Solutions for Tutorial: Continuity

Page 1, first circuit.

$i(0^+) = i(0^-) = 0$

a)

Initial value: $i(0^+) = 0$.

Final value: $i(\infty) = U/R_1$.

Time-constant: $\tau = \frac{L(R_1+R_2)}{R_1 R_2}$,
based on the Thevenin resistance of the circuit that the inductor 'sees'.

So, for $t > 0$,

$$i(t) = \frac{U}{R_1} + \left(0 - \frac{U}{R_1}\right) \exp\left(\frac{-R_1 R_2\, t}{L(R_1+R_2)}\right) = \frac{U}{R_1}\left(1 - \exp\left(\frac{-R_1 R_2\, t}{L(R_1+R_2)}\right)\right).$$

b)

Initial value: $u(0^+) = \frac{R_2}{R_1+R_2} U$.

Final value: $u(\infty) = 0$.

Time-constant: $\tau = \frac{L(R_1+R_2)}{R_1 R_2}$.
Note that this is the same as above. It will be true for any time-changing quantities in this circuit, as there's only the one continuous variable that's undergoing the time-variation. (Consider that integration or differentiation of $\mathrm{e}^{-t/\tau}$ leaves this same term in the result.)

$$u(t) = 0 + \left(\frac{R_2}{R_1+R_2} U - 0\right) \exp\left(\frac{-R_1 R_2\, t}{L(R_1+R_2)}\right) = \frac{U R_2}{R_1+R_2} \exp\left(\frac{-R_1 R_2\, t}{L(R_1+R_2)}\right).$$

c)

Initial value: $i_2(0^+) = \frac{U}{R_1+R_2}$.
This is found by considering that $i(0^+) = 0$, and therefore the right-hand branch can be ignored at $t = 0^+$.

Final value: $i_2(\infty) = 0$.
In this equilibrium, R_2 is 'shorted' by the inductor.

Time-constant: as before (same reason).

$$i_2(t) = \frac{U}{R_1+R_2} \exp\left(\frac{-R_1 R_2\, t}{L(R_1+R_2)}\right).$$

Page 1, second circuit.

$$u(t) = -U + I R_2 \mathrm{e}^{\frac{-t}{C R_1}}$$

$$i(t) = \frac{U + u(t)}{R_1} + \frac{U}{R_0} = \frac{U}{R_0} + \frac{R_2}{R_1} I \mathrm{e}^{\frac{-t}{C R_1}}.$$

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$$i(t) = \frac{U}{R_1 + (1+k)R_2}\left(2\mathrm{e}^{-\frac{R_1+(1+k)R_2}{L}t} - 1\right) \qquad (t \geq 0).$$

This question is 2016-03 EM tenta Q5.
See a solution on that link.

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$$i_c(t) = \frac{-IR_1}{R_1 + R_2}\mathrm{e}^{-\frac{t}{C(R_1+R_2)}} \qquad (t \geq 0).$$

This question is 2014-02 EM ks2 Q1.
See a longer solution on that link.