$$v_{\mathrm{o}} = U_{\mathrm{T}} \cdot \frac{-R_4}{R_3} \cdot \frac{R_3}{R_{\mathrm{T}} + R_3} = -\frac{(U_1 + IR_2)\, R_1}{R_1 + R_2} \cdot \frac{R_4}{R_3} \cdot \frac{R_3}{\frac{R_1 R_2}{R_1 + R_2} + R_3} = -\frac{(U_1 + IR_2)\, R_1 R_4}{R_1 R_2 + R_1 R_3 + R_2 R_3}.$$

4. This is also from 2016-02_EM_ks1



Extended nodal analysis (simple to write, but perhaps not to solve)

We'll define the unknown currents in the two voltage-sources to be going into the source's + terminal.
We'll call them i_α in the independent source U, and i_β in the dependent source hi_y. The current out of the opamp's output can be called i_o.

First we write KCL at all nodes except ground:

$$\begin{aligned}
\mathrm{KCL}(1)_{(\mathrm{out})}: \quad 0 &= \frac{v_1}{R_1} + \frac{v_1 - v_2}{R_2} - i_\mathrm{o} && (3)\\
\mathrm{KCL}(2)_{(\mathrm{out})}: \quad 0 &= \frac{v_2 - v_1}{R_2} - i_\beta - I && (4)\\
\mathrm{KCL}(3)_{(\mathrm{out})}: \quad 0 &= i_\beta + \frac{v_3 - v_4}{R_3} && (5)\\
\mathrm{KCL}(4)_{(\mathrm{out})}: \quad 0 &= \frac{v_4 - v_3}{R_3} - i_\alpha && (6)\\
\mathrm{KCL}(5)_{(\mathrm{out})}: \quad 0 &= \frac{v_5}{R_4} + i_\alpha. && (7)
\end{aligned}$$

These are only 5 equations so far, but with 8 unknowns: v_1, v_2, v_3, v_4, v_5, i_α, i_β, i_o.

We can add the further information given by the voltage sources, which compensates for the extra unknowns caused by their (initially) unknown currents,

$$\begin{aligned}
v_5 - v_4 &= U && (8)\\
v_3 - v_2 &= ku_x. && (9)
\end{aligned}$$

One of those equations introduced a further unknown, u_x, which reminds us that we need to define the marked (but unknown) quantities controlling any dependent sources in the circuit:

$$u_x = v_5. \tag{10}$$

Now there are 8 equations, but 9 unknowns. The opamp is guilty of having introduced an unknown output current i_o *and* an unknown output voltage v_1 that is not directly defined in the way that it would be for a normal dependent voltage source. But the usual opamp assumption (negative feedback and an ideal opamp) lets us state that the opamp input potentials must be equal,

$$v_3 = v_5. \tag{11}$$